Semiconductor Physics and Devices 4th edition Neaman pdf

Electric circuit (Bangladesh University of Business and Technology)

Chapter 1

Problem Solutions

1.1

(a) fcc: 8 corner atoms $\times 1/8 = 1$ atom

6 face atoms $\times 1/2 = 3$ atoms

Total of 4 atoms per unit cell

(b) bcc: 8 corner atoms $\times 1/8 = 1$ atom

1 enclosed atom $= 1$ atom

Total of 2 atoms per unit cell

(c) Diamond: 8 corner atoms $\times 1/8 = 1$ atom

6 face atoms $\times 1/2 = 3$ atoms

4 enclosed atoms $= 4$ atoms

Total of 8 atoms per unit cell

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1.2

(a) Simple cubic lattice: $a = 2r$

Unit cell vol $= a^3 = (2r)^3 = 8r^3$

1 atom per cell, so atom vol $= (1)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{\left(\dfrac{4\pi r^3}{3}\right)}{8r^3} \times 100\% = 52.4\%$$

(b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow a = \frac{d}{\sqrt{2}} = 2\sqrt{2} \cdot r$$

Unit cell vol $= a^3 = \left(2\sqrt{2} \cdot r\right)^3 = 16\sqrt{2} \cdot r^3$

4 atoms per cell, so atom vol $= (4)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(4)\left(\dfrac{4\pi r^3}{3}\right)}{16\sqrt{2} \cdot r^3} \times 100\% = 74\%$$

(c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} \cdot r$$

Unit cell vol $= a^3 = \left(\dfrac{4}{\sqrt{3}} \cdot r\right)^3$

2 atoms per cell, so atom vol $= (2)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(2)\left(\dfrac{4\pi r^3}{3}\right)}{\left(\dfrac{4r}{\sqrt{3}}\right)^3} \times 100\% = 68\%$$

(d) Diamond lattice

Body diagonal $= d = 8r = a\sqrt{3} \Rightarrow a = \dfrac{8}{\sqrt{3}} \cdot r$

Unit cell vol $= a^3 = \left(\dfrac{8r}{\sqrt{3}}\right)^3$

8 atoms per cell, so atom vol $= (8)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(8)\left(\dfrac{4\pi r^3}{3}\right)}{\left(\dfrac{8r}{\sqrt{3}}\right)^3} \times 100\% = 34\%$$

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1.3

(a) $a = 5.43\,\overset{o}{A}$; From Problem 1.2d,

$$a = \frac{8}{\sqrt{3}} \cdot r$$

Then $r = \dfrac{a\sqrt{3}}{8} = \dfrac{(5.43)\sqrt{3}}{8} = 1.176\,\overset{o}{A}$

Center of one silicon atom to center of

nearest neighbor $= 2r = 2.35\,\overset{o}{A}$

(b) Number density

$$= \frac{8}{\left(5.43 \times 10^{-8}\right)^3} = 5 \times 10^{22} \text{ cm}^{-3}$$

(c) Mass density

$$= \rho = \frac{N(At.Wt.)}{N_A} = \frac{\left(5 \times 10^{22}\right)(28.09)}{6.02 \times 10^{23}}$$

$$\Rightarrow \rho = 2.33 \text{ grams/cm}^3$$

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1.4

(a) 4 Ga atoms per unit cell

Number density $= \dfrac{4}{\left(5.65 \times 10^{-8}\right)^3}$

$\Rightarrow$ Density of Ga atoms $= 2.22 \times 10^{22}$ cm^{-3}
4 As atoms per unit cell

$\Rightarrow$ Density of As atoms $= 2.22 \times 10^{22}$ cm^{-3}

(b) 8 Ge atoms per unit cell

Number density $= \dfrac{8}{\left(5.65 \times 10^{-8}\right)^3}$

$\Rightarrow$ Density of Ge atoms $= 4.44 \times 10^{22}$ cm^{-3}

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1.5

From Figure 1.15

(a) $d = \left(\dfrac{a}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) = (0.4330)a$

$= (0.4330)(5.65) \Rightarrow d = 2.447 \overset{o}{A}$

(b) $d = \left(\dfrac{a}{2}\right)\sqrt{2} = (0.7071)a$

$= (0.7071)(5.65) \Rightarrow d = 3.995 \overset{o}{A}$

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1.6

$$\sin\left(\dfrac{\theta}{2}\right) = \dfrac{\dfrac{a}{2}\sqrt{2}}{\dfrac{a}{2}\sqrt{3}} = \sqrt{\dfrac{2}{3}} \Rightarrow \dfrac{\theta}{2} = 54.74°$$

$$\Rightarrow \theta = 109.5°$$

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1.7

(a) Simple cubic: $a = 2r = 3.9 \overset{o}{A}$

(b) fcc: $a = \dfrac{4r}{\sqrt{2}} = 5.515 \overset{o}{A}$

(c) bcc: $a = \dfrac{4r}{\sqrt{3}} = 4.503 \overset{o}{A}$

(d) diamond: $a = \dfrac{2(4r)}{\sqrt{3}} = 9.007 \overset{o}{A}$

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1.8

(a) $2(1.035)\sqrt{2} = 2(1.035) + 2r_B$

$r_B = 0.4287 \overset{o}{A}$

(b) $a = 2(1.035) = 2.07 \overset{o}{A}$

(c) A-atoms: # of atoms $= 8 \times \dfrac{1}{8} = 1$

Density $= \dfrac{1}{\left(2.07 \times 10^{-8}\right)^3}$

$= 1.13 \times 10^{23}$ cm^{-3}

B-atoms: # of atoms $= 6 \times \dfrac{1}{2} = 3$

Density $= \dfrac{3}{\left(2.07 \times 10^{-8}\right)^3}$

$= 3.38 \times 10^{23}$ cm^{-3}

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1.9

(a) $a = 2r = 4.5 \overset{o}{A}$

# of atoms $= 8 \times \dfrac{1}{8} = 1$

Number density $= \dfrac{1}{\left(4.5 \times 10^{-8}\right)^3}$

$= 1.097 \times 10^{22}$ cm^{-3}

Mass density $= \rho = \dfrac{N(At.Wt.)}{N_A}$

$= \dfrac{\left(1.0974 \times 10^{22}\right)(12.5)}{6.02 \times 10^{23}}$

$= 0.228$ gm/cm^3

(b) $a = \dfrac{4r}{\sqrt{3}} = 5.196 \overset{o}{A}$

# of atoms $8 \times \dfrac{1}{8} + 1 = 2$

Number density $= \dfrac{2}{\left(5.196 \times 10^{-8}\right)^3}$

$= 1.4257 \times 10^{22}$ cm^{-3}

Mass density $= \rho = \dfrac{\left(1.4257 \times 10^{22}\right)(12.5)}{6.02 \times 10^{23}}$

$= 0.296$ gm/cm^3

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1.10

From Problem 1.2, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

Volume $= 0.74$ cm^3

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1.11

(b) $a = 1.8 + 1.0 = 2.8 \overset{o}{A}$

(c) Na: Density $= \dfrac{(1/2)}{(2.8 \times 10^{-8})^3}$

$= 2.28 \times 10^{22} \text{ cm}^{-3}$

Cl: Density $= 2.28 \times 10^{22} \text{ cm}^{-3}$

(d) Na: At. Wt. = 22.99
Cl: At. Wt. = 35.45

So, mass per unit cell

$= \dfrac{\left(\dfrac{1}{2}\right)(22.99) + \left(\dfrac{1}{2}\right)(35.45)}{6.02 \times 10^{23}} = 4.85 \times 10^{-23}$

Then mass density

$\rho = \dfrac{4.85 \times 10^{-23}}{(2.8 \times 10^{-8})^3} = 2.21 \text{ grams/cm}^3$

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1.12

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8 \overset{o}{A}$

Then $a = 4.62 \overset{o}{A}$
Density of A:

$= \dfrac{1}{(4.62 \times 10^{-8})^3} = 1.01 \times 10^{22} \text{ cm}^{-3}$

Density of B:

$= \dfrac{1}{(4.62 \times 10^{-8})^3} = 1.01 \times 10^{22} \text{ cm}^{-3}$

(b) Same as (a)
(c) Same material

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1.13

$a = \dfrac{2(2.2) + 2(1.8)}{\sqrt{3}} = 4.619 \overset{o}{A}$

(a) For 1.12(a), A-atoms

Surface density $= \dfrac{1}{a^2} = \dfrac{1}{(4.619 \times 10^{-8})^2}$

$= 4.687 \times 10^{14} \text{ cm}^{-2}$

For 1.12(b), B-atoms: $a = 4.619 \overset{o}{A}$

Surface density $= \dfrac{1}{a^2} = 4.687 \times 10^{14} \text{ cm}^{-2}$

For 1.12(a) and (b), Same material

(b) For 1.12(a), A-atoms; $a = 4.619 \overset{o}{A}$

Surface density

$= \dfrac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$

B-atoms;
Surface density

$= \dfrac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$

For 1.12(b), A-atoms; $a = 4.619 \overset{o}{A}$

Surface density

$= \dfrac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$

B-atoms;
Surface density

$= \dfrac{1}{a^2 \sqrt{2}} = 3.315 \times 10^{14} \text{ cm}^{-2}$

For 1.12(a) and (b), Same material

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1.14

(a) Vol. Density $= \dfrac{1}{a_o^3}$

Surface Density $= \dfrac{1}{a_o^2 \sqrt{2}}$

(b) Same as (a)

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1.15

(i) (110) plane
(see Figure 1.10(b))

(ii) (111) plane
(see Figure 1.10(c))

(iii) (220) plane $\Rightarrow \left(\dfrac{1}{2}, \dfrac{1}{2}, \infty\right) \Rightarrow (1,1,0)$

Same as (110) plane and [110] direction

(iv) (321) plane $\Rightarrow \left(\dfrac{1}{3}, \dfrac{1}{2}, \dfrac{1}{1}\right) \Rightarrow (2,3,6)$

Intercepts of plane at
$p = 2, q = 3, s = 6$

[321] direction is perpendicular to
(321) plane

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1.16

(a)

$$\left(\frac{1}{1},\frac{1}{3},\frac{1}{1}\right)\Rightarrow(313)$$

(b)

$$\left(\frac{1}{4},\frac{1}{2},\frac{1}{4}\right)\Rightarrow(121)$$

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1.17

Intercepts: 2, 4, 3 $\Rightarrow\left(\frac{1}{2},\frac{1}{4},\frac{1}{3}\right)\Rightarrow$

(634) plane

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1.18

(a) $d=a=5.28\,\overset{o}{A}$

(b) $d=\dfrac{a\sqrt{2}}{2}=3.734\,\overset{o}{A}$

(c) $d=\dfrac{a\sqrt{3}}{3}=3.048\,\overset{o}{A}$

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1.19

(a) Simple cubic

(i) (100) plane:

$$\text{Surface density }=\frac{1}{a^2}=\frac{1}{\left(4.73\times10^{-8}\right)^2}$$
$$=4.47\times10^{14}\text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density }=\frac{1}{a^2\sqrt{2}}$$
$$=3.16\times10^{14}\text{ cm}^{-2}$$

(iii) (111) plane:

Area of plane $=\dfrac{1}{2}bh$

where $b=a\sqrt{2}=6.689\,\overset{o}{A}$

Now

$$h^2=\left(a\sqrt{2}\right)^2-\left(\frac{a\sqrt{2}}{2}\right)^2=\frac{3}{4}\left(a\sqrt{2}\right)^2$$

So $h=\dfrac{\sqrt{6}}{2}(4.73)=5.793\,\overset{o}{A}$

Area of plane

$$=\frac{1}{2}\left(6.68923\times10^{-8}\right)\left(5.79304\times10^{-8}\right)$$
$$=19.3755\times10^{-16}\text{ cm}^2$$

$$\text{Surface density }=\frac{3\times\dfrac{1}{6}}{19.3755\times10^{-16}}$$
$$=2.58\times10^{14}\text{ cm}^{-2}$$

(b) bcc

(i) (100) plane:

$$\text{Surface density }=\frac{1}{a^2}=4.47\times10^{14}\text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density }=\frac{2}{a^2\sqrt{2}}$$
$$=6.32\times10^{14}\text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density }=\frac{3\times\dfrac{1}{6}}{19.3755\times10^{-16}}$$
$$=2.58\times10^{14}\text{ cm}^{-2}$$

(c) fcc

(i) (100) plane:

$$\text{Surface density }=\frac{2}{a^2}=8.94\times10^{14}\text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density }=\frac{2}{a^2\sqrt{2}}$$
$$=6.32\times10^{14}\text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density }=\frac{3\times\dfrac{1}{6}+3\times\dfrac{1}{2}}{19.3755\times10^{-16}}$$
$$=1.03\times10^{15}\text{ cm}^{-2}$$

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1.20

(a) (100) plane: - similar to a fcc:

$$\text{Surface density }=\frac{2}{\left(5.43\times10^{-8}\right)^2}$$
$$=6.78\times10^{14}\text{ cm}^{-2}$$

(b) (110) plane:

$$\text{Surface density }=\frac{4}{\sqrt{2}\left(5.43\times10^{-8}\right)^2}$$
$$=9.59\times10^{14}\text{ cm}^{-2}$$

(c) (111) plane:

$$\text{Surface density} = \frac{2}{\left(\sqrt{3}/2\right)\left(5.43\times10^{-8}\right)^2}$$
$$= 7.83\times10^{14}\text{ cm}^{-2}$$

1.21

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.37)}{\sqrt{2}} = 6.703\ \overset{o}{A}$$

(a) $\#/\text{cm}^3 = \dfrac{8\times\dfrac{1}{8}+6\times\dfrac{1}{2}}{a^3} = \dfrac{4}{\left(6.703\times10^{-8}\right)^3}$

$$= 1.328\times10^{22}\text{ cm}^{-3}$$

(b) $\#/\text{cm}^2 = \dfrac{4\times\dfrac{1}{4}+2\times\dfrac{1}{2}}{a^2\sqrt{2}}$

$$= \frac{2}{\left(6.703\times10^{-8}\right)^2\sqrt{2}}$$
$$= 3.148\times10^{14}\text{ cm}^{-2}$$

(c) $d = \dfrac{a\sqrt{2}}{2} = \dfrac{(6.703)\sqrt{2}}{2} = 4.74\ \overset{o}{A}$

(d) # of atoms $= 3\times\dfrac{1}{6}+3\times\dfrac{1}{2} = 2$

Area of plane: (see Problem 1.19)
$$b = a\sqrt{2} = 9.4786\ \overset{o}{A}$$
$$h = \frac{\sqrt{6}a}{2} = 8.2099\ \overset{o}{A}$$

Area
$$= \frac{1}{2}bh = \frac{1}{2}\left(9.4786\times10^{-8}\right)\left(8.2099\times10^{-8}\right)$$
$$= 3.8909\times10^{-15}\text{ cm}^2$$

$$\#/\text{cm}^2 = \frac{2}{3.8909\times10^{-15}}$$
$$= 5.14\times10^{14}\text{ cm}^{-2}$$
$$d = \frac{a\sqrt{3}}{3} = \frac{(6.703)\sqrt{3}}{3} = 3.87\ \overset{o}{A}$$

1.22

Density of silicon atoms $= 5\times10^{22}\text{ cm}^{-3}$ and
    4 valence electrons per atom, so
Density of valence electrons $= 2\times10^{23}\text{ cm}^{-3}$

1.23

Density of GaAs atoms
$$= \frac{8}{\left(5.65\times10^{-8}\right)^3} = 4.44\times10^{22}\text{ cm}^{-3}$$
An average of 4 valence electrons per atom,
    So
Density of valence electrons
$$= 1.77\times10^{23}\text{ cm}^{-3}$$

