Chapter 3

3.1

If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

3.2

Schrodinger's wave equation is:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\cdot\Psi(x,t)$$

$$= j\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Assume the solution is of the form:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Region I: $V(x)=0$. Substituting the assumed solution into the wave equation, we obtain:

$$\frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\left\{ jku(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$\left.+\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$= j\hbar\left(\frac{-jE}{\hbar}\right)\cdot u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

which becomes

$$\frac{-\hbar^2}{2m}\left\{ (jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$\left.+\frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$=+Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation may be written as

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2 u(x)}{\partial x^2}+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_1(x)$ for region I, the equation becomes:

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$

where

$$\alpha^2=\frac{2mE}{\hbar^2}$$

Q.E.D.

In Region II, $V(x)=V_O$. Assume the same form of the solution:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Substituting into Schrodinger's wave equation, we find:

$$\frac{-\hbar^2}{2m}\left\{ (jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$\left.+\frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$+V_Ou(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$=Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation can be written as:

$$-k^2 u(x) + 2jk\frac{\partial u(x)}{\partial x} + \frac{\partial^2 u(x)}{\partial x^2}$$

$$-\frac{2mV_O}{\hbar^2}u(x) + \frac{2mE}{\hbar^2}u(x) = 0$$

Setting $u(x) = u_2(x)$ for region II, this equation becomes

$$\frac{d^2 u_2(x)}{dx^2} + 2jk\frac{du_2(x)}{dx}$$

$$-\left(k^2 - \alpha^2 + \frac{2mV_O}{\hbar^2}\right)u_2(x) = 0$$

where again

$$\alpha^2 = \frac{2mE}{\hbar^2} \qquad \text{Q.E.D.}$$

3.3

We have

$$\frac{d^2 u_1(x)}{dx^2} + 2jk\frac{du_1(x)}{dx} - \left(k^2 - \alpha^2\right)u_1(x) = 0$$

Assume the solution is of the form:

$$u_1(x) = A\exp[j(\alpha - k)x] + B\exp[-j(\alpha + k)x]$$

The first derivative is

$$\frac{du_1(x)}{dx} = j(\alpha - k)A\exp[j(\alpha - k)x]$$

$$-j(\alpha + k)B\exp[-j(\alpha + k)x]$$

and the second derivative becomes

$$\frac{d^2 u_1(x)}{dx^2} = [j(\alpha - k)]^2 A\exp[j(\alpha - k)x]$$

$$+[j(\alpha + k)]^2 B\exp[-j(\alpha + k)x]$$

Substituting these equations into the differential equation, we find

$$-(\alpha - k)^2 A\exp[j(\alpha - k)x]$$
$$-(\alpha + k)^2 B\exp[-j(\alpha + k)x]$$
$$+2jk\{j(\alpha - k)A\exp[j(\alpha - k)x]$$
$$-j(\alpha + k)B\exp[-j(\alpha + k)x]\}$$
$$-\left(k^2 - \alpha^2\right)\{A\exp[j(\alpha - k)x]$$
$$+B\exp[-j(\alpha + k)x]\} = 0$$

Combining terms, we obtain

$$\left[-\left(\alpha^2 - 2\alpha k + k^2\right) - 2k(\alpha - k) - \left(k^2 - \alpha^2\right)\right]$$
$$\times A\exp[j(\alpha - k)x]$$

$$+\left[-\left(\alpha^2 + 2\alpha k + k^2\right) + 2k(\alpha + k) - \left(k^2 - \alpha^2\right)\right]$$
$$\times B\exp[-j(\alpha + k)x] = 0$$

We find that

$$0 = 0 \qquad \text{Q.E.D.}$$

For the differential equation in $u_2(x)$ and the

proposed solution, the procedure is exactly the same as above.

3.4

We have the solutions

$$u_1(x) = A\exp[j(\alpha - k)x] + B\exp[-j(\alpha + k)x]$$

for $0 < x < a$ and

$$u_2(x) = C\exp[j(\beta - k)x] + D\exp[-j(\beta + k)x]$$

for $-b < x < 0$.

The first boundary condition is

$$u_1(0) = u_2(0)$$

which yields

$$A + B - C - D = 0$$

The second boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=0} = \left.\frac{du_2}{dx}\right|_{x=0}$$

which yields

$$(\alpha - k)A - (\alpha + k)B - (\beta - k)C + (\beta + k)D = 0$$

The third boundary condition is

$$u_1(a) = u_2(-b)$$

which yields

$$A\exp[j(\alpha - k)a] + B\exp[-j(\alpha + k)a]$$
$$= C\exp[j(\beta - k)(-b)]$$

$$+D\exp[-j(\beta+k)(-b)]$$

and can be written as

$$A\exp[j(\alpha-k)a]+B\exp[-j(\alpha+k)a]$$
$$-C\exp[-j(\beta-k)b]$$

$$-D\exp[j(\beta+k)b]=0$$

The fourth boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=a}=\left.\frac{du_2}{dx}\right|_{x=-b}$$

which yields

$$j(\alpha-k)A\exp[j(\alpha-k)a]$$
$$-j(\alpha+k)B\exp[-j(\alpha+k)a]$$
$$=j(\beta-k)C\exp[j(\beta-k)(-b)]$$

$$-j(\beta+k)D\exp[-j(\beta+k)(-b)]$$

and can be written as

$$(\alpha-k)A\exp[j(\alpha-k)a]$$
$$-(\alpha+k)B\exp[-j(\alpha+k)a]$$
$$-(\beta-k)C\exp[-j(\beta-k)b]$$
$$+(\beta+k)D\exp[j(\beta+k)b]=0$$

