Chapter 2

2.1
Sketch

2.2
Sketch

2.3
Sketch

2.4
From Problem 2.2, phase $= \dfrac{2\pi x}{\lambda} - \omega t$
= constant

Then

$$\frac{2\pi}{\lambda}\cdot\frac{dx}{dt} - \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = +\omega\left(\frac{\lambda}{2\pi}\right)$$

From Problem 2.3, phase $= \dfrac{2\pi x}{\lambda} + \omega t$
= constant

Then

$$\frac{2\pi}{\lambda}\cdot\frac{dx}{dt} + \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = -\omega\left(\frac{\lambda}{2\pi}\right)$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90$ eV
$= (4.90)(1.6\times10^{-19})$ J
So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(4.90)(1.6\times10^{-19})} = 2.54\times10^{-5}$$

cm
or
$\lambda = 0.254\ \mu$m
Cesium: $E = 1.90$ eV
$= (1.90)(1.6\times10^{-19})$ J
So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(1.90)(1.6\times10^{-19})} = 6.54\times10^{-5}$$

cm
or
$\lambda = 0.654\ \mu$m

2.6
(a) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{550\times10^{-9}}$
$= 1.205\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^{3} \text{ m/s}$$

or $\upsilon = 1.32\times10^{5}$ cm/s
(b) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{440\times10^{-9}}$
$= 1.506\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^{3} \text{ m/s}$$

or $\upsilon = 1.65\times10^{5}$ cm/s
(c) Yes

2.7
(a) (i)

$$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.2)(1.6\times10^{-19})}$$
$$= 5.915\times10^{-25} \text{ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{5.915\times10^{-25}} = 1.12\times10^{-9} \text{ m}$$

or $\lambda = 11.2\ \mathring{A}$
(ii)

$$p = \sqrt{2(9.11\times10^{-31})(12)(1.6\times10^{-19})}$$
$$= 1.87\times10^{-24} \text{ kg-m/s}$$

$$\lambda = \frac{6.625\times10^{-34}}{1.8704\times10^{-24}} = 3.54\times10^{-10}$$
m

or $\lambda = 3.54\ \overset{o}{A}$

(iii)

$$p = \sqrt{2\left(9.11\times10^{-31}\right)(120)\left(1.6\times10^{-19}\right)}$$
$$= 5.915\times10^{-24} \text{ kg-m/s}$$
$$\lambda = \frac{6.625\times10^{-34}}{5.915\times10^{-24}} = 1.12\times10^{-10} \text{ m}$$

or $\lambda = 1.12\ \overset{o}{A}$

(b)

$$p = \sqrt{2\left(1.67\times10^{-27}\right)(1.2)\left(1.6\times10^{-19}\right)}$$
$$= 2.532\times10^{-23} \text{ kg-m/s}$$
$$\lambda = \frac{6.625\times10^{-34}}{2.532\times10^{-23}} = 2.62\times10^{-11}$$
m

or $\lambda = 0.262\ \overset{o}{A}$

2.8

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885 \text{ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$
$$= \sqrt{2\left(9.11\times10^{-31}\right)(0.03885)\left(1.6\times10^{-19}\right)}$$

or

$$p_{avg} = 1.064\times10^{-25} \text{ kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{1.064\times10^{-25}} = 6.225\times10^{-9} \text{ m}$$

or

$$\lambda = 62.25\ \overset{o}{A}$$

2.9

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and}$$
$$p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$
$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$
$$= \frac{2\left(9.11\times10^{-31}\right)\left(3\times10^8\right)^2}{100}$$
$$= 1.64\times10^{-15} \text{ J} = 10.25 \text{ keV}$$

2.10

(a) $$p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{85\times10^{-10}}$$
$$= 7.794\times10^{-26} \text{ kg-m/s}$$

$$\upsilon = \frac{p}{m} = \frac{7.794\times10^{-26}}{9.11\times10^{-31}} = 8.56\times10^4 \text{ m/s}$$

or $\upsilon = 8.56\times10^6$ cm/s

$$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\left(8.56\times10^4\right)^2$$
$$= 3.33\times10^{-21} \text{ J}$$

or $$E = \frac{3.334\times10^{-21}}{1.6\times10^{-19}} = 2.08\times10^{-2}$$
eV

(b) $$E = \frac{1}{2}\left(9.11\times10^{-31}\right)\left(8\times10^3\right)^2$$
$$= 2.915\times10^{-23} \text{ J}$$

or $E = \dfrac{2.915\times10^{-23}}{1.6\times10^{-19}} = 1.82\times10^{-4}$ eV

$$p = m\upsilon = \left(9.11\times10^{-31}\right)\left(8\times10^{3}\right)$$
$$= 7.288\times10^{-27} \text{ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-35}}{7.288\times10^{-27}} - 9.09\times10^{-8} \text{ m}$$

or $\lambda = 909 \overset{o}{A}$

2.11

(a)

