Chapter 4

4.1

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

$$= N_{cO} N_{\upsilon O}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

where N_{cO} and $N_{\upsilon O}$ are the values at 300 K.

(a) Silicon

T (K)	kT (eV)	n_i (cm $^{-3}$)
200	0.01727	7.68×10^4
400	0.03453	2.38×10^{12}
600	0.0518	9.74×10^{14}

(b) Germanium (c) GaAs

T (K)	n_i (cm $^{-3}$)	n_i (cm $^{-3}$)
200	2.16×10^{10}	1.38
400	8.60×10^{14}	3.28×10^9
600	3.82×10^{16}	5.72×10^{12}

4.2

Plot

4.3

(a) $n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$

$$\left(5\times10^{11}\right)^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$2.5\times10^{23} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 367.5$ K

(b)

$$n_i^2 = \left(5\times10^{12}\right)^2 = 2.5\times10^{25}$$

$$= \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 417.5$ K

4.4

At $T = 200$ K, $kT = (0.0259)\left(\frac{200}{300}\right)$
$= 0.017267$ eV

At $T = 400$ K, $kT = (0.0259)\left(\frac{400}{300}\right)$
$= 0.034533$ eV

$$\frac{n_i^2(400)}{n_i^2(200)} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^2\right)^2} = 3.025\times10^{17}$$

$$= \frac{\left(\frac{400}{300}\right)^3}{\left(\frac{200}{300}\right)^3}\times\frac{\exp\left[\frac{-E_g}{0.034533}\right]}{\exp\left[\frac{-E_g}{0.017267}\right]}$$

$$= 8\exp\left[\frac{E_g}{0.017267} - \frac{E_g}{0.034533}\right]$$

$$3.025\times10^{17} = 8\exp\left[E_g(57.9139 - 28.9578)\right]$$

or

$$E_g(28.9561) = \ln\left(\frac{3.025\times10^{17}}{8}\right) = 38.1714$$

or $E_g = 1.318$ eV

Now

$$\left(7.70\times10^{10}\right)^2 = N_{co}N_{\upsilon o}\left(\frac{400}{300}\right)^3$$

$$\times \exp\left(\frac{-1.318}{0.034533}\right)$$

$$5.929\times10^{21} = N_{co}N_{vo}(2.370)(2.658\times10^{-17})$$

so $N_{co}N_{vo} = 9.41\times10^{37}$ cm $^{-6}$

4.5

$$\frac{n_i(B)}{n_i(A)} = \frac{\exp\left(\frac{-1.10}{kT}\right)}{\exp\left(\frac{-0.90}{kT}\right)} = \exp\left(\frac{-0.20}{kT}\right)$$

For $T = 200$ K, $kT = 0.017267$ eV
For $T = 300$ K, $kT = 0.0259$ eV
For $T = 400$ K, $kT = 0.034533$ eV

(a) For $T = 200$ K,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.017267}\right) = 9.325\times10^{-6}$$

(b) For $T = 300$ K,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.0259}\right) = 4.43\times10^{-4}$$

(c) For $T = 400$ K,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.034533}\right) = 3.05\times10^{-3}$$

4.6

(a) $$g_c f_F \propto \sqrt{E-E_c}\,\exp\left[\frac{-(E-E_F)}{kT}\right]$$

$$\propto \sqrt{E-E_c}\,\exp\left[\frac{-(E-E_c)}{kT}\right]$$

$$\times \exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

Let $E - E_c = x$

Then $g_c f_F \propto \sqrt{x}\,\exp\left(\frac{-x}{kT}\right)$

To find the maximum value:

$$\frac{d(g_c f_F)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right) - \frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right) = 0$$

which yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

The maximum value occurs at

$$E = E_c + \frac{kT}{2}$$

(b)

$$g_v(1-f_F) \propto \sqrt{E_v - E}\,\exp\left[\frac{-(E_F-E)}{kT}\right]$$

$$\propto \sqrt{E_v - E}\,\exp\left[\frac{-(E_v-E)}{kT}\right]$$

$$\times \exp\left[\frac{-(E_F-E_v)}{kT}\right]$$

Let $E_v - E = x$

Then $g_v(1-f_F) \propto \sqrt{x}\,\exp\left(\frac{-x}{kT}\right)$

To find the maximum value

$$\frac{d[g_v(1-f_F)]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\,\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2}$$

or

$$E = E_v - \frac{kT}{2}$$

4.7

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_c}\,\exp\left[\frac{-(E_1 - E_c)}{kT}\right]}{\sqrt{E_2 - E_c}\,\exp\left[\frac{-(E_2 - E_c)}{kT}\right]}$$

where

$$E_1 = E_c + 4kT \quad \text{and} \quad E_2 = E_c + \frac{kT}{2}$$

Then

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}}\exp\left[\frac{-(E_1 - E_2)}{kT}\right]$$

$$= 2\sqrt{2}\exp\left[-\left(4 - \frac{1}{2}\right)\right] = 2\sqrt{2}\exp(-3.5)$$

or

$$\frac{n(E_1)}{n(E_2)} = 0.0854$$

4.8

Plot

4.9

Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_o$, $m_n^* = 1.08m_o$

$$E_{Fi} - E_{midgap} = -0.0128 \text{ eV}$$

Germanium: $m_p^* = 0.37m_o$,
$m_n^* = 0.55m_o$

$$E_{Fi} - E_{midgap} = -0.0077 \text{ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_o$,