3.5

(b) (i) First point: $\alpha a=\pi$

Second point: By trial and error,

$$\alpha a=1.729\pi$$

(ii) First point: $\alpha a=2\pi$

Second point: By trial and error,

$$\alpha a=2.617\pi$$

3.6

(b) (i) First point: $\alpha a=\pi$

Second point: By trial and error,

$$\alpha a=1.515\pi$$

(ii) First point: $\alpha a=2\pi$

Second point: By trial and error,

$$\alpha a=2.375\pi$$

3.7

$$P'\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

Let $ka=y$, $\alpha a=x$

Then

$$P'\frac{\sin x}{x}+\cos x=\cos y$$

Consider $\frac{d}{dy}$ of this function.

$$\frac{d}{dy}\{[P'\cdot(x)^{-1}\sin x]+\cos x\}=-\sin y$$

We find

$$P'\left\{(-1)(x)^{-2}\sin x\cdot\frac{dx}{dy}+(x)^{-1}\cos x\cdot\frac{dx}{dy}\right\}$$

$$-\sin x\frac{dx}{dy}=-\sin y$$

Then

$$\frac{dx}{dy}\left\{P'\left[\frac{-1}{x^2}\sin x+\frac{\cos x}{x}\right]-\sin x\right\}=-\sin y$$

For $y=ka=n\pi$, $n=0,1,2,\ldots$

$\Rightarrow \sin y=0$

So that, in general,

$$\frac{dx}{dy}=0=\frac{d(\alpha a)}{d(ka)}=\frac{d\alpha}{dk}$$

And

$$\alpha=\sqrt{\frac{2mE}{\hbar^2}}$$

So

$$\frac{d\alpha}{dk}=\frac{1}{2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\left(\frac{2m}{\hbar^2}\right)\frac{dE}{dk}$$

This implies that

$$\frac{d\alpha}{dk}=0=\frac{dE}{dk}\ \text{ for }\ k=\frac{n\pi}{a}$$

3.8

(a) $\alpha_1 a=\pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{\pi^2 \hbar^2}{2m_o a^2} = \frac{(\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})}$$

$$= 3.4114\times10^{-19} \text{ J}$$

From Problem 3.5

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.0198\times10^{-18} \text{ J}$$

$$\Delta E = E_2 - E_1$$

$$= 1.0198\times10^{-18} - 3.4114\times10^{-19}$$

$$= 6.7868\times10^{-19} \text{ J}$$

or $\Delta E = \dfrac{6.7868\times10^{-19}}{1.6\times10^{-19}} = 4.24$ eV

(b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.3646\times10^{-18} \text{ J}$$

From Problem 3.5,

$$\alpha_4 a = 2.617\pi$$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}} \cdot a = 2.617\pi$$

$$E_4 = \frac{(2.617\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 2.3364\times10^{-18} \text{ J}$$

$$\Delta E = E_4 - E_3$$

$$= 2.3364\times10^{-18} - 1.3646\times10^{-18}$$

$$= 9.718\times10^{-19} \text{ J}$$

or $\Delta E = \dfrac{9.718\times10^{-19}}{1.6\times10^{-19}} = 6.07$ eV

3.9

(a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 3.4114\times10^{-19} \text{ J}$$

At $ka = 0$, By trial and error,

$$\alpha_o a = 0.859\pi$$

$$E_o = \frac{(0.859\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 2.5172\times10^{-19} \text{ J}$$

$$\Delta E = E_1 - E_o$$

$$= 3.4114\times10^{-19} - 2.5172\times10^{-19}$$

$$= 8.942\times10^{-20} \text{ J}$$

or $\Delta E = \dfrac{8.942\times10^{-20}}{1.6\times10^{-19}} = 0.559$ eV

(b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.3646\times10^{-18} \text{ J}$$

At $ka = \pi$. From Problem 3.5,

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.0198\times10^{-18} \text{ J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646\times10^{-18} - 1.0198\times10^{-18}$$

$$= 3.4474\times10^{-19} \text{ J}$$

or $\Delta E = \dfrac{3.4474\times10^{-19}}{1.6\times10^{-19}} = 2.15$ eV

3.10

(a) $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}}\cdot a = \pi$$

$$E_1 = \frac{(\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 3.4114\times10^{-19}$ J

From Problem 3.6, $\alpha_2 a = 1.515\pi$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}}\cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 7.830\times10^{-19}$ J

$\Delta E = E_2 - E_1$

$= 7.830\times10^{-19} - 3.4114\times10^{-19}$

$= 4.4186\times10^{-19}$ J

or $\Delta E = \dfrac{4.4186\times10^{-19}}{1.6\times10^{-19}} = 2.76$ eV

(b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}}\cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 1.3646\times10^{-18}$ J

From Problem 3.6, $\alpha_4 a = 2.375\pi$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}}\cdot a = 2.375\pi$$

$$E_4 = \frac{(2.375\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 1.9242\times10^{-18}$ J

$\Delta E = E_4 - E_3$

$= 1.9242\times10^{-18} - 1.3646\times10^{-18}$

$= 5.597\times10^{-19}$ J

or $\Delta E = \dfrac{5.597\times10^{-19}}{1.6\times10^{-19}} = 3.50$ eV

3.11

(a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}}\cdot a = \pi$$

$$E_1 = \frac{(\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 3.4114\times10^{-19}$ J

At $ka = 0$, By trial and error,

$\alpha_o a = 0.727\pi$

$$\sqrt{\frac{2m_o E_o}{\hbar^2}}\cdot a = 0.727\pi$$

$$E_o = \frac{(0.727\pi)^2(1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$= 1.8030\times10^{-19}$ J