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625\times10^{-34}\right)\left(3\times10^{8}\right)}{1\times10^{-10}}$$
$$= 1.99\times10^{-15} \text{ J}$$

Now

$$E = e\cdot V \Rightarrow V = \frac{E}{e} = \frac{1.99\times10^{-15}}{1.6\times10^{-19}}$$
$$V = 1.24\times10^{4} \text{ V} = 12.4 \text{ kV}$$

(b)

$$p = \sqrt{2mE} = \sqrt{2\left(9.11\times10^{-31}\right)\left(1.99\times10^{-15}\right)}$$
$$= 6.02\times10^{-23} \text{ kg-m/s}$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{6.02\times10^{-23}} = 1.10\times10^{-11} \text{ m}$$

or

$$\lambda = 0.11 \overset{o}{A}$$

2.12

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-6}}$$
$$= 1.054\times10^{-28} \text{ kg-m/s}$$

2.13

(a) (i) $\Delta p \Delta x = \hbar$

$$\Delta p = \frac{1.054\times10^{-34}}{12\times10^{-10}} = 8.783\times10^{-26} \text{ kg-m/s}$$

(ii) $\Delta E = \dfrac{dE}{dp}\cdot\Delta p = \dfrac{d}{dp}\left(\dfrac{p^2}{2m}\right)\cdot\Delta p$

$$= \frac{2p}{2m}\cdot\Delta p = \frac{p\Delta p}{m}$$

Now $p = \sqrt{2mE}$

$$= \sqrt{2\left(9\times10^{-31}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 2.147\times10^{-24} \text{ kg-m/s}$$

so

$$\Delta E = \frac{\left(2.1466\times10^{-24}\right)\left(8.783\times10^{-26}\right)}{9\times10^{-31}}$$
$$= 2.095\times10^{-19} \text{ J}$$

or $\Delta E = \dfrac{2.095\times10^{-19}}{1.6\times10^{-19}} = 1.31$ eV

(b) (i) $\Delta p = 8.783\times10^{-26}$ kg-m/s

(ii)

$$p = \sqrt{2\left(5\times10^{-28}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 5.06\times10^{-23} \text{ kg-m/s}$$

$$\Delta E = \frac{\left(5.06\times10^{-23}\right)\left(8.783\times10^{-26}\right)}{5\times10^{-28}}$$
$$= 8.888\times10^{-21} \text{ J}$$

or $\Delta E = \dfrac{8.888\times10^{-21}}{1.6\times10^{-19}} = 5.55\times10^{-2}$ eV

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-2}} = 1.054\times10^{-32} \text{ kg-m/s}$$

$$p = m\upsilon \Rightarrow \Delta\upsilon = \frac{\Delta p}{m} = \frac{1.054\times10^{-32}}{1500}$$
$$\Delta\upsilon = 7\times10^{-36} \text{ m/s}$$

2.15

(a) $\Delta E \Delta t = \hbar$

$$\Delta t = \frac{1.054\times10^{-34}}{(0.8)(1.6\times10^{-19})} = 8.23\times10^{-16}$$ s

(b) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{1.5\times10^{-10}}$
$= 7.03\times10^{-25}$ kg-m/s

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}[\Psi_1(x,t)+\Psi_2(x,t)] + V(x)[\Psi_1(x,t)+\Psi_2(x,t)]$$

$$= j\hbar\frac{\partial}{\partial t}[\Psi_1(x,t)+\Psi_2(x,t)]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t)+\Psi_2(x,t)$ is also a solution.

(b) If $\Psi_1(x,t)\cdot\Psi_2(x,t)$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}[\Psi_1\cdot\Psi_2] + V(x)[\Psi_1\cdot\Psi_2]$$

$$= j\hbar\frac{\partial}{\partial t}[\Psi_1\cdot\Psi_2]$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1\frac{\partial^2\Psi_2}{\partial x^2} + \Psi_2\frac{\partial^2\Psi_1}{\partial x^2} + 2\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}\right]$$

$$+V(x)[\Psi_1\cdot\Psi_2] = j\hbar\left[\Psi_1\frac{\partial\Psi_2}{\partial t} + \Psi_2\frac{\partial\Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1\cdot\Psi_2$, we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + \frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$+V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial\Psi_1}{\partial t}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + V(x) = j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we have

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$= j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we have