1.24

(a) $\dfrac{5\times10^{17}}{5\times10^{22}}\times100\% = 10^{-3}\%$

(b) $\dfrac{2\times10^{15}}{5\times10^{22}}\times100\% = 4\times10^{-6}\%$

1.25

(a) Fraction by weight
$$\cong \frac{\left(2\times10^{16}\right)(10.82)}{\left(5\times10^{22}\right)(28.06)} = 1.542\times10^{-7}$$

(b) Fraction by weight
$$\cong \frac{\left(10^{18}\right)(30.98)}{\left(5\times10^{22}\right)(28.06)} = 2.208\times10^{-5}$$

1.26

Volume density $= \dfrac{1}{d^3} = 2\times10^{16}\text{ cm}^{-3}$

So $d = 3.684\times10^{-6}\text{ cm} \Rightarrow d = 368.4\ \overset{o}{A}$

We have $a_o = 5.43\ \overset{o}{A}$

Then $\dfrac{d}{a_o} = \dfrac{368.4}{5.43} = 67.85$

1.27

Volume density $= \dfrac{1}{d^3} = 4\times10^{15}\text{ cm}^{-3}$

So $d = 6.30\times10^{-6}\text{ cm} \Rightarrow d = 630\ \overset{o}{A}$

We have $a_o = 5.43\ \overset{o}{A}$

Then $\dfrac{d}{a_o} = \dfrac{630}{5.43} = 116$

Chapter 2

2.1

Sketch

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2.2

Sketch

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2.3

Sketch

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2.4

From Problem 2.2, phase $= \dfrac{2\pi x}{\lambda} - \omega t$

$\qquad\qquad\qquad\qquad = $ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = +\omega\left(\frac{\lambda}{2\pi}\right)$$

From Problem 2.3, phase $= \dfrac{2\pi x}{\lambda} + \omega t$

$\qquad\qquad\qquad\qquad = $ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = -\omega\left(\frac{\lambda}{2\pi}\right)$$

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2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90 \text{ eV} = (4.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(4.90)(1.6\times10^{-19})} = 2.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.254\,\mu\text{m}$

Cesium: $E = 1.90 \text{ eV} = (1.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(1.90)(1.6\times10^{-19})} = 6.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.654\,\mu\text{m}$

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2.6

(a) $\quad p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{550\times10^{-9}}$

$\qquad\quad = 1.205\times10^{-27}$ kg-m/s

$\quad \upsilon = \dfrac{p}{m} = \dfrac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^3$ m/s

or $\upsilon = 1.32\times10^5$ cm/s

(b) $\quad p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{440\times10^{-9}}$

$\qquad\quad = 1.506\times10^{-27}$ kg-m/s

$\quad \upsilon = \dfrac{p}{m} = \dfrac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^3$ m/s

or $\upsilon = 1.65\times10^5$ cm/s

(c) Yes

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2.7

(a) (i)

$$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.2)(1.6\times10^{-19})}$$

$\qquad\quad = 5.915\times10^{-25}$ kg-m/s

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{5.915\times10^{-25}} = 1.12\times10^{-9} \text{ m}$$

or $\lambda = 11.2 \overset{o}{A}$

(ii) $p = \sqrt{2(9.11\times10^{-31})(12)(1.6\times10^{-19})}$

$\qquad\quad = 1.87\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{1.8704\times10^{-24}} = 3.54\times10^{-10} \text{ m}$$

or $\lambda = 3.54 \overset{o}{A}$

(iii) $p = \sqrt{2(9.11\times10^{-31})(120)(1.6\times10^{-19})}$

$\qquad\quad = 5.915\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{5.915\times10^{-24}} = 1.12\times10^{-10} \text{ m}$$

or $\lambda = 1.12 \overset{o}{A}$

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(b)
$$p = \sqrt{2(1.67\times10^{-27})(1.2)(1.6\times10^{-19})}$$
$$= 2.532\times10^{-23}\text{ kg-m/s}$$
$$\lambda = \frac{6.625\times10^{-34}}{2.532\times10^{-23}} = 2.62\times10^{-11}\text{ m}$$

or $\lambda = 0.262\ \overset{o}{A}$

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2.8

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885\text{ eV}$$

Now
$$p_{avg} = \sqrt{2mE_{avg}}$$
$$= \sqrt{2(9.11\times10^{-31})(0.03885)(1.6\times10^{-19})}$$

or
$$p_{avg} = 1.064\times10^{-25}\text{ kg-m/s}$$

Now
$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{1.064\times10^{-25}} = 6.225\times10^{-9}\text{ m}$$

or
$$\lambda = 62.25\ \overset{o}{A}$$

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2.9

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now
$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then
$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields
$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2(9.11\times10^{-31})(3\times10^8)^2}{100}$$

$$= 1.64\times10^{-15}\text{ J} = 10.25\text{ keV}$$

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2.10

(a) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{85\times10^{-10}}$

$\quad = 7.794\times10^{-26}$ kg-m/s

$\quad \upsilon = \dfrac{p}{m} = \dfrac{7.794\times10^{-26}}{9.11\times10^{-31}} = 8.56\times10^4$ m/s

or $\upsilon = 8.56\times10^6$ cm/s

$E = \dfrac{1}{2}m\upsilon^2 = \dfrac{1}{2}(9.11\times10^{-31})(8.56\times10^4)^2$

$\quad = 3.33\times10^{-21}$ J

or $E = \dfrac{3.334\times10^{-21}}{1.6\times10^{-19}} = 2.08\times10^{-2}$ eV

(b) $E = \dfrac{1}{2}(9.11\times10^{-31})(8\times10^3)^2$

$\quad = 2.915\times10^{-23}$ J

or $E = \dfrac{2.915\times10^{-23}}{1.6\times10^{-19}} = 1.82\times10^{-4}$ eV

$p = m\upsilon = (9.11\times10^{-31})(8\times10^3)$

$\quad = 7.288\times10^{-27}$ kg-m/s

$\lambda = \dfrac{h}{p} = \dfrac{6.625\times10^{-35}}{7.288\times10^{-27}} - 9.09\times10^{-8}$ m

or $\lambda = 909\ \overset{o}{A}$

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2.11

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{(6.625\times10^{-34})(3\times10^8)}{1\times10^{-10}}$

$\quad = 1.99\times10^{-15}$ J

Now
$E = e\cdot V \Rightarrow V = \dfrac{E}{e} = \dfrac{1.99\times10^{-15}}{1.6\times10^{-19}}$

$V = 1.24\times10^4$ V $= 12.4$ kV

(b) $p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.99\times10^{-15})}$

$\quad = 6.02\times10^{-23}$ kg-m/s

Then
$\lambda = \dfrac{h}{p} = \dfrac{6.625\times10^{-34}}{6.02\times10^{-23}} = 1.10\times10^{-11}$ m

or
$\lambda = 0.11\ \overset{o}{A}$

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2.12

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-6}}$$

$$= 1.054 \times 10^{-28} \text{ kg-m/s}$$

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2.13

(a) (i) $\Delta p \Delta x = \hbar$

$$\Delta p = \frac{1.054 \times 10^{-34}}{12 \times 10^{-10}} = 8.783 \times 10^{-26} \text{ kg-m/s}$$

(ii) $\Delta E = \frac{dE}{dp} \cdot \Delta p = \frac{d}{dp}\left(\frac{p^2}{2m}\right) \cdot \Delta p$

$$= \frac{2p}{2m} \cdot \Delta p = \frac{p \Delta p}{m}$$

Now $p = \sqrt{2mE}$

$$= \sqrt{2(9 \times 10^{-31})(16)(1.6 \times 10^{-19})}$$

$$= 2.147 \times 10^{-24} \text{ kg-m/s}$$

so $\Delta E = \frac{(2.1466 \times 10^{-24})(8.783 \times 10^{-26})}{9 \times 10^{-31}}$

$$= 2.095 \times 10^{-19} \text{ J}$$

or $\Delta E = \frac{2.095 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.31 \text{ eV}$