$\Delta E = E_1 - E_o$

$= 3.4114\times10^{-19} - 1.8030\times10^{-19}$

$= 1.6084\times10^{-19}$ J

or

$\Delta E = \dfrac{1.6084\times10^{-19}}{1.6\times10^{-19}} = 1.005$ eV

(b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_oE_3}{\hbar^2}}\cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2\left(1.054\times10^{-34}\right)^2}{2\left(9.11\times10^{-31}\right)\left(4.2\times10^{-10}\right)^2}$$

$$= 1.3646\times10^{-18} \text{ J}$$

At $ka = \pi$, From Problem 3.6,

$$\alpha_2 a = 1.515\pi$$

$$\sqrt{\frac{2m_oE_2}{\hbar^2}}\cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2\left(1.054\times10^{-34}\right)^2}{2\left(9.11\times10^{-34}\right)\left(4.2\times10^{-10}\right)^2}$$

$$= 7.830\times10^{-19} \text{ J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646\times10^{-18} - 7.830\times10^{-19}$$

$$= 5.816\times10^{-19} \text{ J}$$

or $\Delta E = \dfrac{5.816\times10^{-19}}{1.6\times10^{-19}} = 3.635$ eV

3.12

For $T = 100$ K,

$$E_g = 1.170 - \frac{\left(4.73\times10^{-4}\right)(100)^2}{636+100} \Rightarrow$$

$$E_g = 1.164 \text{ eV}$$

$T = 200$ K, $E_g = 1.147$ eV

$T = 300$ K, $E_g = 1.125$ eV

$T = 400$ K, $E_g = 1.097$ eV

$T = 500$ K, $E_g = 1.066$ eV

$T = 600$ K, $E_g = 1.032$ eV

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\right)^{-1}$$

We have

$$\frac{d^2E}{dk^2}(curve\ A) > \frac{d^2E}{dk^2}(curve\ B)$$

so that $m^*(curve\ A) < m^*(curve\ B)$

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2}\cdot\left|\frac{d^2E}{dk^2}\right|\right)^{-1}$$

We have that

$$\left|\frac{d^2E}{dk^2}\right|(curve\ A) > \left|\frac{d^2E}{dk^2}\right|(curve\ B)$$

so that $m_p^*(curve\ A) < m_p^*(curve\ B)$

3.15

Points A,B: $\dfrac{dE}{dk} < 0 \Rightarrow$ velocity in -x direction

Points C,D: $\dfrac{dE}{dk} > 0 \Rightarrow$ velocity in +x direction

Points A,D: $\dfrac{d^2E}{dk^2} < 0 \Rightarrow$ negative effective mass

Points B,C: $\dfrac{d^2E}{dk^2} > 0 \Rightarrow$ positive effective mass

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3.16

For A: $E = C_i k^2$

At $k = 0.08\times10^{+10}$ m $^{-1}$, $E = 0.05$ eV

Or

$$E = (0.05)\left(1.6\times10^{-19}\right) = 8\times10^{-21} \text{ J}$$

So $8\times10^{-21} = C_1\left(0.08\times10^{10}\right)^2$

$$\Rightarrow C_1 = 1.25\times10^{-38}$$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{\left(1.054\times10^{-34}\right)^2}{2\left(1.25\times10^{-38}\right)}$

$$= 4.44\times10^{-31} \text{ kg}$$

or $m^* = \dfrac{4.4437\times10^{-31}}{9.11\times10^{-31}}\cdot m_o$

$$m^* = 0.488\, m_o$$

For B: $E = C_i k^2$

At $k = 0.08 \times 10^{+10}$ m $^{-1}$, $E = 0.5$ eV

Or

$$E = (0.5)(1.6 \times 10^{-19}) = 8 \times 10^{-20} \text{ J}$$

So $8 \times 10^{-20} = C_1 (0.08 \times 10^{10})^2$

$$\Rightarrow C_1 = 1.25 \times 10^{-37}$$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{(1.054 \times 10^{-34})^2}{2(1.25 \times 10^{-37})}$

$$= 4.44 \times 10^{-32} \text{ kg}$$

or $m^* = \dfrac{4.4437 \times 10^{-32}}{9.11 \times 10^{-31}} \cdot m_o$

$$m^* = 0.0488 m_o$$

3.17

For A: $E - E_\upsilon = -C_2 k^2$

$$-(0.025)(1.6 \times 10^{-19}) = -C_2 (0.08 \times 10^{10})^2$$

$$\Rightarrow C_2 = 6.25 \times 10^{-39}$$

$$m^* = \frac{-\hbar^2}{2C_2} = \frac{-(1.054 \times 10^{-34})^2}{2(6.25 \times 10^{-39})}$$

$$= -8.8873 \times 10^{-31} \text{ kg}$$

or $m^* = \dfrac{-8.8873 \times 10^{-31}}{9.11 \times 10^{-31}} \cdot m_o$

$$m^* = -\,-0.976 m_o$$

For B: $E - E_\upsilon = -C_2 k^2$

$$-(0.3)(1.6 \times 10^{-19}) = -C_2 (0.08 \times 10^{10})^2$$

$$\Rightarrow C_2 = 7.5 \times 10^{-38}$$

$$m^* = \frac{-\hbar^2}{2C_2} = \frac{-(1.054 \times 10^{-34})^2}{2(7.5 \times 10^{-38})}$$

$$= -7.406 \times 10^{-32} \text{ kg}$$

or $m^* = \dfrac{-7.406 \times 10^{-32}}{9.11 \times 10^{-31}} \cdot m_o$

$$m^* = -0.0813 m_o$$

3.18

(a) (i) $E = h\nu$

or $\nu = \dfrac{E}{h} = \dfrac{(1.42)(1.6 \times 10^{-19})}{6.625 \times 10^{-34}}$

