Homework Assignment 3

Exercise 1: Suppose $f(x)$ is a continuous function on $[a, b]$. Show that if $f(x)$ is one-to-one, then $f(x)$ is either increasing or decreasing on $[a, b]$.

Exercise 2: Let $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x^2} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

1. Find $f'(x)$ for $x \neq 0$.
2. Find $f'(0)$.
3. Show that $f'(x)$ is not continuous at 0.

Exercise 3: If f and g have n-th derivatives. Use induction to show that

$$(fg)^n(x) = \sum_{k=0}^{n} C_k^n \, f^{(n-k)}(x) g^{(k)}(x),$$

where $C_k^n = \frac{n!}{k!(n-k)!}$.

Exercise 4: Let $P(x) = \sum_{k=0}^{n} a_k x^k$ be a polynomial and c be some real number. Show that if $P(x) = \sum_{k=0}^{n} b_k (x-c)^k$, then $b_k = \dfrac{P^{(k)}(c)}{k!}$ for $k = 0, 1, \ldots, n-1$.

Exercise 5: Do the following exercise problems in the textbook,
Sec 1.8: 72, 74
Sec 2.1: 8, 21, 25
Sec 2.2: 3, 25, 26, 30, 30, 45, 47, 56, 59, 61, 63
Sec 2.3: 7, 18, 26, 43, 51, 59, 81, 86(d), 89, 115
Sec 2.4: 7, 12, 14, 19, 20, 22, 23, 27, 35, 37, 38, 44, 50, 53, 57, 58, 62, 67