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + V(x) = j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x} - V(x) = 0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\int_{-1}^{+3} A^2\cos^2\left(\frac{\pi x}{2}\right)dx = 1$$

$$A^2\left[\frac{x}{2} + \frac{\sin(\pi x)}{2\pi}\right]\Bigg|_{-1}^{+3} = 1$$

$$A^2\left[\frac{3}{2} - \left(\frac{-1}{2}\right)\right] = 1$$

so $A^2 = \frac{1}{2}$

or $|A| = \dfrac{1}{\sqrt{2}}$

2.18

$$\int_{-1/2}^{+1/2} A^2 \cos^2(n\pi x)\,dx = 1$$

$$A^2\left[\frac{x}{2} + \frac{\sin(2n\pi x)}{4n\pi}\right]\Bigg|_{-1/2}^{+1/2} = 1$$

$$A^2\left[\frac{1}{4} - \left(-\frac{1}{4}\right)\right] = 1 = A^2\left(\frac{1}{2}\right)$$

or $|A| = \sqrt{2}$

2.19

Note that $\int_0^\infty \Psi \cdot \Psi^* dx = 1$

Function has been normalized.

(a) Now

$$P = \int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$= \frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$= \frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P = (-1)\left[\exp\left(\frac{-2a_o}{4a_o}\right) - 1\right] = 1 - \exp\left(\frac{-1}{2}\right)$$

which yields

$P = 0.393$

(b)

$$P = \int_{a_o/4}^{a_o/2}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$= \frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$= \frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P = (-1)\left[\exp(-1) - \exp\left(\frac{-1}{2}\right)\right]$$

which yields

$P = 0.239$

(c)

$$P = \int_0^{a_o}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$= \frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$= \frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

$$= (-1)[\exp(-2) - 1]$$

which yields

$P = 0.865$

2.20

$$P = \int |\psi(x)|^2\,dx$$

(a) $$\int_0^{a/4}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{2}\right)dx$$

$$= \left(\frac{2}{a}\right)\left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_0^{a/4}$$

$$= \left(\frac{2}{a}\right)\left[\frac{\left(\frac{a}{4}\right)}{2} + \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$= \left(\frac{2}{a}\right)\left[\frac{a}{8} + \frac{(1)(a)}{4\pi}\right]$$

or $P = 0.409$

(b) $$P = \int_{a/4}^{a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\frac{a}{8}-\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$=2\left[\frac{1}{4}+0-\frac{1}{8}-\frac{1}{4\pi}\right]$$

or $P=0.0908$

(c) $P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\left(\frac{-a}{4}\right)-\frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)}\right]$$

or $P=1$

2.21

(a) $P=\int_{0}^{a/4}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{0}^{a/4}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{8}-\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(b) $P=\int_{a/4}^{a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{a}{8}\right)+\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(c) $P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{-a}{4}\right)+\frac{\sin(-2\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=1$

2.22

(a) (i) $\upsilon_p=\frac{\omega}{k}=\frac{8\times10^{12}}{8\times10^{8}}=10^4$ m/s

or $\upsilon_p=10^6$ cm/s

$$\lambda=\frac{2\pi}{k}=\frac{2\pi}{8\times10^8}=7.854\times10^{-9}\text{ m}$$

or $\lambda=78.54\ \overset{o}{A}$

(ii) $p=m\upsilon=\left(9.11\times10^{-31}\right)\left(10^4\right)$

$=9.11\times10^{-27}$ kg-m/s

$E=\frac{1}{2}m\upsilon^2=\frac{1}{2}\left(9.11\times10^{-31}\right)\left(10^4\right)^2$

$=4.555\times10^{-23}$ J

or $E=\frac{4.555\times10^{-23}}{1.6\times10^{-19}}=2.85\times10^{-4}$ eV

(b) (i) $\upsilon_p=\frac{\omega}{k}=\frac{1.5\times10^{13}}{-1.5\times10^{9}}=-10^4$ m/s or $\upsilon_p = -10^6$ cm/s

$$\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{1.5\times10^9} = 4.19\times10^{-9}$$
m

or $\lambda = 41.9\ \overset{o}{A}$

(ii) $p = -9.11\times10^{-27}$ kg-m/s

$E = 2.85\times10^{-4}$ eV

2.23

(a) $\Psi(x,t) = Ae^{-j(kx+\omega t)}$

(b) $E = (0.025)(1.6\times10^{-19}) = \frac{1}{2}m\upsilon^2$

$$= \frac{1}{2}(9.11\times10^{-31})\upsilon^2$$

so $|\upsilon| = 9.37\times10^4$ m/s $= 9.37\times10^6$ cm/s

For electron traveling in $-x$ direction,

$\upsilon = -9.37\times10^6$ cm/s

$$p = m\upsilon = (9.11\times10^{-31})(-9.37\times10^4)$$
$$= -8.537\times10^{-26} \text{ kg-m/s}$$

$$\lambda = \frac{h}{|p|} = \frac{6.625\times10^{-34}}{8.537\times10^{-26}} = 7.76\times10^{-9}$$
m