$$= 3.429 \times 10^{14} \text{ Hz}$$

(ii) $\lambda = \dfrac{hc}{E} = \dfrac{c}{\nu} = \dfrac{3 \times 10^{10}}{3.429 \times 10^{14}}$

$$= 8.75 \times 10^{-5} \text{ cm} = 875 \text{ nm}$$

(b) (i) $\nu = \dfrac{E}{h} = \dfrac{(1.12)(1.6 \times 10^{-19})}{6.625 \times 10^{-34}}$

$$= 2.705 \times 10^{14} \text{ Hz}$$

(ii) $\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{10}}{2.705 \times 10^{14}}$

$$= 1.109 \times 10^{-4} \text{ cm} = 1109 \text{ nm}$$

3.19

(c) Curve A: Effective mass is a constant
Curve B: Effective mass is positive around $k = 0$, and is negative around $k = \pm \dfrac{\pi}{2}$.

3.20

$$E = E_O - E_1 \cos[\alpha(k - k_O)]$$

Then

$$\frac{dE}{dk} = (-E_1)(-\alpha)\sin[\alpha(k - k_O)]$$

$$= +E_1 \alpha \sin[\alpha(k - k_O)]$$

and

$$\frac{d^2E}{dk^2} = E_1 \alpha^2 \cos[\alpha(k - k_O)]$$

Then

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \cdot \left.\frac{d^2E}{dk^2}\right|_{k=k_o} = \frac{E_1 \alpha^2}{\hbar^2}$$

or

$$m^* = \frac{\hbar^2}{E_1 \alpha^2}$$

3.21

(a) $m_{dn}^* = 4^{2/3}\left[(m_t)^2 m_l\right]^{1/3}$

$= 4^{2/3}\left[(0.082m_o)^2(1.64m_o)\right]^{1/3}$

$m_{dn}^* = 0.56m_o$

(b)

$$\frac{3}{m_{cn}^*} = \frac{2}{m_t} + \frac{1}{m_l} = \frac{2}{0.082m_o} + \frac{1}{1.64m_o}$$

$$= \frac{24.39}{m_o} + \frac{0.6098}{m_o}$$

$m_{cn}^* = 0.12m_o$

3.22

(a) $m_{dp}^* = \left[(m_{hh})^{3/2} + (m_{lh})^{3/2}\right]^{2/3}$

$= \left[(0.45m_o)^{3/2} + (0.082m_o)^{3/2}\right]^{2/3}$

$= [0.30187 + 0.02348]^{2/3} \cdot m_o$

$m_{dp}^* = 0.473m_o$

(b) $$m_{cp}^* = \frac{(m_{hh})^{3/2} + (m_{lh})^{3/2}}{(m_{hh})^{1/2} + (m_{lh})^{1/2}}$$

$$= \frac{(0.45)^{3/2} + (0.082)^{3/2}}{(0.45)^{1/2} + (0.082)^{1/2}} \cdot m_o$$

$m_{cp}^* = 0.34m_o$

3.23

For the 3-dimensional infinite potential well, $V(x) = 0$ when $0 < x < a$, $0 < y < a$, and $0 < z < a$. In this region, the wave equation is:

$$\frac{\partial^2 \psi(x,y,z)}{\partial x^2} + \frac{\partial^2 \psi(x,y,z)}{\partial y^2} + \frac{\partial^2 \psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables technique, so let

$\psi(x,y,z) = X(x)Y(y)Z(z)$

Substituting into the wave equation, we have

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} \cdot XYZ = 0$$

Dividing by XYZ, we obtain

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

The solution is of the form:

$X(x) = A\sin k_x x + B\cos k_x x$

Since $\psi(x,y,z) = 0$ at $x = 0$, then $X(0) = 0$

so that $B = 0$.

Also, $\psi(x,y,z) = 0$ at $x = a$, so that

$X(a) = 0$. Then $k_x a = n_x \pi$ where

$n_x = 1, 2, 3, \ldots$

Similarly, we have

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2 \quad \text{and} \quad \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

From the boundary conditions, we find

$k_y a = n_y \pi$ and $k_z a = n_z \pi$

where

$n_y = 1, 2, 3, \ldots$ and $n_z = 1, 2, 3, \ldots$

From the wave equation, we can write

$$-k_x^2 - k_y^2 - k_z^2 + \frac{2mE}{\hbar^2} = 0$$

The energy can be written as

$$E = E_{n_x n_y n_z} = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

3.24

The total number of quantum states in the 3-dimensional potential well is given (in k-space) by

$$g_T(k)dk = \frac{\pi k^2 dk}{\pi^3} \cdot a^3$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

We can then write

$$k = \frac{\sqrt{2mE}}{\hbar}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar}\cdot\sqrt{2m}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{E}}\cdot dE = \frac{1}{\hbar}\cdot\sqrt{\frac{m}{2E}}\cdot dE$$

Substituting these expressions into the density of states function, we have

$$g_T(E)dE = \frac{\pi a^3}{\pi^3}\left(\frac{2mE}{\hbar^2}\right)\cdot\frac{1}{\hbar}\cdot\sqrt{\frac{m}{2E}}\cdot dE$$

Noting that

$$\hbar = \frac{h}{2\pi}$$

this density of states function can be simplified and written as

$$g_T(E)dE = \frac{4\pi a^3}{h^3}(2m)^{3/2}\cdot\sqrt{E}\cdot dE$$

Dividing by a^3 will yield the density of states so that

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\cdot\sqrt{E}$$

3.25

For a one-dimensional infinite potential well,

$$\frac{2m_n^* E}{\hbar^2} = \frac{n^2\pi^2}{a^2} = k^2$$

Distance between quantum states

$$k_{n+1} - k_n = (n+1)\left(\frac{\pi}{a}\right) = (n)\left(\frac{\pi}{a}\right) = \frac{\pi}{a}$$