(b) (i) $\Delta p = 8.783 \times 10^{-26} \text{ kg-m/s}$

(ii) $p = \sqrt{2(5 \times 10^{-28})(16)(1.6 \times 10^{-19})}$

$$= 5.06 \times 10^{-23} \text{ kg-m/s}$$

$$\Delta E = \frac{(5.06 \times 10^{-23})(8.783 \times 10^{-26})}{5 \times 10^{-28}}$$

$$= 8.888 \times 10^{-21} \text{ J}$$

or $\Delta E = \frac{8.888 \times 10^{-21}}{1.6 \times 10^{-19}} = 5.55 \times 10^{-2} \text{ eV}$

_____

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{10^{-2}} = 1.054 \times 10^{-32} \text{ kg-m/s}$$

$$p = mv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054 \times 10^{-32}}{1500}$$

$$\Delta v = 7 \times 10^{-36} \text{ m/s}$$

_____

2.15

(a) $\Delta E \Delta t = \hbar$

$$\Delta t = \frac{1.054 \times 10^{-34}}{(0.8)(1.6 \times 10^{-19})} = 8.23 \times 10^{-16} \text{ s}$$

(b) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}}{1.5 \times 10^{-10}}$

$$= 7.03 \times 10^{-25} \text{ kg-m/s}$$

_____

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar \frac{\partial \Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar \frac{\partial \Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}[\Psi_1(x,t) + \Psi_2(x,t)]$$

$$+ V(x)[\Psi_1(x,t) + \Psi_2(x,t)]$$

$$= j\hbar \frac{\partial}{\partial t}[\Psi_1(x,t) + \Psi_2(x,t)]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t) + \Psi_2(x,t)$ is also a solution.

(b) If $\Psi_1(x,t) \cdot \Psi_2(x,t)$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}[\Psi_1 \cdot \Psi_2] + V(x)[\Psi_1 \cdot \Psi_2]$$

$$= j\hbar \frac{\partial}{\partial t}[\Psi_1 \cdot \Psi_2]$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1 \frac{\partial^2 \Psi_2}{\partial x^2} + \Psi_2 \frac{\partial^2 \Psi_1}{\partial x^2} + 2\frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x}\right]$$

$$+ V(x)[\Psi_1 \cdot \Psi_2] = j\hbar\left[\Psi_1 \frac{\partial \Psi_2}{\partial t} + \Psi_2 \frac{\partial \Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1 \cdot \Psi_2$, we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + \frac{2}{\Psi_1 \Psi_2}\frac{\partial \Psi_1}{\partial x}\frac{\partial \Psi_2}{\partial x}\right]$$

$$+ V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial \Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial \Psi_1}{\partial t}\right]$$

_____

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we have

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$
$$=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we have

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

_____

2.17

$$\int_{-1}^{+3}A^2\cos^2\left(\frac{\pi x}{2}\right)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(\pi x)}{2\pi}\right]\Big|_{-1}^{+3}=1$$

$$A^2\left[\frac{3}{2}-\left(\frac{-1}{2}\right)\right]=1$$

so $A^2=\frac{1}{2}$

or $|A|=\frac{1}{\sqrt{2}}$

_____

2.18

$$\int_{-1/2}^{+1/2}A^2\cos^2(n\pi x)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(2n\pi x)}{4n\pi}\right]\Big|_{-1/2}^{+1/2}=1$$

$$A^2\left[\frac{1}{4}-\left(-\frac{1}{4}\right)\right]=1=A^2\left(\frac{1}{2}\right)$$

or $|A|=\sqrt{2}$

_____

2.19

Note that $\int_0^\infty \Psi\cdot\Psi^* dx=1$

Function has been normalized.
(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Big|_0^{a_o/4}$$

or

$$P=(-1)\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields
$P=0.393$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Big|_{a_o/4}^{a_o/2}$$

or

$$P=(-1)\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields
$P=0.239$

(c)

$$P=\int_0^{a_o}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2 dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Big|_0^{a_o}$$

$$=(-1)[\exp(-2)-1]$$

which yields
$P=0.865$

_____

2.20

$$P = \int |\psi(x)|^2 \, dx$$

(a) $\displaystyle \int_0^{a/4} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{2}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]_0^{a/4}$$

$$= \left(\frac{2}{a}\right) \left[\frac{\left(\frac{a}{4}\right)}{2} + \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{8} + \frac{(1)(a)}{4\pi}\right]$$

or $P = 0.409$

(b) $\displaystyle P = \int_{a/4}^{a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]_{a/4}^{a/2}$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \frac{a}{8} - \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$= 2\left[\frac{1}{4} + 0 - \frac{1}{8} - \frac{1}{4\pi}\right]$$

or $P = 0.0908$

(c) $\displaystyle P = \int_{-a/2}^{+a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)}\right]_{-a/2}^{+a/2}$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \left(\frac{-a}{4}\right) - \frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)}\right]$$

or $P = 1$

2.21

(a) $\displaystyle P = \int_0^{a/4} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]_0^{a/4}$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{8} - \frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P = 0.25$

(b) $\displaystyle P = \int_{a/4}^{a/2} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]_{a/4}^{a/2}$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{4} - \frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)} - \left(\frac{a}{8}\right) + \frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P = 0.25$

(c) $\displaystyle P = \int_{-a/2}^{+a/2} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx$

$$= \left(\frac{2}{a}\right) \left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]_{-a/2}^{+a/2}$$

$$= \left(\frac{2}{a}\right) \left[\frac{a}{4} - \frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)} - \left(\frac{-a}{4}\right) + \frac{\sin(-2\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P = 1$

2.22

(a) (i) $\displaystyle \upsilon_p = \frac{\omega}{k} = \frac{8 \times 10^{12}}{8 \times 10^8} = 10^4 \, \text{m/s}$

or $\upsilon_p = 10^6 \, \text{cm/s}$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{8 \times 10^8} = 7.854 \times 10^{-9} \, \text{m}$$

_____

or $\lambda = 78.54 \overset{o}{A}$

(ii) $p = m\upsilon = \left(9.11 \times 10^{-31}\right)\left(10^4\right)$

$\qquad = 9.11 \times 10^{-27}$ kg-m/s

$\qquad E = \frac{1}{2} m\upsilon^2 = \frac{1}{2}\left(9.11 \times 10^{-31}\right)\left(10^4\right)^2$

$\qquad\quad = 4.555 \times 10^{-23}$ J

or $E = \frac{4.555 \times 10^{-23}}{1.6 \times 10^{-19}} = 2.85 \times 10^{-4}$ eV

(b) (i) $\upsilon_p = \frac{\omega}{k} = \frac{1.5 \times 10^{13}}{-1.5 \times 10^9} = -10^4$ m/s

or $\upsilon_p = -10^6$ cm/s

$\qquad \lambda = \frac{2\pi}{|k|} = \frac{2\pi}{1.5 \times 10^9} = 4.19 \times 10^{-9}$ m

or $\lambda = 41.9 \overset{o}{A}$

(ii) $p = -9.11 \times 10^{-27}$ kg-m/s

$\qquad E = 2.85 \times 10^{-4}$ eV

_____

2.23

(a) $\Psi(x,t) = A e^{-j(kx+\omega t)}$

(b) $E = (0.025)\left(1.6 \times 10^{-19}\right) = \frac{1}{2} m\upsilon^2$

$\qquad\qquad\qquad\qquad = \frac{1}{2}\left(9.11 \times 10^{-31}\right)\upsilon^2$

so $|\upsilon| = 9.37 \times 10^4$ m/s $= 9.37 \times 10^6$ cm/s

For electron traveling in $-x$ direction,

$\qquad \upsilon = -9.37 \times 10^6$ cm/s

$\qquad p = m\upsilon = \left(9.11 \times 10^{-31}\right)\left(-9.37 \times 10^4\right)$