$m_n^* = 0.067m_o$

$$E_{Fi} - E_{midgap} = +0.0382 \text{ eV}$$

4.11

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_v}{N_c}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04\times10^{19}}{2.8\times10^{19}}\right) = -0.4952(kT)$$

T (K)	kT (eV)	($E_{Fi} - E_{midgap}$) (eV)
200	0.01727	−0.0086
400	0.03453	−0.0171
600	0.0518	−0.0257

4.12

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{0.70}{1.21}\right)$$

$\Rightarrow -10.63$ meV

(b)

$$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.75}{0.080}\right)$$

$\Rightarrow +43.47$ meV

4.13

Let $g_c(E) = K =$ constant

Then

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E)\,dE$$

$$= K\int_{E_c}^{\infty}\frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}dE$$

$$\cong K\int_{E_c}^{\infty}\exp\left[\frac{-(E - E_F)}{kT}\right]dE$$

Let

$$\eta = \frac{E - E_c}{kT} \quad \text{so that} \quad dE = kT\cdot d\eta$$

We can write

$$E - E_F = (E_c - E_F) + (E - E_c)$$

so that

$$\exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(E_c - E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c - E_F)}{kT}\right] \int_0^\infty \exp(-\eta) d\eta$$

which becomes

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

4.14

Let $g_c(E) = C_1 (E - E_c)$ for $E \geq E_c$

Then

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= C_1 \int_{E_c}^{\infty} \frac{(E - E_c)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

$$\cong C_1 \int_{E_c}^{\infty} (E - E_C) \exp\left[\frac{-(E - E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E - E_c}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E - E_c) + (E_c - E_F)$$

Then

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$\times \int_{E_c}^{\infty} (E - E_c) \exp\left[\frac{-(E - E_c)}{kT}\right] dE$$

or

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$\times \int_0^\infty (kT)(\eta)[\exp(-\eta)](kT) d\eta$$

We find that

$$\int_0^\infty \eta \exp(-\eta) d\eta = \exp(-\eta)(-\eta - 1)\Big|_0^\infty = +1$$

So

$$n_o = C_1 (kT)^2 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

4.15

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For germanium, $\in_r = 16$, $m^* = 0.55 m_o$

Then

$$r_1 = (16)\left(\frac{1}{0.55}\right) a_o = (29)(0.53)$$

or

$$r_1 = 15.4 \overset{o}{A}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) \text{ eV}$$

$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow E = 0.029 \text{ eV}$$

4.16

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For gallium arsenide, $\in_r = 13.1$,

$m^* = 0.067 m_o$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53) = 104 \overset{o}{A}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or

$$E = 0.0053 \text{ eV}$$

4.17

(a) $E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$

$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{7\times10^{15}}\right)$

$= 0.2148$ eV

(b) $E_F - E_\upsilon = E_g - (E_c - E_F)$

$= 1.12 - 0.2148 = 0.90518$ eV

(c) $p_o = N_\upsilon \exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$

$= (1.04\times10^{19})\exp\left[\frac{-0.90518}{0.0259}\right]$

$= 6.90\times10^{3}$ cm $^{-3}$

(d) Holes

(e) $E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{7\times10^{15}}{1.5\times10^{10}}\right)$

$= 0.338$ eV

4.18

(a) $E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{2\times10^{16}}\right)$

$= 0.162$ eV

(b) $E_c - E_F = E_g - (E_F - E_\upsilon)$

$= 1.12 - 0.162 = 0.958$ eV

(c) $n_o = (2.8\times10^{19})\exp\left(\frac{-0.958}{0.0259}\right)$

$= 2.41\times10^{3}$ cm $^{-3}$

(d) $E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right)$

$= 0.365$ eV

4.19

(a) $E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$

$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{2\times10^{5}}\right)$

$= 0.8436$ eV

$E_F - E_\upsilon = E_g - (E_c - E_F)$

$= 1.12 - 0.8436$

$E_F - E_\upsilon = 0.2764$ eV

(b)

$p_o = (1.04\times10^{19})\exp\left(\frac{-0.27637}{0.0259}\right)$

$= 2.414\times10^{14}$ cm $^{-3}$

(c) p-type

4.20

(a) $kT = (0.0259)\left(\frac{375}{300}\right) = 0.032375$ eV

$n_o = (4.7\times10^{17})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-0.28}{0.032375}\right]$

$= 1.15\times10^{14}$ cm $^{-3}$

$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.28$

$= 1.14$ eV

$p_o = (7\times10^{18})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-1.14}{0.032375}\right]$

$= 4.99\times10^{3}$ cm $^{-3}$

(b)

$E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{1.15\times10^{14}}\right)$

$= 0.2154$ eV

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.2154$$
$$= 1.2046 \text{ eV}$$
$$p_o = (7\times10^{18})\exp\left[\frac{-1.2046}{0.0259}\right]$$
$$= 4.42\times10^{-2} \text{ cm}^{-3}$$

4.21

(a) $kT = (0.0259)\left(\frac{375}{300}\right) = 0.032375$ eV

$$n_o = (2.8\times10^{19})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-0.28}{0.032375}\right]$$
$$= 6.86\times10^{15} \text{ cm}^{-3}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.12 - 0.28$$
$$= 0.840 \text{ eV}$$

$$p_o = (1.04\times10^{19})\left(\frac{375}{300}\right)^{3/2} \exp\left[\frac{-0.840}{0.032375}\right]$$
$$= 7.84\times10^{7} \text{ cm}^{-3}$$