Now

$$g_T(k)dk = \frac{2\cdot dk}{\left(\frac{\pi}{a}\right)}$$

Now

$$k = \frac{1}{\hbar}\cdot\sqrt{2m_n^* E}$$

$$dk = \frac{1}{\hbar}\cdot\frac{1}{2}\cdot\sqrt{\frac{2m_n^*}{E}}\cdot dE$$

Then

$$g_T(E)dE = \frac{2a}{\pi}\cdot\frac{1}{2\hbar}\cdot\sqrt{\frac{2m_n^*}{E}}\cdot dE$$

Divide by the "volume" a, so

$$g(E) = \frac{1}{\hbar\pi}\cdot\sqrt{\frac{2m_n^*}{E}}$$

So

$$g(E) = \frac{1}{(1.054\times10^{-34})(\pi)}\cdot\frac{\sqrt{2(0.067)(9.11\times10^{-31})}}{\sqrt{E}}$$

$$g(E) = \frac{1.055\times10^{18}}{\sqrt{E}}\ \text{m}^{-3}\ \text{J}^{-1}$$

3.26

(a) Silicon, $m_n^* = 1.08m_o$

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

$$g_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2kT}\sqrt{E-E_c}\cdot dE$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(E-E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= \frac{4\pi[2(1.08)(9.11\times10^{-31})]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= (7.953\times10^{55})(2kT)^{3/2}$$

(i) At $T = 300$ K, $kT = 0.0259$ eV

$$= (0.0259)(1.6\times10^{-19})$$

$$= 4.144\times10^{-21}\ \text{J}$$

Then

$$g_c = (7.953\times10^{55})[2(4.144\times10^{-21})]^{3/2}$$

$$= 6.0\times10^{25}\ \text{m}^{-3}$$

or $g_c = 6.0\times10^{19}$ cm $^{-3}$

(ii) At $T = 400$ K,

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533 \text{ eV}$$

$$= (0.034533)(1.6\times10^{-19}) = 5.5253\times10^{-21} \text{ J}$$

Then

$$g_c = (7.953\times10^{55})\left[2(5.5253\times10^{-21})\right]^{3/2} = 9.239\times10^{25} \text{ m}^{-3}$$

or $g_c = 9.24\times10^{19}$ cm $^{-3}$

(b) GaAs, $m_n^* = 0.067m_o$

$$g_c = \frac{4\pi\left[2(0.067)(9.11\times10^{-31})\right]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= (1.2288\times10^{54})(2kT)^{3/2}$$

(i) At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_c = (1.2288\times10^{54})\left[2(4.144\times10^{-21})\right]^{3/2} = 9.272\times10^{23} \text{ m}^{-3}$$

or $g_c = 9.27\times10^{17}$ cm $^{-3}$

(ii) At $T = 400$ K,

$kT = 5.5253\times10^{-21}$ J

$$g_c = (1.2288\times10^{54})\left[2(5.5253\times10^{-21})\right]^{3/2} = 1.427\times10^{24} \text{ m}^{-3}$$

$g_c = 1.43\times10^{18}$ cm $^{-3}$

3.27

(a) Silicon, $m_p^* = 0.56m_o$

$$g_\upsilon(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_\upsilon - E}$$

$$g_\upsilon = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\int_{E_\upsilon-3kT}^{E_\upsilon}\sqrt{E_\upsilon - E}\cdot dE$$

$$= \frac{4\pi(2m_p^*)^{3/2}}{h^3}\left(\frac{-2}{3}\right)(E_\upsilon - E)^{3/2}\Big|_{E_\upsilon-3kT}^{E_\upsilon}$$

$$= \frac{4\pi(2m_p^*)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[-(3kT)^{3/2}\right]$$

$$= \frac{4\pi\left[2(0.56)(9.11\times10^{-31})\right]^{3/2}}{(6.625\times10^{-34})^3}\left(\frac{2}{3}\right)(3kT)^{3/2}$$

$$= (2.969\times10^{55})(3kT)^{3/2}$$

(i)At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_\upsilon = (2.969\times10^{55})\left[3(4.144\times10^{-21})\right]^{3/2} = 4.116\times10^{25} \text{ m}^{-3}$$

or $g_\upsilon = 4.12\times10^{19}$ cm $^{-3}$

(ii)At $T = 400$ K, $kT = 5.5253\times10^{-21}$ J

$$g_\upsilon = (2.969\times10^{55})\left[3(5.5253\times10^{-21})\right]^{3/2} = 6.337\times10^{25} \text{ m}^{-3}$$

or $g_\upsilon = 6.34\times10^{19}$ cm $^{-3}$

(b) GaAs, $m_p^* = 0.48m_o$

$$g_\upsilon = \frac{4\pi\left[2(0.48)(9.11\times10^{-31})\right]^{3/2}}{(6.625\times10^{-34})^3}\left(\frac{2}{3}\right)(3kT)^{3/2}$$

$$= (2.3564\times10^{55})(3kT)^{3/2}$$

(i)At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_\upsilon = (2.3564\times10^{55})\left[3(4.144\times10^{-21})\right]^{3/2} = 3.266\times10^{25} \text{ m}^{-3}$$

or $g_\upsilon = 3.27\times10^{19}$ cm $^{-3}$

(ii)At $T = 400$ K,

$kT = 5.5253\times10^{-21}$ J

$$g_\upsilon = (2.3564\times10^{55})\left[3(5.5253\times10^{-21})\right]^{3/2} = 5.029\times10^{25} \text{ m}^{-3}$$

or $g_\upsilon = 5.03\times10^{19}$ cm $^{-3}$

3.28

(a) $g_c(E) = \dfrac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E-E_c}$

$$= \frac{4\pi\left[2(1.08)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\sqrt{E-E_c}$$