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.76\times10^{-9}} = 8.097\times10^8 \text{ m}^{-1}$$

$$\omega = k\cdot|\upsilon| = (8.097\times10^8)(9.37\times10^4)$$

or $\omega = 7.586\times10^{13}$ rad/s

2.24

(a) $p = m\upsilon = (9.11\times10^{-31})(5\times10^4)$

$= 4.555\times10^{-26}$ kg-m/s

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{4.555\times10^{-26}} = 1.454\times10^{-8}$$
m

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.454\times10^{-8}} = 4.32\times10^8 \text{ m}^{-1}$$

$$\omega = k\upsilon = (4.32\times10^8)(5\times10^4)$$
$$= 2.16\times10^{13} \text{ rad/s}$$

(b) $p = (9.11\times10^{-31})(10^6)$

$= 9.11\times10^{-25}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{9.11\times10^{-25}} = 7.27\times10^{-10}$$
m

$$k = \frac{2\pi}{7.272\times10^{-10}} = 8.64\times10^9 \text{ m}^{-1}$$

$$\omega = (8.64\times10^9)(10^6) = 8.64\times10^{15}$$
rad/s

2.25

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2(1.054\times10^{-34})^2\pi^2}{2(9.11\times10^{-31})(75\times10^{-10})^2}$$

$E_n = n^2(1.0698\times10^{-21})$ J

or

$$E_n = \frac{n^2(1.0698\times10^{-21})}{1.6\times10^{-19}}$$

or $E_n = n^2(6.686\times10^{-3})$ eV

Then

$E_1 = 6.69\times10^{-3}$ eV

$E_2 = 2.67\times10^{-2}$ eV

$E_3 = 6.02\times10^{-2}$ eV

2.26

(a)

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2(1.054\times10^{-34})^2\pi^2}{2(9.11\times10^{-31})(10\times10^{-10})^2}$$
$$= n^2(6.018\times10^{-20}) \text{ J}$$

or

$$E_n = \frac{n^2(6.018\times10^{-20})}{1.6\times10^{-19}} = n^2(0.3761) \text{ eV}$$

Then

$E_1 = 0.376$ eV
$E_2 = 1.504$ eV
$E_3 = 3.385$ eV

(b) $\lambda = \dfrac{hc}{\Delta E}$

$\Delta E = (3.385 - 1.504)(1.6\times 10^{-19})$
$= 3.01\times 10^{-19}$ J

$$\lambda = \frac{(6.625\times 10^{-34})(3\times 10^{8})}{3.01\times 10^{-19}}$$
$= 6.604\times 10^{-7}$ m

or $\lambda = 660.4$ nm

2.27

(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$15\times 10^{-3} = \frac{n^2(1.054\times 10^{-34})^2 \pi^2}{2(15\times 10^{-3})(1.2\times 10^{-2})^2}$$
$15\times 10^{-3} = n^2(2.538\times 10^{-62})$

or $n = 7.688\times 10^{29}$

(b) $E_{n+1} \cong 15$ mJ

(c) No

2.28

For a neutron and $n=1$:

$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{(1.054\times 10^{-34})^2\pi^2}{2(1.66\times 10^{-27})(10^{-14})^2}$$
$= 3.3025\times 10^{-13}$ J

or

$$E_1 = \frac{3.3025\times 10^{-13}}{1.6\times 10^{-19}} = 2.06\times 10^{6} \text{ eV}$$

For an electron in the same potential well:

$$E_1 = \frac{(1.054\times 10^{-34})^2\pi^2}{2(9.11\times 10^{-31})(10^{-14})^2}$$
$= 6.0177\times 10^{-10}$ J

or

$$E_1 = \frac{6.0177\times 10^{-10}}{1.6\times 10^{-19}} = 3.76\times 10^{9} \text{ eV}$$

2.29

Schrodinger's time-independent wave equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

We know that

$\psi(x) = 0$ for $x \ge \dfrac{a}{2}$ and $x \le \dfrac{-a}{2}$

We have

$V(x) = 0$ for $\dfrac{-a}{2} < x < \dfrac{+a}{2}$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