(b) $E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6.862\times10^{15}}\right)$$
$$= 0.2153 \text{ eV}$$

$$E_F - E_\upsilon = 1.12 - 0.2153 = 0.9047 \text{ eV}$$

$$p_o = (1.04\times10^{19})\exp\left[\frac{-0.904668}{0.0259}\right]$$
$$= 7.04\times10^{3} \text{ cm}^{-3}$$

4.22

(a) p-type

(b) $E_F - E_\upsilon = \frac{E_g}{4} = \frac{1.12}{4} = 0.28$ eV

$$p_o = N_\upsilon \exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$
$$= (1.04\times10^{19})\exp\left[\frac{-0.28}{0.0259}\right]$$
$$= 2.10\times10^{14} \text{ cm}^{-3}$$
$$E_c - E_F = E_g - (E_F - E_\upsilon)$$
$$= 1.12 - 0.28 = 0.84 \text{ eV}$$
$$n_o = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
$$= (2.8\times10^{19})\exp\left[\frac{-0.84}{0.0259}\right]$$
$$= 2.30\times10^{5} \text{ cm}^{-3}$$

4.23

(a) $n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$

$$= (1.5\times10^{10})\exp\left[\frac{0.22}{0.0259}\right]$$
$$= 7.33\times10^{13} \text{ cm}^{-3}$$
$$p_o = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$$
$$= (1.5\times10^{10})\exp\left[\frac{-0.22}{0.0259}\right]$$
$$= 3.07\times10^{6} \text{ cm}^{-3}$$

(b) $n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$

$$= (1.8\times10^{6})\exp\left[\frac{0.22}{0.0259}\right]$$
$$= 8.80\times10^{9} \text{ cm}^{-3}$$
$$p_o = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$$
$$= (1.8\times10^{6})\exp\left[\frac{-0.22}{0.0259}\right]$$
$$= 3.68\times10^{2} \text{ cm}^{-3}$$

4.24

(a) $E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{15}}\right)$

$= 0.1979$ eV

(b) $E_c - E_F = E_g - (E_F - E_\upsilon)$

$= 1.12 - 0.19788 = 0.92212$ eV

(c) $n_o = (2.8\times10^{19})\exp\left[\frac{-0.92212}{0.0259}\right]$

$= 9.66\times10^{3}$ cm $^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$

$= 0.3294$ eV

4.25

$kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533$ eV

$N_\upsilon = (1.04\times10^{19})\left(\frac{400}{300}\right)^{3/2}$

$= 1.601\times10^{19}$ cm $^{-3}$

$N_c = (2.8\times10^{19})\left(\frac{400}{300}\right)^{3/2}$

$= 4.3109\times10^{19}$ cm $^{-3}$

$n_i^2 = (4.3109\times10^{19})(1.601\times10^{19})$

$\times\exp\left[\frac{-1.12}{0.034533}\right]$

$= 5.6702\times10^{24}$

$\Rightarrow n_i = 2.381\times10^{12}$ cm $^{-3}$

(a) $E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.034533)\ln\left(\frac{1.601\times10^{19}}{5\times10^{15}}\right)$

$= 0.2787$ eV

(b)

$E_c - E_F = 1.12 - 0.27873 = 0.84127$ eV

(c)

$n_o = (4.3109\times10^{19})\exp\left[\frac{-0.84127}{0.034533}\right]$

$= 1.134\times10^{9}$ cm $^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$= (0.034533)\ln\left(\frac{5\times10^{15}}{2.381\times10^{12}}\right)$

$= 0.2642$ eV

4.26

(a) $p_o = (7\times10^{18})\exp\left[\frac{-0.25}{0.0259}\right]$

$= 4.50\times10^{14}$ cm $^{-3}$

$E_c - E_F = 1.42 - 0.25 = 1.17$ eV

$n_o = (4.7\times10^{17})\exp\left[\frac{-1.17}{0.0259}\right]$

$= 1.13\times10^{-2}$ cm $^{-3}$

(b) $kT = 0.034533$ eV

$N_\upsilon = (7\times10^{18})\left(\frac{400}{300}\right)^{3/2}$

$= 1.078\times10^{19}$ cm $^{-3}$

$N_c = (4.7\times10^{17})\left(\frac{400}{300}\right)^{3/2}$

$= 7.236\times10^{17}$ cm $^{-3}$

$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.034533)\ln\left(\frac{1.078\times10^{19}}{4.50\times10^{14}}\right)$

$= 0.3482$ eV

$E_c - E_F = 1.42 - 0.3482 = 1.072$ eV

$$n_o = (7.236\times10^{17})\exp\left[\frac{-1.07177}{0.034533}\right]$$
$$= 2.40\times10^4 \text{ cm}^{-3}$$

4.27

(a) $p_o = (1.04\times10^{19})\exp\left[\frac{-0.25}{0.0259}\right]$
$$= 6.68\times10^{14} \text{ cm}^{-3}$$
$$E_c - E_F = 1.12 - 0.25 = 0.870 \text{ eV}$$
$$n_o = (2.8\times10^{19})\exp\left[\frac{-0.870}{0.0259}\right]$$
$$n_o = 7.23\times10^4 \text{ cm}^{-3}$$

