Homework Assignment 2

Exercise 1: Suppose $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = M$ and there exists a positive number δ such that

$$f(x) \geq g(x) \quad \forall x \in (c-\delta, c+\delta)\backslash\{c\}.$$

Prove that $L \geq M$. Give an example such that $f(x) > g(x)$ $\forall x \in \mathbb{R}$ but $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both exist and are equal.

Exercise 2: Suppose $\lim_{x\to c} f(x) = 0$ and there exists positive numbers p and M such that

$$|g(x)| \leq M \quad \forall x \in (c-p, c+p)\backslash\{c\}.$$

Prove that $\lim_{x\to c} f(x)g(x) = 0$. Use this result to show that $\lim_{x\to 0} \tan(x)\, e^{\sin\frac{1}{x}} = 0$

Exercise 3: Suppose $\lim_{x\to 0} f(x) = L$. Use the definition to show that
(i) $\lim_{x\to 0} |f(x) - L| = 0$;
(ii) $\lim_{x\to 0} f(cx) = L$ for any $c \in \mathbb{R}$ and $c \neq 0$.

Exercise 4: Give an example such that f and g are discontinuous everywhere but $f \circ g$ is continuous everywhere.

Exercise 5: Do the following exercise problems in the textbook by Stewart,
Sec 1.5: 30, 31, 44
Sec 1.6: 30, 31, 34, 41, 44, 55, 57, 59, 62, 63, 65
Sec 1.7: 28, 30, 44
Sec 1.8: 17, 22, 26, 36, 50