$= 1.1929\times10^{56}\sqrt{E-E_c}$

For $E = E_c$; $g_c = 0$

$E = E_c + 0.1$ eV; $g_c = 1.509\times10^{46}$ m $^{-3}$ J $^{-1}$

$E = E_c + 0.2$ eV; $= 2.134\times10^{46}$ m $^{-3}$ J $^{-1}$

$E = E_c + 0.3$ eV; $= 2.614\times10^{46}$ m $^{-3}$ J $^{-1}$

$E = E_c + 0.4$ eV; $= 3.018\times10^{46}$ m $^{-3}$ J $^{-1}$

(b) $g_\upsilon = \dfrac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_\upsilon - E}$

$$= \frac{4\pi\left[2(0.56)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\sqrt{E_\upsilon - E}$$

$= 4.4541\times10^{55}\sqrt{E_\upsilon - E}$

For $E = E_\upsilon$; $g_\upsilon = 0$

$E = E_\upsilon - 0.1$ eV; $g_\upsilon = 5.634\times10^{45}$ m $^{-3}$ J $^{-1}$

$E = E_\upsilon - 0.2$ eV; $= 7.968\times10^{45}$ m $^{-3}$ J $^{-1}$

$E = E_\upsilon - 0.3$ eV; $= 9.758\times10^{45}$ m $^{-3}$ J $^{-1}$

$E = E_\upsilon - 0.4$ eV; $= 1.127\times10^{46}$ m $^{-3}$ J $^{-1}$

3.29

(a) $\dfrac{g_c}{g_\upsilon} = \dfrac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\dfrac{1.08}{0.56}\right)^{3/2} = 2.68$

(b)

$\dfrac{g_c}{g_\upsilon} = \dfrac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\dfrac{0.067}{0.48}\right)^{3/2} = 0.0521$

3.30

Plot

3.31

(a) $W_i = \dfrac{g_i!}{N_i!\left(g_i - N_i\right)!} = \dfrac{10!}{(7!)(10-7)!}$

$= \dfrac{(10)(9)(8)(7!)}{(7!)(3!)} = \dfrac{(10)(9)(8)}{(3)(2)(1)} = 120$

(b) (i)

$W_i = \dfrac{12!}{(10!)(12-10)!} = \dfrac{(12)(11)(10!)}{(10!)(2)(1)}$

$= 66$

(ii)

$W_i = \dfrac{12!}{(8!)(12-8)!} = \dfrac{(12)(11)(10)(9)(8!)}{(8!)(4)(3)(2)(1)}$

$= 495$

3.32

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

(a) $E - E_F = kT$, $f(E) = \dfrac{1}{1+\exp(1)} \Rightarrow$

$f(E) = 0.269$

(b) $E - E_F = 5kT$,

$f(E) = \dfrac{1}{1+\exp(5)} \Rightarrow$

$f(E) = 6.69\times10^{-3}$

(c) $E - E_F = 10kT$,

$f(E) = \dfrac{1}{1+\exp(10)} \Rightarrow$

$f(E) = 4.54\times10^{-5}$

3.33

$$1 - f(E) = 1 - \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

or

$$1-f(E)=\frac{1}{1+\exp\left(\frac{E_F-E}{kT}\right)}$$

(a) $E_F-E=kT$, $1-f(E)=0.269$

(b) $E_F-E=5kT$, $1-f(E)=6.69\times10^{-3}$

(c) $E_F-E=10kT$,

$1-f(E)=4.54\times10^{-5}$

3.34

(a) $f_F\cong\exp\left[\frac{-(E-E_F)}{kT}\right]$

$E=E_c$;

$f_F=\exp\left[\frac{-0.30}{0.0259}\right]=9.32\times10^{-6}$

$E_c+\frac{kT}{2}$;

$f_F=\exp\left[\frac{-(0.30+0.0259/2)}{0.0259}\right]$

$=5.66\times10^{-6}$

E_c+kT ;

$f_F=\exp\left[\frac{-(0.30+0.0259)}{0.0259}\right]$

$=3.43\times10^{-6}$

$E_c+\frac{3kT}{2}$;

$f_F=\exp\left[\frac{-(0.30+3(0.0259/2))}{0.0259}\right]$

$=2.08\times10^{-6}$

E_c+2kT ;

$f_F=\exp\left[\frac{-(0.30+2(0.0259))}{0.0259}\right]$

$=1.26\times10^{-6}$

(b) $1-f_F=1-\frac{1}{1+\exp\left[\frac{E-E_F}{kT}\right]}$

$\cong\exp\left[\frac{-(E_F-E)}{kT}\right]$

$E=E_\upsilon$; $1-f_F=\exp\left[\frac{-0.25}{0.0259}\right]$

$=6.43\times10^{-5}$

$E_\upsilon-\frac{kT}{2}$;

$1-f_F=\exp\left[\frac{-(0.25+0.0259/2)}{0.0259}\right]$

$=3.90\times10^{-5}$

$E_\upsilon-kT$;

$1-f_F=\exp\left[\frac{-(0.25+0.0259)}{0.0259}\right]$

$=2.36\times10^{-5}$

$E_\upsilon-\frac{3kT}{2}$;

$1-f_F=\exp\left[\frac{-(0.25+3(0.0259/2))}{0.0259}\right]$

$=1.43\times10^{-5}$

$E_\upsilon-2kT$;