(b) $kT = 0.034533$ eV
$$N_\upsilon = (1.04\times10^{19})\left(\frac{400}{300}\right)^{3/2}$$
$$= 1.601\times10^{19} \text{ cm}^{-3}$$
$$N_c = (2.8\times10^{19})\left(\frac{400}{300}\right)^{3/2}$$
$$= 4.311\times10^{19} \text{ cm}^{-3}$$
$$E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$$
$$= (0.034533)\ln\left(\frac{1.601\times10^{19}}{6.68\times10^{14}}\right)$$
$$= 0.3482 \text{ eV}$$
$$E_c - E_F = 1.12 - 0.3482 = 0.7718 \text{ eV}$$
$$n_o = (4.311\times10^{19})\exp\left[\frac{-0.77175}{0.034533}\right]$$
$$= 8.49\times10^9 \text{ cm}^{-3}$$

4.28

(a) $n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$

For $E_F = E_c + kT/2$,
$$\eta_F = \frac{E_F - E_c}{kT} = \frac{kT/2}{kT} = 0.5$$
Then $F_{1/2}(\eta_F) \cong 1.0$
$$n_o = \frac{2}{\sqrt{\pi}}(2.8\times10^{19})(1.0)$$
$$= 3.16\times10^{19} \text{ cm}^{-3}$$

(b) $n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$
$$= \frac{2}{\sqrt{\pi}}(4.7\times10^{17})(1.0)$$
$$= 5.30\times10^{17} \text{ cm}^{-3}$$

4.29

$$p_o = \frac{2}{\sqrt{\pi}} N_\upsilon F_{1/2}(\eta'_F)$$
$$5\times10^{19} = \frac{2}{\sqrt{\pi}}(1.04\times10^{19})F_{1/2}(\eta'_F)$$
So $F_{1/2}(\eta'_F) = 4.26$

We find $\eta'_F \cong 3.0 = \frac{E_\upsilon - E_F}{kT}$
$$E_\upsilon - E_F = (3.0)(0.0259) = 0.0777 \text{ eV}$$

4.30

(a) $\eta_F = \frac{E_F - E_c}{kT} = \frac{4kT}{kT} = 4$

Then $F_{1/2}(\eta_F) \cong 6.0$
$$n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$
$$= \frac{2}{\sqrt{\pi}}(2.8\times10^{19})(6.0)$$
$$= 1.90\times10^{20} \text{ cm}^{-3}$$

(b) $n_o = \frac{2}{\sqrt{\pi}}(4.7\times10^{17})(6.0)$
$$= 3.18\times10^{18} \text{ cm}^{-3}$$

4.31

For the electron concentration

$$n(E) = g_c(E) f_F(E)$$

The Boltzmann approximation applies, so

$$n(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c} \times \exp\left[\frac{-(E - E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E - E_c}{kT}}\exp\left[\frac{-(E - E_c)}{kT}\right]$$

Define

$$x = \frac{E - E_c}{kT}$$

Then

$$n(E) \to n(x) = K\sqrt{x}\exp(-x)$$

To find maximum $n(E) \to n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x) + x^{1/2}(-1)\exp(-x)\right]$$

or

$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2} - x\right]$$

which yields

$$x = \frac{1}{2} = \frac{E - E_c}{kT} \Rightarrow E = E_c + \frac{1}{2}kT$$

For the hole concentration

$$p(E) = g_\upsilon(E)[1 - f_F(E)]$$

Using the Boltzmann approximation

$$p(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_\upsilon - E} \times \exp\left[\frac{-(E_F - E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E_\upsilon - E}{kT}}\exp\left[\frac{-(E_\upsilon - E)}{kT}\right]$$

Define

$$x' = \frac{E_\upsilon - E}{kT}$$

Then

$$p(x') = K'\sqrt{x'}\exp(-x')$$

To find maximum value of $p(E) \to p(x')$, set

$$\frac{dp(x')}{dx'} = 0$$ Using the results from above, we find the maximum at

$$E = E_\upsilon - \frac{1}{2}kT$$

4.32

(a) Silicon: We have

$$n_o = N_c\exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

We can write

$$E_c - E_F = (E_c - E_d) + (E_d - E_F)$$

For

$E_c - E_d = 0.045$ eV and $E_d - E_F = 3kT$ eV

we can write

$$n_o = (2.8\times10^{19})\exp\left[\frac{-0.045}{0.0259} - 3\right]$$

$$= (2.8\times10^{19})\exp(-4.737)$$

or

$$n_o = 2.45\times10^{17}\text{ cm}^{-3}$$

We also have

$$p_o = N_\upsilon\exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

Again, we can write

$$E_F - E_\upsilon = (E_F - E_a) + (E_a - E_\upsilon)$$

For

$E_F - E_a = 3kT$ and $E_a - E_\upsilon = 0.045$ eV

Then

$$p_o = (1.04\times10^{19})\exp\left[-3-\frac{0.045}{0.0259}\right]$$
$$= (1.04\times10^{19})\exp(-4.737)$$
or
$$p_o = 9.12\times10^{16} \text{ cm}^{-3}$$

(b) GaAs: assume $E_c - E_d = 0.0058$ eV
Then
$$n_o = (4.7\times10^{17})\exp\left[\frac{-0.0058}{0.0259}-3\right]$$
$$= (4.7\times10^{17})\exp(-3.224)$$
or
$$n_o = 1.87\times10^{16} \text{ cm}^{-3}$$
Assume $E_a - E_v = 0.0345$ eV
Then
$$p_o = (7\times10^{18})\exp\left[\frac{-0.0345}{0.0259}-3\right]$$
$$= (7\times10^{18})\exp(-4.332)$$
or
$$p_o = 9.20\times10^{16} \text{ cm}^{-3}$$