The solution is of the form

$\psi(x) = A\cos kx + B\sin kx$

where

$k = \sqrt{\dfrac{2mE}{\hbar^2}}$

Boundary conditions:

$\psi(x) = 0$ at $x = \dfrac{-a}{2}$, $x = \dfrac{+a}{2}$

First mode solution:

$\psi_1(x) = A_1 \cos k_1 x$

where

$k_1 = \dfrac{\pi}{a} \Rightarrow E_1 = \dfrac{\pi^2\hbar^2}{2ma^2}$

Second mode solution:

$\psi_2(x) = B_2 \sin k_2 x$

where

$k_2 = \dfrac{2\pi}{a} \Rightarrow E_2 = \dfrac{4\pi^2\hbar^2}{2ma^2}$

Third mode solution:

$\psi_3(x) = A_3 \cos k_3 x$

where

$k_3 = \dfrac{3\pi}{a} \Rightarrow E_3 = \dfrac{9\pi^2\hbar^2}{2ma^2}$

Fourth mode solution:

$\psi_4(x) = B_4 \sin k_4 x$

where

$k_4 = \dfrac{4\pi}{a} \Rightarrow E_4 = \dfrac{16\pi^2\hbar^2}{2ma^2}$

2.30

The 3-D time-independent wave equation in cartesian coordinates for $V(x, y, z) = 0$ is:

$$\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2}$$

$$+ \frac{2mE}{\hbar^2} \psi(x, y, z) = 0$$

Use separation of variables, so let

$$\psi(x, y, z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we obtain

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2}$$

$$+ \frac{2mE}{\hbar^2} XYZ = 0$$

Dividing by XYZ and letting $k^2 = \frac{2mE}{\hbar^2}$,

we

find

(1)

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

We may set

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form

$$X(x) = A \sin(k_x x) + B \cos(k_x x)$$

Boundary conditions: $X(0) = 0 \Rightarrow B = 0$

and $X(x = a) = 0 \Rightarrow k_x = \frac{n_x \pi}{a}$

where $n_x = 1, 2, 3 \ldots.$

Similarly, let

$$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

Applying the boundary conditions, we find

$$k_y = \frac{n_y \pi}{a}, \quad n_y = 1, 2, 3 \ldots.$$

$$k_z = \frac{n_z \pi}{a}, \quad n_z = 1, 2, 3 \ldots$$

From Equation (1) above, we have

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

or

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$$

so that

$$E \rightarrow E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2} \left(n_x^2 + n_y^2 + n_z^2 \right)$$

2.31

(a)

$$\frac{\partial^2 \psi(x, y)}{\partial x^2} + \frac{\partial^2 \psi(x, y)}{\partial y^2} + \frac{2mE}{\hbar^2} \cdot \psi(x, y) = 0$$

Solution is of the form:

$$\psi(x, y) = A \sin k_x x \cdot \sin k_y y$$

We find

$$\frac{\partial \psi(x, y)}{\partial x} = Ak_x \cos k_x x \cdot \sin k_y y$$

$$\frac{\partial^2 \psi(x, y)}{\partial x^2} = -Ak_x^2 \sin k_x x \cdot \sin k_y y$$

$$\frac{\partial \psi(x, y)}{\partial y} = Ak_y \sin k_x x \cdot \cos k_y y$$

$$\frac{\partial^2 \psi(x, y)}{\partial y^2} = -Ak_y^2 \sin k_x x \cdot \sin k_y y$$

Substituting into the original equation, we find:

(1) $$-k_x^2 - k_y^2 + \frac{2mE}{\hbar^2} = 0$$

From the boundary conditions,

$A \sin k_x a = 0$, where $a = 40 \overset{o}{A}$

So $k_x = \frac{n_x \pi}{a}, \quad n_x = 1, 2, 3, \ldots$

Also $A \sin k_y b = 0$, where $b = 20 \overset{o}{A}$

So $k_y = \frac{n_y \pi}{b}, \quad n_y = 1, 2, 3, \ldots$

Substituting into Eq. (1) above

$$E_{n_x n_y} = \frac{\hbar^2}{2m} \left(\frac{n_x^2 \pi^2}{a^2} + \frac{n_y^2 \pi^2}{b^2} \right)$$

(b)Energy is quantized - similar to 1-D result. There can be more than one quantum state per given energy - different than 1-D result.