$\qquad\quad = -8.537 \times 10^{-26}$ kg-m/s

$\qquad \lambda = \frac{h}{|p|} = \frac{6.625 \times 10^{-34}}{8.537 \times 10^{-26}} = 7.76 \times 10^{-9}$ m

$\qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.76 \times 10^{-9}} = 8.097 \times 10^8$ m^{-1}

$\qquad \omega = k \cdot |\upsilon| = \left(8.097 \times 10^8\right)\left(9.37 \times 10^4\right)$

or $\omega = 7.586 \times 10^{13}$ rad/s

_____

2.24

(a) $p = m\upsilon = \left(9.11 \times 10^{-31}\right)\left(5 \times 10^4\right)$

$\qquad\quad = 4.555 \times 10^{-26}$ kg-m/s

$\qquad \lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{4.555 \times 10^{-26}} = 1.454 \times 10^{-8}$ m

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.454 \times 10^{-8}} = 4.32 \times 10^8$ m^{-1}

$\omega = k\upsilon = \left(4.32 \times 10^8\right)\left(5 \times 10^4\right)$

$\qquad = 2.16 \times 10^{13}$ rad/s

(b) $p = \left(9.11 \times 10^{-31}\right)\left(10^6\right)$

$\qquad = 9.11 \times 10^{-25}$ kg-m/s

$\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-25}} = 7.27 \times 10^{-10}$ m

$k = \frac{2\pi}{7.272 \times 10^{-10}} = 8.64 \times 10^9$ m^{-1}

$\omega = \left(8.64 \times 10^9\right)\left(10^6\right) = 8.64 \times 10^{15}$ rad/s

_____

2.25

$\qquad E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2 \left(1.054 \times 10^{-34}\right)^2 \pi^2}{2\left(9.11 \times 10^{-31}\right)\left(75 \times 10^{-10}\right)^2}$

$\qquad E_n = n^2 \left(1.0698 \times 10^{-21}\right)$ J

or

$\qquad E_n = \frac{n^2 \left(1.0698 \times 10^{-21}\right)}{1.6 \times 10^{-19}}$

or $E_n = n^2 \left(6.686 \times 10^{-3}\right)$ eV

Then

$\qquad E_1 = 6.69 \times 10^{-3}$ eV

$\qquad E_2 = 2.67 \times 10^{-2}$ eV

$\qquad E_3 = 6.02 \times 10^{-2}$ eV

_____

2.26

(a) $E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2 \left(1.054 \times 10^{-34}\right)^2 \pi^2}{2\left(9.11 \times 10^{-31}\right)\left(10 \times 10^{-10}\right)^2}$

$\qquad\quad = n^2 \left(6.018 \times 10^{-20}\right)$ J

or $E_n = \frac{n^2 \left(6.018 \times 10^{-20}\right)}{1.6 \times 10^{-19}} = n^2 \left(0.3761\right)$ eV

Then

$\qquad E_1 = 0.376$ eV

$\qquad E_2 = 1.504$ eV

$\qquad E_3 = 3.385$ eV

(b) $\lambda = \frac{hc}{\Delta E}$

$\qquad \Delta E = (3.385 - 1.504)\left(1.6 \times 10^{-19}\right)$

$\qquad\quad = 3.01 \times 10^{-19}$ J

_____

$$\lambda = \frac{\left(6.625\times10^{-34}\right)\left(3\times10^{8}\right)}{3.01\times10^{-19}}$$

$$= 6.604\times10^{-7}\text{ m}$$

or $\quad \lambda = 660.4$ nm

_____

2.27

(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$15\times10^{-3} = \frac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(15\times10^{-3}\right)\left(1.2\times10^{-2}\right)^2}$$

$$15\times10^{-3} = n^2\left(2.538\times10^{-62}\right)$$

or $\quad n = 7.688\times10^{29}$

(b) $E_{n+1} \cong 15$ mJ

(c) No

_____

2.28

For a neutron and $n = 1$:

$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.66\times10^{-27}\right)\left(10^{-14}\right)^2}$$

$$= 3.3025\times10^{-13}\text{ J}$$

or

$$E_1 = \frac{3.3025\times10^{-13}}{1.6\times10^{-19}} = 2.06\times10^{6}\text{ eV}$$

For an electron in the same potential well:

$$E_1 = \frac{\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(10^{-14}\right)^2}$$

$$= 6.0177\times10^{-10}\text{ J}$$

or

$$E_1 = \frac{6.0177\times10^{-10}}{1.6\times10^{-19}} = 3.76\times10^{9}\text{ eV}$$

_____

2.29

Schrodinger's time-independent wave
equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \geq \frac{a}{2} \text{ and } x \leq \frac{-a}{2}$$

We have

$$V(x) = 0 \text{ for } \frac{-a}{2} < x < \frac{+a}{2}$$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

The solution is of the form

$$\psi(x) = A\cos kx + B\sin kx$$

where

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{-a}{2},\ x = \frac{+a}{2}$$

First mode solution:

$$\psi_1(x) = A_1\cos k_1 x$$

where

$$k_1 = \frac{\pi}{a} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$

Second mode solution:

$$\psi_2(x) = B_2\sin k_2 x$$

where

$$k_2 = \frac{2\pi}{a} \Rightarrow E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$$

Third mode solution:

$$\psi_3(x) = A_3\cos k_3 x$$

where

$$k_3 = \frac{3\pi}{a} \Rightarrow E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$$

Fourth mode solution:

$$\psi_4(x) = B_4\sin k_4 x$$

where

$$k_4 = \frac{4\pi}{a} \Rightarrow E_4 = \frac{16\pi^2\hbar^2}{2ma^2}$$

_____

2.30

The 3-D time-independent wave equation in
cartesian coordinates for $V(x, y, z) = 0$ is:

$$\frac{\partial^2\psi(x, y, z)}{\partial x^2} + \frac{\partial^2\psi(x, y, z)}{\partial y^2} + \frac{\partial^2\psi(x, y, z)}{\partial z^2}$$

$$+ \frac{2mE}{\hbar^2}\psi(x, y, z) = 0$$

Use separation of variables, so let

$$\psi(x, y, z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we
obtain

_____

$$YZ\frac{\partial^2 X}{\partial x^2}+XZ\frac{\partial^2 Y}{\partial y^2}+XY\frac{\partial^2 Z}{\partial z^2}$$

$$+\frac{2mE}{\hbar^2}XYZ=0$$

Dividing by XYZ and letting $k^2=\frac{2mE}{\hbar^2}$, we

find

(1) $\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}+k^2=0$

We may set

$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}=-k_x^2\Rightarrow\frac{\partial^2 X}{\partial x^2}+k_x^2 X=0$

Solution is of the form

$X(x)=A\sin(k_x x)+B\cos(k_x x)$

Boundary conditions: $X(0)=0\Rightarrow B=0$

and $X(x=a)=0\Rightarrow k_x=\frac{n_x\pi}{a}$

where $n_x=1,2,3....$

Similarly, let

$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}=-k_y^2$ and $\frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}=-k_z^2$

Applying the boundary conditions, we find

$k_y=\frac{n_y\pi}{a}$, $n_y=1,2,3....$

$k_z=\frac{n_z\pi}{a}$, $n_z=1,2,3...$

From Equation (1) above, we have

$-k_x^2-k_y^2-k_z^2+k^2=0$

or

$k_x^2+k_y^2+k_z^2=k^2=\frac{2mE}{\hbar^2}$

so that

$E\to E_{n_x n_y n_z}=\frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2+n_y^2+n_z^2\right)$

_____

2.31

(a) $\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial y^2}+\frac{2mE}{\hbar^2}\cdot\psi(x,y)=0$

Solution is of the form:

$\psi(x,y)=A\sin k_x x\cdot\sin k_y y$

We find

$\frac{\partial\psi(x,y)}{\partial x}=Ak_x\cos k_x x\cdot\sin k_y y$

$\frac{\partial^2\psi(x,y)}{\partial x^2}=-Ak_x^2\sin k_x x\cdot\sin k_y y$

$\frac{\partial\psi(x,y)}{\partial y}=Ak_y\sin k_x x\cdot\cos k_y y$

$\frac{\partial^2\psi(x,y)}{\partial y^2}=-Ak_y^2\sin k_x x\cdot\sin k_y y$

Substituting into the original equation, we
find:

(1) $-k_x^2-k_y^2+\frac{2mE}{\hbar^2}=0$

From the boundary conditions,

$A\sin k_x a=0$, where $a=40\overset{o}{A}$

So $k_x=\frac{n_x\pi}{a}$, $n_x=1,2,3,...$

Also $A\sin k_y b=0$, where $b=20\overset{o}{A}$

So $k_y=\frac{n_y\pi}{b}$, $n_y=1,2,3,...$

Substituting into Eq. (1) above

$E_{n_x n_y}=\frac{\hbar^2}{2m}\left(\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{b^2}\right)$

(b) Energy is quantized - similar to 1-D result.
There can be more than one quantum state
per given energy - different than 1-D result.