4.33
Plot

4.34
(a) $p_o = 4\times15 - 10^{15} = 3\times10^{15}$ cm $^{-3}$
$$n_o = \frac{(1.5\times10^{10})^2}{3\times10^{15}} = 7.5\times10^4 \text{ cm}^{-3}$$
(b) $n_o = N_d = 3\times10^{16}$ cm $^{-3}$
$$p_o = \frac{(1.5\times10^{10})^2}{3\times10^{16}} = 7.5\times10^3 \text{ cm}^{-3}$$
(c) $n_o = p_o = n_i = 1.5\times10^{10}$ cm $^{-3}$
(d)
$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{375}{300}\right)^3$$
$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(375)}\right]$$
$$\Rightarrow n_i = 7.334\times10^{11} \text{ cm}^{-3}$$
$$p_o = N_a = 4\times10^{15} \text{ cm}^{-3}$$
$$n_o = \frac{(7.334\times10^{11})^2}{4\times10^{15}} = 1.34\times10^8 \text{ cm}^{-3}$$
(e)
$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{450}{300}\right)^3$$
$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(450)}\right]$$
$$\Rightarrow n_i = 1.722\times10^{13} \text{ cm}^{-3}$$

$$n_o = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + (1.722\times10^{13})^2}$$
$$= 1.029\times10^{14} \text{ cm}^{-3}$$
$$p_o = \frac{(1.722\times10^{13})^2}{1.029\times10^{14}} = 2.88\times10^{12} \text{ cm}^{-3}$$

4.35
(a) $p_o = N_a - N_d = 4\times10^{15} - 10^{15}$
$= 3\times10^{15}$ cm $^{-3}$
$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8\times10^6)^2}{3\times10^{15}} = 1.08\times10^{-3} \text{ cm}^{-3}$$
(b) $n_o = N_d = 3\times10^{16}$ cm $^{-3}$
$$p_o = \frac{(1.8\times10^6)^2}{3\times10^{16}} = 1.08\times10^{-4} \text{ cm}^{-3}$$
(c) $n_o = p_o = n_i = 1.8\times10^6$ cm $^{-3}$
(d)
$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\left(\frac{375}{300}\right)^3$$
$$\times\exp\left[\frac{-(1.42)(300)}{(0.0259)(375)}\right]$$

$\Rightarrow n_i = 7.580\times10^8$ cm $^{-3}$

$p_o = N_a = 4\times10^{15}$ cm $^{-3}$

$$n_o = \frac{(7.580\times10^8)^2}{4\times10^{15}} = 1.44\times10^2$$

cm $^{-3}$

(e)

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\left(\frac{450}{300}\right)^3$$

$$\times\exp\left[\frac{-(1.42)(300)}{(0.0259)(450)}\right]$$

$\Rightarrow n_i = 3.853\times10^{10}$ cm $^{-3}$

$n_o = N_d = 10^{14}$ cm $^{-3}$

$$p_o = \frac{(3.853\times10^{10})^2}{10^{14}} = 1.48\times10^7$$

cm $^{-3}$

4.36

(a) Ge: $n_i = 2.4\times10^{13}$ cm $^{-3}$

(i) $n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$

$$= \frac{2\times10^{15}}{2} + \sqrt{\left(\frac{2\times10^{15}}{2}\right)^2 + (2.4\times10^{13})^2}$$

or

$n_o \cong N_d = 2\times10^{15}$ cm $^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(2.4\times10^{13})^2}{2\times10^{15}}$$

$= 2.88\times10^{11}$ cm $^{-3}$

(ii) $p_o \cong N_a - N_d = 10^{16} - 7\times10^{15}$

$= 3\times10^{15}$ cm $^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.4\times10^{13})^2}{3\times10^{15}}$$

$= 1.92\times10^{11}$ cm $^{-3}$

(b) GaAs: $n_i = 1.8\times10^6$ cm $^{-3}$

(i) $n_o \cong N_d = 2\times10^{15}$ cm

$$p_o = \frac{(1.8\times10^6)^2}{2\times10^{15}} = 1.62\times10^{-3} \text{ cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 3\times10^{15}$ cm $^{-3}$

$$n_o = \frac{(1.8\times10^6)^2}{3\times10^{15}} = 1.08\times10^{-3} \text{ cm}$$

-3

(c) The result implies that there is only one minority carrier in a volume of 10^3 cm ^{3} .

4.37

(a) For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1+\frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\frac{n_d}{N_d} = 8.85\times10^{-4}$$

(b) We have

$$f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_c) + (E_c - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1+\exp\left(1+\frac{0.245}{0.0259}\right)}$$

or

$$f_F(E) = 2.87\times10^{-5}$$

4.38

(a) $N_a > N_d \Rightarrow$ p-type

(b) Silicon:

$$p_o = N_a - N_d = 2.5\times10^{13} - 1\times10^{13}$$

or

$$p_o = 1.5\times10^{13} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{13}} = 1.5\times10^{7}$$

cm $^{-3}$

Germanium:

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5\times10^{13}}{2}\right) + \sqrt{\left(\frac{1.5\times10^{13}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$$

or

$$p_o = 3.26\times10^{13} \text{ cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.4\times10^{13}\right)^2}{3.264\times10^{13}} = 1.76\times10^{13} \text{ cm}^{-3}$$