2.32

(a) Derivation of energy levels exactly the same as in the text

(b) $\Delta E = \dfrac{\hbar^2\pi^2}{2ma^2}\left(n_2^2 - n_1^2\right)$

For $n_2 = 2, n_1 = 1$

Then

$$\Delta E = \frac{3\hbar^2\pi^2}{2ma^2}$$

(i) For $a = 4\overset{o}{A}$

$$\Delta E = \frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(4\times10^{-10}\right)^2}$$
$$= 6.155\times10^{-22} \text{ J}$$

or

$$\Delta E = \frac{6.155\times10^{-22}}{1.6\times10^{-19}} = 3.85\times10^{-3} \text{ eV}$$

(ii) For $a = 0.5$ cm

$$\Delta E = \frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(0.5\times10^{-2}\right)^2}$$
$$= 3.939\times10^{-36} \text{ J}$$

or

$$\Delta E = \frac{3.939\times10^{-36}}{1.6\times10^{-19}} = 2.46\times10^{-17} \text{ eV}$$

2.33

(a) For region II, $x > 0$

$$\frac{\partial^2\psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - V_O\right)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp\left(jk_2x\right) + B_2 \exp\left(-jk_2x\right)$$

where

$$k_2 = \sqrt{\frac{2m}{\hbar^2}\left(E - V_O\right)}$$

Term with B_2 represents incident wave and term with A_2 represents reflected wave.

Region I, $x < 0$

$$\frac{\partial^2\psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

General form of the solution is

$$\psi_1(x) = A_1 \exp\left(jk_1x\right) + B_1 \exp\left(-jk_1x\right)$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave and the term involving A_1 represents reflected wave: but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1 \exp\left(-jk_1x\right)$$

$$\psi_2(x) = A_2 \exp\left(jk_2x\right) + B_2 \exp\left(-jk_2x\right)$$

(b)

Boundary conditions:

(1) $\psi_1(x=0) = \psi_2(x=0)$

(2) $\left.\dfrac{\partial\psi_1}{\partial x}\right|_{x=0} = \left.\dfrac{\partial\psi_2}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$
$$k_2A_2 - k_2B_2 = -k_1B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\cdot B_2$$

$$B_1 = \left(\frac{2k_2}{k_2 + k_1}\right)\cdot B_2$$

The reflection coefficient is

$$R = \frac{A_2A_2^*}{B_2B_2^*} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4k_1k_2}{(k_1+k_2)^2}$$

2.34

$$\psi_2(x) = A_2 \exp(-k_2 x)$$

$$P = \frac{|\psi(x)|^2}{A_2 A_2^*} = \exp(-2k_2 x)$$

where $k_2 = \sqrt{\dfrac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2(9.11\times10^{-31})(3.5-2.8)(1.6\times10^{-19})}}{1.054\times10^{-34}}$$

$$k_2 = 4.286\times10^9 \text{ m}^{-1}$$

(a) For $x = 5\overset{o}{A} = 5\times10^{-10}$ m

$$P = \exp(-2k_2 x)$$

$$= \exp[-2(4.2859\times10^9)(5\times10^{-10})]$$

$$= 0.0138$$

(b) For $x = 15\overset{o}{A} = 15\times10^{-10}$ m

$$P = \exp[-2(4.2859\times10^9)(15\times10^{-10})]$$

$$= 2.61\times10^{-6}$$

(c) For $x = 40\overset{o}{A} = 40\times10^{-10}$ m

$$P = \exp[-2(4.2859\times10^9)(40\times10^{-10})]$$

$$= 1.29\times10^{-15}$$

2.35

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1-\frac{E}{V_o}\right)\exp(-2k_2 a)$$

where $k_2 = \sqrt{\dfrac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2(9.11\times10^{-31})(1.0-0.1)(1.6\times10^{-19})}}{1.054\times10^{-34}}$$

or $k_2 = 4.860\times10^9 \text{ m}^{-1}$

(a) For $a = 4\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp[-2(4.85976\times10^9)(4\times10^{-10})]$$

$$= 0.0295$$

(b) For $a = 12\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp[-2(4.85976\times10^9)(12\times10^{-10}$$

$$= 1.24\times10^{-5}$$

(c) $J = N_t e \upsilon$, where N_t is the density of transmitted electrons.

$$E = 0.1 \text{ eV} = 1.6\times10^{-20} \text{ J}$$

$$= \frac{1}{2}m\upsilon^2 = \frac{1}{2}(9.11\times10^{-31})\upsilon^2$$

$$\Rightarrow \upsilon = 1.874\times10^5 \text{ m/s}$$

$$= 1.874\times10^7 \text{ cm/s}$$

$$1.2\times10^{-3} = N_t(1.6\times10^{-19})(1.874\times10^7)$$

$$N_t = 4.002\times10^8 \text{ electrons/cm}^3$$

Density of incident electrons,

$$N_i = \frac{4.002\times10^8}{0.0295} = 1.357\times10^{10} \text{ cm}^{-3}$$

2.36

$$T \cong 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp(-2k_2 a)$$

(a) For $m = (0.067)m_o$

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)(9.11\times10^{-31})(0.8-0.2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}\right\}^{1/2}$$

or

$$k_2 = 1.027\times10^9 \text{ m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$

$$\times\exp[-2(1.027\times10^9)(15\times10^{-10})]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$