_____

2.32

(a) Derivation of energy levels exactly the
same as in the text

(b) $\Delta E=\frac{\hbar^2\pi^2}{2ma^2}\left(n_2^2-n_1^2\right)$

For $n_2=2,n_1=1$

Then

$\Delta E=\frac{3\hbar^2\pi^2}{2ma^2}$

(i) For $a=4\overset{o}{A}$

$\Delta E=\frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(4\times10^{-10}\right)^2}$

$=6.155\times10^{-22}$ J

or $\Delta E=\frac{6.155\times10^{-22}}{1.6\times10^{-19}}=3.85\times10^{-3}$ eV

_____

(ii) For $a = 0.5$ cm

$$\Delta E = \frac{3(1.054 \times 10^{-34})^2 \pi^2}{2(1.67 \times 10^{-27})(0.5 \times 10^{-2})^2}$$

$$= 3.939 \times 10^{-36} \text{ J}$$

or

$$\Delta E = \frac{3.939 \times 10^{-36}}{1.6 \times 10^{-19}} = 2.46 \times 10^{-17} \text{ eV}$$

_____

2.33

(a) For region II, $x > 0$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

where

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave and term with A_2 represents reflected wave.

Region I, $x < 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

General form of the solution is

$$\psi_1(x) = A_1 \exp(jk_1 x) + B_1 \exp(-jk_1 x)$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave and the term involving A_1 represents reflected wave: but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1 \exp(-jk_1 x)$$

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

(b)

Boundary conditions:

(1) $\psi_1(x = 0) = \psi_2(x = 0)$

(2) $\left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$k_2 A_2 - k_2 B_2 = -k_1 B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right) \cdot B_2$$

$$B_1 = \left(\frac{2k_2}{k_2 + k_1}\right) \cdot B_2$$

The reflection coefficient is

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4k_1 k_2}{(k_1 + k_2)^2}$$

_____

2.34

$$\psi_2(x) = A_2 \exp(-k_2 x)$$

$$P = \frac{|\psi(x)|^2}{A_2 A_2^*} = \exp(-2k_2 x)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2(9.11 \times 10^{-31})(3.5 - 2.8)(1.6 \times 10^{-19})}}{1.054 \times 10^{-34}}$$

$$k_2 = 4.286 \times 10^9 \text{ m}^{-1}$$

(a) For $x = 5 \overset{o}{A} = 5 \times 10^{-10}$ m

$$P = \exp(-2k_2 x)$$

$$= \exp[-2(4.2859 \times 10^9)(5 \times 10^{-10})]$$

$$= 0.0138$$

(b) For $x = 15 \overset{o}{A} = 15 \times 10^{-10}$ m

$$P = \exp[-2(4.2859 \times 10^9)(15 \times 10^{-10})]$$

$$= 2.61 \times 10^{-6}$$

(c) For $x = 40 \overset{o}{A} = 40 \times 10^{-10}$ m

$$P = \exp[-2(4.2859 \times 10^9)(40 \times 10^{-10})]$$

$$= 1.29 \times 10^{-15}$$

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2.35

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1 - \frac{E}{V_o}\right)\exp(-2k_2 a)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2(9.11 \times 10^{-31})(1.0 - 0.1)(1.6 \times 10^{-19})}}{1.054 \times 10^{-34}}$$

_____

or $k_2 = 4.860 \times 10^9 \, \text{m}^{-1}$

(a) For $a = 4 \times 10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1 - \frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976 \times 10^9\right)\left(4 \times 10^{-10}\right)\right]$$

$$= 0.0295$$

(b) For $a = 12 \times 10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1 - \frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976 \times 10^9\right)\left(12 \times 10^{-10}\right)\right]$$

$$= 1.24 \times 10^{-5}$$

(c) $J = N_t e \upsilon$, where N_t is the density of transmitted electrons.

$$E = 0.1 \, \text{eV} = 1.6 \times 10^{-20} \, \text{J}$$

$$= \frac{1}{2}m\upsilon^2 = \frac{1}{2}\left(9.11 \times 10^{-31}\right)\upsilon^2$$

$$\Rightarrow \upsilon = 1.874 \times 10^5 \, \text{m/s} = 1.874 \times 10^7 \, \text{cm/s}$$

$$1.2 \times 10^{-3} = N_t\left(1.6 \times 10^{-19}\right)\left(1.874 \times 10^7\right)$$

$$N_t = 4.002 \times 10^8 \, \text{electrons/cm}^3$$

Density of incident electrons,

$$N_i = \frac{4.002 \times 10^8}{0.0295} = 1.357 \times 10^{10} \, \text{cm}^{-3}$$

_____

2.36

$$T \cong 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp\left(-2k_2 a\right)$$

(a) For $m = (0.067)m_o$

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)\left(9.11 \times 10^{-31}\right)\left(0.8 - 0.2\right)\left(1.6 \times 10^{-19}\right)}{\left(1.054 \times 10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 1.027 \times 10^9 \, \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1 - \frac{0.2}{0.8}\right)$$

$$\times \exp\left[-2\left(1.027 \times 10^9\right)\left(15 \times 10^{-10}\right)\right]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$

$k_2 =$

$$\left\{\frac{2(1.08)\left(9.11 \times 10^{-31}\right)\left(0.8 - 0.2\right)\left(1.6 \times 10^{-19}\right)}{\left(1.054 \times 10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 4.124 \times 10^9 \, \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1 - \frac{0.2}{0.8}\right)$$

$$\times \exp\left[-2\left(4.124 \times 10^9\right)\left(15 \times 10^{-10}\right)\right]$$

or

$$T = 1.27 \times 10^{-5}$$

_____

2.37

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1 - \frac{E}{V_o}\right)\exp\left(-2k_2 a\right)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(1.67 \times 10^{-27}\right)(12 - 1) \times 10^6 \times \left(1.6 \times 10^{-19}\right)}}{1.054 \times 10^{-34}}$$

$$= 7.274 \times 10^{14} \, \text{m}^{-1}$$

(a)

$$T \cong 16\left(\frac{1}{12}\right)\left(1 - \frac{1}{12}\right)\exp\left[-2\left(7.274 \times 10^{14}\right)\left(10^{-14}\right)\right]$$

$$= 1.222\exp\left[-14.548\right]$$

$$= 5.875 \times 10^{-7}$$

(b)

$$T = (10)\left(5.875 \times 10^{-7}\right)$$

$$= 1.222\exp\left[-2\left(7.274 \times 10^{14}\right)a\right]$$

$$2\left(7.274 \times 10^{14}\right)a = \ln\left(\frac{1.222}{5.875 \times 10^{-6}}\right)$$

or $a = 0.842 \times 10^{-14}$ m

_____

2.38

Region I $(x < 0)$, $V = 0$;

Region II $(0 < x < a)$, $V = V_o$

Region III $(x > a)$, $V = 0$

(a) Region I:

$$\psi_1(x) = A_1 \exp(jk_1 x) + B_1 \exp(-jk_1 x)$$

       (incident)   (reflected)

_____

where
$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:
$$\psi_2(x) = A_2 \exp(k_2 x) + B_2 \exp(-k_2 x)$$
where
$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

Region III:
$$\psi_3(x) = A_3 \exp(jk_1 x) + B_3 \exp(-jk_1 x)$$

(b)

In Region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into Region III, there will not be a reflected wave so that $B_3 = 0$.