Gallium Arsenide:

$$p_o = N_a - N_d = 1.5\times10^{13} \text{ cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^{6}\right)^2}{1.5\times10^{13}} = 0.216 \text{ cm}^{-3}$$

4.39

(a) $N_d > N_a \Rightarrow$ n-type

(b)

$$n_o \cong N_d - N_a = 2\times10^{15} - 1.2\times10^{15} = 8\times10^{14} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{8\times10^{14}} = 2.81\times10^{5} \text{ cm}^{-3}$$

(c) $p_o \cong \left(N_a' + N_a\right) - N_d$

$$4\times10^{15} = N_a' + 1.2\times10^{15} - 2\times10^{15}$$

$$\Rightarrow N_a' = 4.8\times10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{4\times10^{15}} = 5.625\times10^{4}$$

cm $^{-3}$

4.40

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{5}} = 1.125\times10^{15} \text{ cm}^{-3}$$

$n_o > p_o \Rightarrow$ n-type

4.41

$$n_i^2 = \left(1.04\times10^{19}\right)\left(6.0\times10^{18}\right)\left(\frac{250}{300}\right)^3$$

$$\times \exp\left[\frac{-0.66}{(0.0259)(250/300)}\right]$$

$$= 1.8936\times10^{24}$$

$$\Rightarrow n_i = 1.376\times10^{12} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{n_i^2}{4n_o} \Rightarrow n_o^2 = \frac{1}{4}n_i^2$$

$$\Rightarrow n_o = \frac{1}{2}n_i$$

So $n_o = 6.88\times10^{11}$ cm $^{-3}$,

Then $p_o = 2.75\times10^{12}$ cm $^{-3}$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$\left(2.752\times10^{12} - \frac{N_a}{2}\right)^2 = \left(\frac{N_a}{2}\right)^2 + 1.8936\times10^{24}$$

$$7.5735\times10^{24} - \left(2.752\times10^{12}\right)N_a + \left(\frac{N_a}{2}\right)^2$$

$$= \left(\frac{N_a}{2}\right)^2 + 1.8936\times10^{24}$$

so that $N_a = 2.064\times10^{12}$ cm $^{-3}$

4.42
Plot

4.43
Plot

4.44
Plot

4.45

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$1.1\times10^{14} = \frac{2\times10^{14} - 1.2\times10^{14}}{2}$$

$$+ \sqrt{\left(\frac{2\times10^{14} - 1.2\times10^{14}}{2}\right)^2 + n_i^2}$$

$$\left(1.1\times10^{14} - 4\times10^{13}\right)^2 = \left(4\times10^{13}\right)^2 + n_i^2$$

$$4.9\times10^{27} = 1.6\times10^{27} + n_i^2$$

so $n_i = 5.74\times10^{13}$ cm $^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{3.3\times10^{27}}{1.1\times10^{14}} = 3\times10^{13} \text{ cm}$$

$^{-3}$

4.46

(a) $N_a > N_d \Rightarrow$ p-type
Majority carriers are holes

$$p_o = N_a - N_d = 3\times10^{16} - 1.5\times10^{16}$$
$$= 1.5\times10^{16} \text{ cm}^{-3}$$

Minority carriers are electrons

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}} = 1.5\times10^4$$

cm $^{-3}$

(b) Boron atoms must be added

$$p_o = N_a' + N_a - N_d$$

$$5\times10^{16} = N_a' + 3\times10^{16} - 1.5\times10^{16}$$

So $N_a' = 3.5\times10^{16}$ cm $^{-3}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3 \text{ cm}$$

$^{-3}$

4.47

(a) $p_o << n_i \Rightarrow$ n-type

(b) $p_o = \frac{n_i^2}{n_o} \Rightarrow n_o = \frac{n_i^2}{p_o}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^4} = 1.125\times10^{16} \text{ cm}^{-3}$$

$\Rightarrow$ electrons are majority carriers

$p_o = 2\times10^4$ cm $^{-3}$

$\Rightarrow$ holes are minority carriers

(c) $n_o = N_d - N_a$

$$1.125\times10^{16} = N_d - 7\times10^{15}$$

so $N_d = 1.825\times10^{16}$ cm $^{-3}$

4.48

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$$

For Germanium

T (K)	kT (eV)	n_i (cm $^{-3}$)
200	0.01727	2.16×10^{10}
400	0.03453	8.60×10^{14}
600	0.0518	3.82×10^{16}

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$ and

$N_a = 10^{15}$ cm

$^{-3}$

T (K)	p_o (cm $^{-3}$)	$\left(E_{Fi} - E_F\right)$ (eV)

200	1.0×10^{15}	0.1855
400	1.49×10^{15}	0.01898
600	3.87×10^{16}	0.000674

4.49

(a) $E_c - E_F = kT\ln\left(\dfrac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

For 10^{14} cm^{-3}, $E_c - E_F = 0.3249$ eV
10^{15} cm^{-3}, $E_c - E_F = 0.2652$ eV
10^{16} cm^{-3}, $E_c - E_F = 0.2056$ eV
10^{17} cm^{-3}, $E_c - E_F = 0.1459$ eV

(b) $E_F - E_{Fi} = kT\ln\left(\dfrac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

For 10^{14} cm^{-3}, $E_F - E_{Fi} = 0.2280$ eV
10^{15} cm^{-3}, $E_F - E_{Fi} = 0.2877$ eV
10^{16} cm^{-3}, $E_F - E_{Fi} = 0.3473$ eV
10^{17} cm^{-3}, $E_F - E_{Fi} = 0.4070$ eV