$1-f_F=\exp\left[\frac{-(0.25+2(0.0259))}{0.0259}\right]$

$=8.70\times10^{-6}$

3.35

$f_F=\exp\left[\frac{-(E-E_F)}{kT}\right]=\exp\left[\frac{-(E_c+kT-E_F)}{kT}\right]$

and

$1-f_F=\exp\left[\frac{-(E_F-E)}{kT}\right]$

$=\exp\left[\frac{-(E_F-(E_\upsilon-kT))}{kT}\right]$

So $\exp\left[\frac{-(E_c+kT-E_F)}{kT}\right]$

$=\exp\left[\frac{-(E_F-E_\upsilon+kT)}{kT}\right]$

Then $E_c + kT - E_F = E_F - E_v + kT$

Or $E_F = \dfrac{E_c + E_v}{2} = E_{midgap}$

3.36

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

For $n = 6$, Filled state

$$E_6 = \frac{(1.054\times10^{-34})^2 (6)^2 (\pi)^2}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$$= 1.5044\times10^{-18} \text{ J}$$

or $E_6 = \dfrac{1.5044\times10^{-18}}{1.6\times10^{-19}} = 9.40$ eV

For $n = 7$, Empty state

$$E_7 = \frac{(1.054\times10^{-34})^2 (7)^2 (\pi)^2}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$$= 2.048\times10^{-18} \text{ J}$$

or $E_7 = \dfrac{2.048\times10^{-18}}{1.6\times10^{-19}} = 12.8$ eV

Therefore $9.40 < E_F < 12.8$ eV

3.37

(a) For a 3-D infinite potential well

$$\frac{2mE}{\hbar^2} = (n_x^2 + n_y^2 + n_z^2)\left(\frac{\pi}{a}\right)^2$$

For 5 electrons, the 5th electron occupies the quantum state $n_x = 2, n_y = 2, n_z = 1$; so

$$E_5 = \frac{\hbar^2}{2m}(n_x^2 + n_y^2 + n_z^2)\left(\frac{\pi}{a}\right)^2$$

$$= \frac{(1.054\times10^{-34})^2 (\pi)^2 (2^2 + 2^2 + 1^2)}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$$= 3.761\times10^{-19} \text{ J}$$

or $E_5 = \dfrac{3.761\times10^{-19}}{1.6\times10^{-19}} = 2.35$ eV

For the next quantum state, which is empty, the quantum state is $n_x = 1, n_y = 2, n_z = 2$. This quantum state is at the same energy, so

$E_F = 2.35$ eV

(b) For 13 electrons, the 13th electron occupies the quantum state $n_x = 3, n_y = 2, n_z = 3$; so

$$E_{13} = \frac{(1.054\times10^{-34})^2 (\pi)^2 (3^2 + 2^2 + 3^2)}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$$= 9.194\times10^{-19} \text{ J}$$

or $E_{13} = \dfrac{9.194\times10^{-19}}{1.6\times10^{-19}} = 5.746$ eV

The 14th electron would occupy the quantum state $n_x = 2, n_y = 3, n_z = 3$. This state is at the same energy, so

$E_F = 5.746$ eV

3.38

The probability of a state at $E_1 = E_F + \Delta E$ being occupied is

$$f_1(E_1) = \frac{1}{1+\exp\left(\dfrac{E_1 - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2 = E_F - \Delta E$ being empty is

$$1 - f_2(E_2) = 1 - \frac{1}{1+\exp\left(\dfrac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1+\exp\left(\dfrac{-\Delta E}{kT}\right)} = \frac{\exp\left(\dfrac{-\Delta E}{kT}\right)}{1+\exp\left(\dfrac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1+\exp\left(\dfrac{\Delta E}{kT}\right)}$$

so $f_1(E_1) = 1 - f_2(E_2)$ Q.E.D.

3.39

(a) At energy E_1, we want

$$\frac{\dfrac{1}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}{\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}=0.01$$

This expression can be written as

$$\frac{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-1=0.01$$

or

$$1=(0.01)\exp\left(\frac{E_1-E_F}{kT}\right)$$

Then

$$E_1=E_F+kT\ln(100)$$

or

$$E_1=E_F+4.6kT$$

(b)

At $E=E_F+4.6kT$,

$$f(E_1)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}=\frac{1}{1+\exp(4.6)}$$

which yields

$$f(E_1)=0.00990\cong 0.01$$

3.40

(a)

$$f_F=\exp\left[\frac{-(E-E_F)}{kT}\right]=\exp\left[\frac{-(5.80-5.50)}{0.0259}\right]$$

$$=9.32\times 10^{-6}$$

(b) $kT=(0.0259)\left(\dfrac{700}{300}\right)=0.060433$

eV

$$f_F=\exp\left[\frac{-0.30}{0.060433}\right]=6.98\times 10^{-3}$$

(c) $1-f_F\cong\exp\left[\dfrac{-(E_F-E)}{kT}\right]$

$$0.02=\exp\left[\frac{-0.25}{kT}\right]$$

or $\exp\left[\dfrac{+0.25}{kT}\right]=\dfrac{1}{0.02}=50$

$$\frac{0.25}{kT}=\ln(50)$$

or

$$kT=\frac{0.25}{\ln(50)}=0.063906=(0.0259)\left(\frac{T}{300}\right)$$

which yields $T=740$ K

3.41

(a)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.0259}\right)}=0.00304$$

or 0.304%

(b) At $T=1000$ K, $kT=0.08633$ eV

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.08633}\right)}=0.1496$$

or 14.96%

(c)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.85-7.0}{0.0259}\right)}=0.997$$

or 99.7%

(d)