$k_2 =$

$$\left\{\frac{2(1.08)(9.11\times10^{-31})(0.8-0.2)(1.6\times10^{-19}}{(1.054\times10^{-34})^2}\right.$$

or

$k_2 = 4.124\times10^{9}\ \text{m}^{-1}$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$

$$\times\exp\left[-2(4.124\times10^{9})(15\times10^{-10})\right]$$

or

$T = 1.27\times10^{-5}$

2.37

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1-\frac{E}{V_o}\right)\exp(-2k_2 a)$$

where $k_2 = \sqrt{\dfrac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2(1.67\times10^{-27})(12-1)\times10^{6}\times(1.6\times10^{-1}}}{1.054\times10^{-34}}$$

$= 7.274\times10^{14}\ \text{m}^{-1}$

(a)

$$T \cong 16\left(\frac{1}{12}\right)\left(1-\frac{1}{12}\right)\exp\left[-2(7.274\times10^{14})(1\right.$$

$= 1.222\exp[-14.548]$

$= 5.875\times10^{-7}$

(b)

$T = (10)(5.875\times10^{-7})$

$$= 1.222\exp\left[-2(7.274\times10^{14})a\right]$$

$$2(7.274\times10^{14})a = \ln\left(\frac{1.222}{5.875\times10^{-6}}\right)$$

or $a = 0.842\times10^{-14}$ m

2.38

Region I $(x<0)$, $V=0$;
Region II $(0<x<a)$, $V=V_O$
Region III $(x>a)$, $V=0$

(a) Region I:

$$\psi_1(x) = A_1\exp(jk_1x) + B_1\exp(-jk_1x)$$

(incident) (reflected)

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:

$$\psi_2(x) = A_2\exp(k_2x) + B_2\exp(-k_2x)$$

where

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

Region III:

$$\psi_3(x) = A_3\exp(jk_1x) + B_3\exp(-jk_1x)$$

(b)

In Region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into Region III, there will not be a reflected wave so that $B_3 = 0$.

(c) Boundary conditions:

At $x=0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow$$

$$jk_1A_1 - jk_1B_1 = k_2A_2 - k_2B_2$$

At $x=a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\exp(k_2a) + B_2\exp(-k_2a)$$

$$= A_3\exp(jk_1a)$$

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$k_2A_2\exp(k_2a) - k_2B_2\exp(-k_2a)$$

$= jk_1 A_3 \exp(jk_1 a)$

The transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1.

Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4k_1 k_2}\left\{\left(k_2^2 - k_1^2\right)\left[\exp(k_2 a) - \exp(-k_2 a)\right]\right.$$

$$\left. -2jk_1 k_2\left[\exp(k_2 a) + \exp(-k_2 a)\right]\right\} \times \exp(jk_1 a)$$

We then find

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2}\left\{\left(k_2^2 - k_1^2\right)\left[\exp(k_2 a) - \exp(-k_2 a)\right]^2\right.$$

$$\left. +4k_1^2 k_2^2\left[\exp(k_2 a) + \exp(-k_2 a)\right]^2\right\}$$

We have

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

If we assume that $V_O >> E$, then $k_2 a$ will be large so that

$$\exp(k_2 a) >> \exp(-k_2 a)$$

We can then write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2}\left\{\left(k_2^2 - k_1^2\right)\left[\exp(k_2 a)\right]^2\right.$$

$$\left. +4k_1^2 k_2^2\left[\exp(k_2 a)\right]^2\right\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2}\left(k_2^2 + k_1^2\right)\exp(2k_2 a)$$

Substituting the expressions for k_1 and k_2, we find

$$k_1^2 + k_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$k_1^2 k_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O - E)(E)$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O)\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^* \left(\frac{2mV_O}{\hbar^2}\right)^2 \exp(2k_2 a)}{16\left[\left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)}$$

Finally,

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)$$

2.39

Region I: $V = 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0 \Rightarrow$$

$$\psi_1(x) = \underbrace{A_1 \exp(jk_1 x)}_{\text{incident}} + \underbrace{B_1 \exp(-jk_1 x)}_{\text{reflected}}$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: $V = V_1$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2(x) = 0 \Rightarrow$$