4.50

(a) $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$n_o = 1.05N_d = 1.05\times10^{15}$ cm^{-3}

$$\left(1.05\times10^{15} - 0.5\times10^{15}\right)^2$$

$$= \left(0.5\times10^{15}\right)^2 + n_i^2$$

so $n_i^2 = 5.25\times10^{28}$

Now

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$5.25\times10^{28} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-12972.973}{T}\right]$$

By trial and error, $T = 536.5$ K

(b) At $T = 300$ K,

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$$

$$= 0.2652 \text{ eV}$$

At $T = 536.5$ K,

$$kT = (0.0259)\left(\frac{536.5}{300}\right) = 0.046318 \text{ eV}$$

$$N_c = \left(2.8\times10^{19}\right)\left(\frac{536.5}{300}\right)^{3/2}$$

$$= 6.696\times10^{19} \text{ cm}^{-3}$$

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.046318)\ln\left(\frac{6.696\times10^{19}}{1.05\times10^{15}}\right)$$

$$= 0.5124 \text{ eV}$$

then $\Delta\left(E_c - E_F\right) = 0.2472$ eV

(c) Closer to the intrinsic energy level.

4.51

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$$

At $T = 200$ K, $kT = 0.017267$ eV
$T = 400$ K, $kT = 0.034533$ eV
$T = 600$ K, $kT = 0.0518$ eV

At $T = 200$ K,

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{200}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.017267}\right]$$

$\Rightarrow n_i = 7.638\times10^4$ cm $^{-3}$

At $T = 400$ K,

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{400}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.034533}\right]$$

$\Rightarrow n_i = 2.381\times10^{12}$ cm $^{-3}$

At $T = 600$ K,

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{600}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.0518}\right]$$

$\Rightarrow n_i = 9.740\times10^{14}$ cm $^{-3}$

At $T = 200$ K and $T = 400$ K,

$p_o = N_a = 3\times10^{15}$ cm $^{-3}$

At $T = 600$ K,

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= \frac{3\times10^{15}}{2} + \sqrt{\left(\frac{3\times10^{15}}{2}\right)^2 + \left(9.740\times10^{14}\right)^2}$$

$= 3.288\times10^{15}$ cm $^{-3}$

Then, $T = 200$ K, $E_{Fi} - E_F = 0.4212$ eV

$T = 400$ K, $E_{Fi} - E_F = 0.2465$ eV

$T = 600$ K, $E_{Fi} - E_F = 0.0630$ eV

4.52

(a)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{N_a}{1.8\times10^6}\right)$$

For $N_a = 10^{14}$ cm $^{-3}$,

$E_{Fi} - E_F = 0.4619$ eV

$N_a = 10^{15}$ cm $^{-3}$,

$E_{Fi} - E_F = 0.5215$ eV

$N_a = 10^{16}$ cm $^{-3}$,

$E_{Fi} - E_F = 0.5811$ eV

$N_a = 10^{17}$ cm $^{-3}$,

$E_{Fi} - E_F = 0.6408$ eV

(b)

$$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{N_a}\right) = (0.0259)\ln\left(\frac{7.0\times10^{18}}{N_a}\right)$$

For $N_a = 10^{14}$ cm $^{-3}$,

$E_F - E_\upsilon = 0.2889$ eV

$N_a = 10^{15}$ cm $^{-3}$,

$E_F - E_\upsilon = 0.2293$ eV

$N_a = 10^{16}$ cm $^{-3}$,

$E_F - E_\upsilon = 0.1697$ eV

$N_a = 10^{17}$ cm $^{-3}$,

$E_F - E_\upsilon = 0.1100$ eV

4.53

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln(10)$$

or

$E_{Fi} - E_{midgap} = +0.0447$ eV

(b) Impurity atoms to be added so

$E_{midgap} - E_F = 0.45$ eV

(i) p-type, so add acceptor atoms

(ii)

$E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947$ eV

Then

$$p_o = n_i\exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= \left(10^5\right)\exp\left(\frac{0.4947}{0.0259}\right)$$

or

$p_o = N_a = 1.97\times10^{13}$ cm $^{-3}$

4.54

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5\times10^{15} + (2.8\times10^{19})\exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5\times10^{15} + 6.95\times10^{15}$$

or

$$N_d = 1.2\times10^{16} \text{ cm}^{-3}$$

4.55

(a) Silicon

(i) $E_c - E_F = kT\ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6\times10^{15}}\right) = 0.2188$$

eV

(ii)

$E_c - E_F = 0.2188 - 0.0259 = 0.1929$ eV

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= (2.8\times10^{19})\exp\left[\frac{-0.1929}{0.0259}\right]$$

$$N_d = 1.631\times10^{16} \text{ cm}^{-3}$$

$$= N_d' + 6\times10^{15}$$

$$\Rightarrow N_d' = 1.031\times10^{16} \text{ cm}^{-3}$$

Additional donor atoms

(b) GaAs

(i) $E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{15}}\right)$

$$= 0.15936 \text{ eV}$$

(ii)

$E_c - E_F = 0.15936 - 0.0259 = 0.13346$ eV

$$N_d = (4.7\times10^{17})\exp\left[\frac{-0.13346}{0.0259}\right]$$

$$= 2.718\times10^{15} \text{ cm}^{-3}$$

$$= N_d' + 10^{15}$$

$$\Rightarrow N_d' = 1.718\times10^{15} \text{ cm}^{-3}$$

Additional donor atoms

4.56

(a) $E_{Fi} - E_F = kT\ln\left(\frac{N_\upsilon}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{2\times10^{16}}\right)$$

$$= 0.1620 \text{ eV}$$

(b) $E_F - E_{Fi} = kT\ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{2\times10^{16}}\right) = 0.1876 \text{ eV}$$