(c) Boundary conditions:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$
$$A_1 + B_1 = A_2 + B_2$$
$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow$$
$$jk_1 A_1 - jk_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$
$$A_2 \exp(k_2 a) + B_2 \exp(-k_2 a)$$
$$= A_3 \exp(jk_1 a)$$
$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$
$$k_2 A_2 \exp(k_2 a) - k_2 B_2 \exp(-k_2 a)$$
$$= jk_1 A_3 \exp(jk_1 a)$$

The transmission coefficient is defined as
$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find
$$A_1 = \frac{+jA_3}{4k_1 k_2} \left\{ (k_2^2 - k_1^2)[\exp(k_2 a) - \exp(-k_2 a)] \right.$$
$$\left. - 2jk_1 k_2 [\exp(k_2 a) + \exp(-k_2 a)] \right\}$$
$$\times \exp(jk_1 a)$$

We then find

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \left\{ (k_2^2 - k_1^2)[\exp(k_2 a) \right.$$
$$- \exp(-k_2 a)]^2$$
$$\left. + 4k_1^2 k_2^2 [\exp(k_2 a) + \exp(-k_2 a)]^2 \right\}$$

We have
$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

If we assume that $V_O \gg E$, then $k_2 a$ will be large so that
$$\exp(k_2 a) \gg \exp(-k_2 a)$$

We can then write
$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \left\{ (k_2^2 - k_1^2)[\exp(k_2 a)]^2 \right.$$
$$\left. + 4k_1^2 k_2^2 [\exp(k_2 a)]^2 \right\}$$

which becomes
$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} (k_2^2 + k_1^2) \exp(2k_2 a)$$

Substituting the expressions for k_1 and k_2, we find
$$k_1^2 + k_2^2 = \frac{2mV_O}{\hbar^2}$$

and
$$k_1^2 k_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2} \right]\left[\frac{2mE}{\hbar^2} \right]$$
$$= \left(\frac{2m}{\hbar^2} \right)^2 (V_O - E)(E)$$
$$= \left(\frac{2m}{\hbar^2} \right)^2 (V_O)\left(1 - \frac{E}{V_O} \right)(E)$$

Then
$$A_1 A_1^* = \frac{A_3 A_3^* \left(\frac{2mV_O}{\hbar^2} \right)^2 \exp(2k_2 a)}{16\left[\left(\frac{2m}{\hbar^2} \right)^2 V_O \left(1 - \frac{E}{V_O} \right)(E) \right]}$$
$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O} \right)\left(1 - \frac{E}{V_O} \right)\exp(-2k_2 a)}$$

Finally,
$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O} \right)\left(1 - \frac{E}{V_O} \right)\exp(-2k_2 a)$$

_____

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2.39

Region I: $V = 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_1(x) = 0 \Rightarrow$$

$$\psi_1(x) = A_1 \exp(jk_1 x) + B_1 \exp(-jk_1 x)$$
$$\text{incident} \qquad \text{reflected}$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: $V = V_1$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2} \psi_2(x) = 0 \Rightarrow$$

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$
$$\text{transmitted} \qquad \text{reflected}$$

where

$$k_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$$

Region III: $V = V_2$

$$\frac{\partial^2 \psi_3(x)}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2} \psi_3(x) = 0 \Rightarrow$$

$$\psi_3(x) = A_3 \exp(jk_3 x)$$
$$\text{transmitted}$$

where

$$k_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$$

There is no reflected wave in Region III.
The transmission coefficient is defined as:

$$T = \frac{v_3}{v_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{k_3}{k_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From the boundary conditions, solve for A_3
in terms of A_1. The boundary conditions are:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow$$

$$k_1 A_1 - k_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jk_2 a) + B_2 \exp(-jk_2 a)$$
$$= A_3 \exp(jk_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$k_2 A_2 \exp(jk_2 a) - k_2 B_2 \exp(-jk_2 a)$$
$$= k_3 A_3 \exp(jk_3 a)$$

But $k_2 a = 2n\pi \Rightarrow$

$$\exp(jk_2 a) = \exp(-jk_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the
boundary condition equations, we find

$$T = \frac{k_3}{k_1} \cdot \frac{4k_1^2}{(k_1 + k_3)^2} = \frac{4k_1 k_3}{(k_1 + k_3)^2}$$

_____

2.40

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2} \psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written,
keeping in mind that ψ_1 must remain
finite for $x < 0$, as

$$\psi_1(x) = B_1 \exp(k_1 x)$$
$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$$
$$\psi_3(x) = 0$$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right) B_2$$

_____

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right) B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right) \tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

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2.41

$$E_n = \frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \quad \text{(J)}$$

$$= \frac{-m_o e^3}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \quad \text{(eV)}$$

$$= \frac{-(9.11\times10^{-31})(1.6\times10^{-19})^3}{[4\pi(8.85\times10^{-12})]^2 2(1.054\times10^{-34})^2 n^2}$$

or

$$E_n = \frac{-13.58}{n^2} \quad \text{(eV)}$$

$$n = 1 \Rightarrow E_1 = -13.58\,\text{eV}$$
$$n = 2 \Rightarrow E_2 = -3.395\,\text{eV}$$
$$n = 3 \Rightarrow E_3 = -1.51\,\text{eV}$$
$$n = 4 \Rightarrow E_4 = -0.849\,\text{eV}$$

_____

2.42

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100}\psi_{100}^*$$

$$= 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3} \left\{ \left(\frac{-2}{a_o}\right)(r^2)\exp\left(\frac{-2r}{a_o}\right) \right.$$

$$\left. + 2r \exp\left(\frac{-2r}{a_o}\right) \right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow r = a_o$$

or $r = a_o$ is the radius that gives the greatest probability.

_____

2.43

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}(E - V(r))\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi\epsilon_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

Then

$$\frac{\partial \psi_{100}}{\partial r} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left(\frac{-1}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)$$

so

$$r^2 \frac{\partial \psi_{100}}{\partial r} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

We then obtain

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi_{100}}{\partial r}\right) = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2}$$

$$\times \left[2r \exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)\right]$$

Substituting into the wave equation, we have

_____

$$\frac{-1}{r^2\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right)-\frac{r^2}{a_o}\exp\left(\frac{-r}{a_o}\right)\right]$$

$$+\frac{2m_o}{\hbar^2}\left[E+\frac{\hbar^2}{m_o a_o r}\right]$$

$$\times\left(\frac{1}{\sqrt{\pi}}\right)\cdot\left(\frac{1}{a_o}\right)^{3/2}\exp\left(\frac{-r}{a_o}\right)=0$$

where

$$E=E_1=\frac{-m_o e^4}{\left(4\pi\in_o\right)^2 2\hbar^2}=\frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]\left\{\frac{-1}{r^2 a_o}\left[2r-\frac{r^2}{a_o}\right]\right.$$

$$\left.+\frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o}+\frac{\hbar^2}{m_o a_o r}\right)\right\}=0$$

or

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]$$

$$\times\left\{\frac{-2}{a_o r}+\frac{1}{a_o^2}+\left(\frac{-1}{a_o^2}+\frac{2}{a_o r}\right)\right\}=0$$

which gives $0=0$ and shows that ψ_{100} is indeed a solution to the wave equation.

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2.44

All elements are from the Group I column of the periodic table. All have one valence electron in the outer shell.