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

transmitted reflected

where

$$k_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$$

Region III: $V = V_2$

$$\frac{\partial^2 \psi_3(x)}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3(x) = 0 \Rightarrow$$

$$\psi_3(x) = A_3 \exp(jk_3 x)$$

transmitted

where

$$k_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$$

There is no reflected wave in Region III.
The transmission coefficient is defined as:

$$T = \frac{\upsilon_3}{\upsilon_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{k_3}{k_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From the boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow$$

$$k_1 A_1 - k_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jk_2 a) + B_2 \exp(-jk_2 a)$$

$$= A_3 \exp(jk_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$k_2 A_2 \exp(jk_2 a) - k_2 B_2 \exp(-jk_2 a)$$

$$= k_3 A_3 \exp(jk_3 a)$$

But $k_2 a = 2n\pi \Rightarrow$

$$\exp(jk_2 a) = \exp(-jk_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the boundary condition equations, we find

$$T = \frac{k_3}{k_1} \cdot \frac{4k_1^2}{(k_1 + k_3)^2} = \frac{4k_1 k_3}{(k_1 + k_3)^2}$$

2.40

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1(x) = B_1 \exp(k_1 x)$$

$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$$

$$\psi_3(x) = 0$$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$ and

$$k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right) B_2$$

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right) B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right) \tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.41

$$E_n = \frac{-m_0 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} \text{ (J)}$$

$$= \frac{-m_0 e^3}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} \text{ (eV)}$$

$$= \frac{-(9.11\times10^{-31})(1.6\times10^{-19})^3}{[4\pi(8.85\times10^{-12})]^2\, 2(1.054\times10^{-34})^2 n^2}$$

or

$$E_n = \frac{-13.58}{n^2} \text{ (eV)}$$

$n = 1 \Rightarrow E_1 = -13.58$ eV

$n = 2 \Rightarrow E_2 = -3.395$ eV

$n = 3 \Rightarrow E_3 = -1.51$ eV

$n = 4 \Rightarrow E_4 = -0.849$ eV

2.42

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^*$$

$$= 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3} \left\{ \left(\frac{-2}{a_o}\right)(r^2) \exp\left(\frac{-2r}{a_o}\right) + 2r \exp\left(\frac{-2r}{a_o}\right) \right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow r = a_o$$

or $r = a_o$ is the radius that gives the greatest probability.

2.43

ψ_{100} is independent of θ and ϕ, so the wave
equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{2m_o}{\hbar^2}\left(E - V(r) \right)\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi\epsilon_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{3/2} \exp\left(\frac{-r}{a_o} \right)$$

Then

$$\frac{\partial \psi_{100}}{\partial r} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{3/2} \left(\frac{-1}{a_o} \right) \exp\left(\frac{-r}{a_o} \right)$$

so

$$r^2 \frac{\partial \psi_{100}}{\partial r} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{5/2} r^2 \exp\left(\frac{-r}{a_o} \right)$$

We then obtain

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi_{100}}{\partial r} \right) = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{5/2}$$

$$\times \left[2r \exp\left(\frac{-r}{a_o} \right) - \left(\frac{r^2}{a_o} \right) \exp\left(\frac{-r}{a_o} \right) \right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2 \sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{5/2} \left[2r \exp\left(\frac{-r}{a_o} \right) - \frac{r^2}{a_o} \exp\left(\frac{-r}{a_o} \right) \right]$$

$$+ \frac{2m_o}{\hbar^2}\left[E + \frac{\hbar^2}{m_o a_o r} \right]$$

$$\times \left(\frac{1}{\sqrt{\pi}} \right) \cdot \left(\frac{1}{a_o} \right)^{3/2} \exp\left(\frac{-r}{a_o} \right) = 0$$

where

$$E = E_1 = \frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2} = \frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{3/2} \left[\exp\left(\frac{-r}{a_o} \right) \right] \left\{ \frac{-1}{r^2 a_o} \left[2r - \frac{r^2}{a_o} \right] \right.$$

$$\left. + \frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o} + \frac{\hbar^2}{m_o a_o r} \right) \right\} = 0$$

or

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o} \right)^{3/2} \left[\exp\left(\frac{-r}{a_o} \right) \right]$$

$$\times \left\{ \frac{-2}{a_o r} + \frac{1}{a_o^2} + \left(\frac{-1}{a_o^2} + \frac{2}{a_o r} \right) \right\} = 0$$

which gives 0 = 0 and shows that ψ_{100} is indeed a solution to the wave equation.

2.44

All elements are from the Group I column of the periodic table. All have one valence electron in the outer shell.