(c) For part (a);

$$p_o = 2\times10^{16} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{2\times10^{16}}$$

$$= 1.125\times10^{4} \text{ cm}^{-3}$$

For part (b):

$$n_o = 2\times10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10})^2}{2\times10^{16}}$$

$$= 1.125\times10^{4} \text{ cm}^{-3}$$

4.57

$$n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$= (1.8\times10^{6})\exp\left[\frac{0.55}{0.0259}\right]$$

$$= 3.0\times10^{15} \text{ cm}^{-3}$$

Add additional acceptor impurities

$$n_o = N_d - N_a$$

$$3\times10^{15} = 7\times10^{15} - N_a$$

$$\Rightarrow N_a = 4\times10^{15}\ \text{cm}^{-3}$$

4.58

(a) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3\times10^{15}}{1.5\times10^{10}}\right) = 0.3161\ \text{eV}$$

(b) $E_F - E_{Fi} = kT\ln\left(\frac{n_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\ \text{eV}$$

(c) $E_F = E_{Fi}$

(d) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$$= (0.0259)\left(\frac{375}{300}\right)\ln\left(\frac{4\times10^{15}}{7.334\times10^{11}}\right)$$

$$= 0.2786\ \text{eV}$$

(e) $E_F - E_{Fi} = kT\ln\left(\frac{n_o}{n_i}\right)$

$$= (0.0259)\left(\frac{450}{300}\right)\ln\left(\frac{1.029\times10^{14}}{1.722\times10^{13}}\right)$$

$$= 0.06945\ \text{eV}$$

4.59

(a) $E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$

$$= (0.0259)\ln\left(\frac{7.0\times10^{18}}{3\times10^{15}}\right) = 0.2009\ \text{eV}$$

(b)

$$E_F - E_\upsilon = (0.0259)\ln\left(\frac{7.0\times10^{18}}{1.08\times10^{-4}}\right)$$

$$= 1.360\ \text{eV}$$

(c) $E_F - E_\upsilon = (0.0259)\ln\left(\frac{7.0\times10^{18}}{1.8\times10^{6}}\right)$

$$= 0.7508\ \text{eV}$$

(d) $E_F - E_\upsilon = (0.0259)\left(\frac{375}{300}\right)$

$$\times\ln\left[\frac{(7.0\times10^{18})(375/300)^{3/2}}{4\times10^{15}}\right]$$

$$= 0.2526\ \text{eV}$$

(e) $E_F - E_\upsilon = (0.0259)\left(\frac{450}{300}\right)$

$$\times\ln\left[\frac{(7.0\times10^{18})(450/300)^{3/2}}{1.48\times10^{7}}\right]$$

$$= 1.068\ \text{eV}$$

4.60

n-type

$$E_F - E_{Fi} = kT\ln\left(\frac{n_o}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{1.125\times10^{16}}{1.5\times10^{10}}\right) = 0.3504\ \text{eV}$$

4.61

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$5.08\times10^{15} = \frac{5\times10^{15}}{2} + \sqrt{\left(\frac{5\times10^{15}}{2}\right)^2 + n_i^2}$$

$$\left(5.08\times10^{15} - 2.5\times10^{15}\right)^2$$

$$= \left(2.5\times10^{15}\right)^2 + n_i^2$$

$$6.6564\times10^{30} = 6.25\times10^{30} + n_i^2$$

$$\Rightarrow n_i^2 = 4.064\times10^{29}$$

$$n_i^2 = N_c N_\upsilon \exp\left[\frac{-E_g}{kT}\right]$$

$$kT = (0.0259)\left(\frac{350}{300}\right) = 0.030217 \text{ eV}$$

$$N_c = \left(1.2\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 1.633\times10^{19} \text{ cm}^{-3}$$

$$N_\upsilon = \left(1.8\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 2.45\times10^{19} \text{ cm}^{-3}$$

Now

$$4.064\times10^{29} = \left(1.633\times10^{19}\right)\left(2.45\times10^{19}\right) \times\exp\left[\frac{-E_g}{0.030217}\right]$$

So

$$E_g = (0.030217)\ln\left[\frac{\left(1.633\times10^{19}\right)\left(2.45\times10^{1}\right.}{4.064\times10^{29}}\right.$$

$$\Rightarrow E_g = 0.6257 \text{ eV}$$

4.62

(a) Replace Ga atoms $\Rightarrow$ Silicon acts as a donor

$$N_d = (0.05)\left(7\times10^{15}\right) = 3.5\times10^{14} \text{ cm}^{-3}$$

Replace As atoms $\Rightarrow$ Silicon acts as an acceptor

$$N_a = (0.95)\left(7\times10^{15}\right) = 6.65\times10^{15} \text{ cm}^{-3}$$

(b) $N_a > N_d \Rightarrow$ p-type

(c)

$$p_o = N_a - N_d = 6.65\times10^{15} - 3.5\times10^{14} = 6.3\times10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^6\right)^2}{6.3\times10^{15}} = 5.14\times10^{-4} \text{ cm}^{-3}$$

(d) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{6.3\times10^{15}}{1.8\times10^6}\right) = 0.5692 \text{ eV}$$