At $E=E_F$, $f(E)=\dfrac{1}{2}$ for all temperatures

3.42

(a) For $E=E_1$

$$f(E)=\frac{1}{1+\exp\left(\frac{E_1-E_F}{kT}\right)}\cong\exp\left[\frac{-(E_1-E_F)}{kT}\right]$$

Then

$$f(E_1)=\exp\left(\frac{-0.30}{0.0259}\right)=9.32\times10^{-6}$$

For $E=E_2$,

$E_F-E_2=1.12-0.30=0.82$ eV

Then

$$1-f(E)=1-\frac{1}{1+\exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1-f(E)\cong1-\left[1-\exp\left(\frac{-0.82}{0.0259}\right)\right]$$

$$=\exp\left(\frac{-0.82}{0.0259}\right)=1.78\times10^{-14}$$

(b) For $E_F-E_2=0.4$ eV,

$E_1-E_F=0.72$ eV

At $E=E_1$,

$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-0.72}{0.0259}\right)$$

or

$$f(E)=8.45\times10^{-13}$$

At $E=E_2$,

$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$

$$=\exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$1-f(E)=1.96\times10^{-7}$$

3.43

(a) At $E=E_1$

$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$f(E)=9.32\times10^{-6}$$

At $E=E_2$,

$E_F-E_2=1.42-0.3=1.12$ eV

So

$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$

$$=\exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$1-f(E)=1.66\times10^{-19}$$

(b) For $E_F-E_2=0.4$,

$E_1-E_F=1.02$ eV

At $E=E_1$,

$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-1.02}{0.0259}\right)$$

or

$$f(E)=7.88\times10^{-18}$$

At $E=E_2$,

$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$

$$=\exp\left(\frac{-0.4}{0.0259}\right)$$

or $1-f(E)=1.96\times10^{-7}$

3.44

$$f(E)=\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^{-1}$$

so

$$\frac{df(E)}{dE}=(-1)\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^{-2}$$

$$\times\left(\frac{1}{kT}\right)\exp\left(\frac{E-E_F}{kT}\right)$$

or

$$\frac{df(E)}{dE} = \frac{\left(\frac{-1}{kT}\right)\exp\left(\frac{E-E_F}{kT}\right)}{\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^2}$$

(a) At $T = 0$ K, For

$E < E_F \Rightarrow \exp(-\infty) = 0 \Rightarrow \frac{df}{dE} = 0$

$E > E_F \Rightarrow \exp(+\infty) = +\infty \Rightarrow \frac{df}{dE} = 0$

At $E = E_F \Rightarrow \frac{df}{dE} = -\infty$

(b) At $T = 300$ K, $kT = 0.0259$ eV

For $E << E_F$, $\frac{df}{dE} = 0$

For $E >> E_F$, $\frac{df}{dE} = 0$

At $E = E_F$,

$$\frac{df}{dE} = \frac{\left(\frac{-1}{0.0259}\right)(1)}{(1+1)^2} = -9.65$$

(eV) $^{-1}$

(c) At $T = 500$ K, $kT = 0.04317$ eV

For $E << E_F$, $\frac{df}{dE} = 0$

For $E >> E_F$, $\frac{df}{dE} = 0$

At $E = E_F$,

$$\frac{df}{dE} = \frac{\left(\frac{-1}{0.04317}\right)(1)}{(1+1)^2} = -5.79 \text{ (eV)}^{-1}$$

3.45

(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12$ eV,

$$f(E) = \frac{1}{1+\exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$f(E) = 4.07\times10^{-10}$

Ge: $E_g = 0.66$ eV

$$f(E) = \frac{1}{1+\exp\left[\frac{0.66}{2(0.0259)}\right]}$$

or

$f(E) = 2.93\times10^{-6}$

GaAs: $E_g = 1.42$ eV

$$f(E) = \frac{1}{1+\exp\left[\frac{1.42}{2(0.0259)}\right]}$$

or

$f(E) = 1.24\times10^{-12}$

(b) Using the results of Problem 3.38, the answers to part (b) are exactly the same as those given in part (a).

3.46

(a) $f_F = \exp\left[\frac{-(E-E_F)}{kT}\right]$

$10^{-8} = \exp\left[\frac{-0.60}{kT}\right]$

or $\frac{0.60}{kT} = \ln(10^{+8})$

$$kT = \frac{0.60}{\ln(10^8)} = 0.032572 \text{ eV}$$

$$0.032572 = (0.0259)\left(\frac{T}{300}\right)$$

so $T = 377$ K

(b) $10^{-6} = \exp\left[\frac{-0.60}{kT}\right]$

$$\frac{0.60}{kT} = \ln\left(10^{+6}\right)$$

$$kT = \frac{0.60}{\ln\left(10^{6}\right)} = 0.043429$$

$$0.043429 = (0.0259)\left(\frac{T}{300}\right)$$

or $T = 503$ K

3.47

(a) At $T = 200$ K,

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267 \text{ eV}$$

$$f_F = 0.05 = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$\exp\left(\frac{E - E_F}{kT}\right) = \frac{1}{0.05} - 1 = 19$$

$$E - E_F = kT \ln(19) = (0.017267)\ln(19)$$
$$= 0.05084 \text{ eV}$$

By symmetry, for $f_F = 0.95$,

$$E - E_F = -0.05084 \text{ eV}$$

Then $\Delta E = 2(0.05084) = 0.1017$ eV

(b) $T = 400$ K, $kT = 0.034533$ eV

For $f_F = 0.05$, from part (a),

$$E - E_F = kT \ln(19) = (0.034533)\ln(19)$$
$$= 0.10168 \text{ eV}$$

Then $\Delta E = 2(0.10168) = 0.2034$ eV