_____

Chapter 3

3.1

If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

_____

3.2

Schrodinger's wave equation is:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\cdot\Psi(x,t)$$

$$= j\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Assume the solution is of the form:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Region I: $V(x)=0$. Substituting the assumed solution into the wave equation, we obtain:

$$\frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\left\{jku(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$\left.+\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$= j\hbar\left(\frac{-jE}{\hbar}\right)\cdot u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

which becomes

$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$\left.+\frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$= +Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation may be written as

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2 u(x)}{\partial x^2}+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_1(x)$ for region I, the equation becomes:

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$

where

$$\alpha^2=\frac{2mE}{\hbar^2}\qquad\qquad\text{Q.E.D.}$$

In Region II, $V(x)=V_O$. Assume the same form of the solution:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Substituting into Schrodinger's wave equation, we find:

$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right.$$

$$+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$\left.+\frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$+V_Ou(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$= Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation can be written as:

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2 u(x)}{\partial x^2}$$

$$-\frac{2mV_O}{\hbar^2}u(x)+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_2(x)$ for region II, this equation becomes

$$\frac{d^2u_2(x)}{dx^2}+2jk\frac{du_2(x)}{dx}$$

$$-\left(k^2-\alpha^2+\frac{2mV_O}{\hbar^2}\right)u_2(x)=0$$

where again

$$\alpha^2=\frac{2mE}{\hbar^2}\qquad\qquad\text{Q.E.D.}$$

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3.3

We have

$$\frac{d^2 u_1(x)}{dx^2} + 2jk\frac{du_1(x)}{dx} - \left(k^2 - \alpha^2\right)u_1(x) = 0$$

Assume the solution is of the form:

$$u_1(x) = A\exp[j(\alpha - k)x]$$
$$+ B\exp[-j(\alpha + k)x]$$

The first derivative is

$$\frac{du_1(x)}{dx} = j(\alpha - k)A\exp[j(\alpha - k)x]$$
$$- j(\alpha + k)B\exp[-j(\alpha + k)x]$$

and the second derivative becomes

$$\frac{d^2 u_1(x)}{dx^2} = [j(\alpha - k)]^2 A\exp[j(\alpha - k)x]$$
$$+ [j(\alpha + k)]^2 B\exp[-j(\alpha + k)x]$$

Substituting these equations into the differential equation, we find

$$-(\alpha - k)^2 A\exp[j(\alpha - k)x]$$
$$-(\alpha + k)^2 B\exp[-j(\alpha + k)x]$$
$$+ 2jk\{j(\alpha - k)A\exp[j(\alpha - k)x]$$
$$- j(\alpha + k)B\exp[-j(\alpha + k)x]\}$$
$$-(k^2 - \alpha^2)\{A\exp[j(\alpha - k)x]$$
$$+ B\exp[-j(\alpha + k)x]\} = 0$$

Combining terms, we obtain

$$\left[-\left(\alpha^2 - 2\alpha k + k^2\right) - 2k(\alpha - k) - \left(k^2 - \alpha^2\right)\right]$$
$$\times A\exp[j(\alpha - k)x]$$
$$+\left[-\left(\alpha^2 + 2\alpha k + k^2\right) + 2k(\alpha + k) - \left(k^2 - \alpha^2\right)\right]$$
$$\times B\exp[-j(\alpha + k)x] = 0$$

We find that

$$0 = 0 \qquad\qquad\qquad \text{Q.E.D.}$$

For the differential equation in $u_2(x)$ and the proposed solution, the procedure is exactly the same as above.

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3.4

We have the solutions

$$u_1(x) = A\exp[j(\alpha - k)x]$$
$$+ B\exp[-j(\alpha + k)x]$$

for $0 < x < a$ and

$$u_2(x) = C\exp[j(\beta - k)x]$$
$$+ D\exp[-j(\beta + k)x]$$

for $-b < x < 0$.

The first boundary condition is

$$u_1(0) = u_2(0)$$

which yields

$$A + B - C - D = 0$$

The second boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=0} = \left.\frac{du_2}{dx}\right|_{x=0}$$

which yields

$$(\alpha - k)A - (\alpha + k)B - (\beta - k)C$$
$$+ (\beta + k)D = 0$$

The third boundary condition is

$$u_1(a) = u_2(-b)$$

which yields

$$A\exp[j(\alpha - k)a] + B\exp[-j(\alpha + k)a]$$
$$= C\exp[j(\beta - k)(-b)]$$
$$+ D\exp[-j(\beta + k)(-b)]$$

and can be written as

$$A\exp[j(\alpha - k)a] + B\exp[-j(\alpha + k)a]$$
$$- C\exp[-j(\beta - k)b]$$
$$- D\exp[j(\beta + k)b] = 0$$

The fourth boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=a} = \left.\frac{du_2}{dx}\right|_{x=-b}$$

which yields

$$j(\alpha - k)A\exp[j(\alpha - k)a]$$
$$- j(\alpha + k)B\exp[-j(\alpha + k)a]$$
$$= j(\beta - k)C\exp[j(\beta - k)(-b)]$$
$$- j(\beta + k)D\exp[-j(\beta + k)(-b)]$$

and can be written as

$$(\alpha - k)A\exp[j(\alpha - k)a]$$
$$-(\alpha + k)B\exp[-j(\alpha + k)a]$$
$$-(\beta - k)C\exp[-j(\beta - k)b]$$
$$+(\beta + k)D\exp[j(\beta + k)b] = 0$$

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3.5

(b) (i) First point: $\alpha a = \pi$

Second point: By trial and error,
$$\alpha a = 1.729\pi$$

(ii) First point: $\alpha a = 2\pi$

Second point: By trial and error,
$$\alpha a = 2.617\pi$$

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3.6

(b) (i) First point: $\alpha a = \pi$

Second point: By trial and error,
$$\alpha a = 1.515\pi$$

(ii) First point: $\alpha a = 2\pi$

Second point: By trial and error,
$$\alpha a = 2.375\pi$$

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3.7

$$P' \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Let $ka = y$, $\alpha a = x$

Then

$$P' \frac{\sin x}{x} + \cos x = \cos y$$

Consider $\dfrac{d}{dy}$ of this function.

$$\frac{d}{dy}\left\{ \left[P' \cdot (x)^{-1} \sin x \right] + \cos x \right\} = -\sin y$$

We find

$$P' \left\{ (-1)(x)^{-2} \sin x \cdot \frac{dx}{dy} + (x)^{-1} \cos x \cdot \frac{dx}{dy} \right\}$$

$$- \sin x \frac{dx}{dy} = -\sin y$$

Then

$$\frac{dx}{dy}\left\{ P'\left[\frac{-1}{x^2} \sin x + \frac{\cos x}{x} \right] - \sin x \right\} = -\sin y$$

For $y = ka = n\pi$, $n = 0, 1, 2, \ldots \Rightarrow \sin y = 0$

So that, in general,

$$\frac{dx}{dy} = 0 = \frac{d(\alpha a)}{d(ka)} = \frac{d\alpha}{dk}$$

And

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

So

$$\frac{d\alpha}{dk} = \frac{1}{2}\left(\frac{2mE}{\hbar^2} \right)^{-1/2}\left(\frac{2m}{\hbar^2} \right)\frac{dE}{dk}$$

This implies that

$$\frac{d\alpha}{dk} = 0 = \frac{dE}{dk} \text{ for } k = \frac{n\pi}{a}$$

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3.8

(a) $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{\pi^2 \hbar^2}{2m_o a^2} = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19} \text{ J}$$

From Problem 3.5

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.0198 \times 10^{-18} \text{ J}$$

$$\Delta E = E_2 - E_1$$

$$= 1.0198 \times 10^{-18} - 3.4114 \times 10^{-19}$$

$$= 6.7868 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{6.7868 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.24 \text{ eV}$

(b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18} \text{ J}$$

From Problem 3.5,

$$\alpha_4 a = 2.617\pi$$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}} \cdot a = 2.617\pi$$

$$E_4 = \frac{(2.617\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 2.3364 \times 10^{-18} \text{ J}$$

$$\Delta E = E_4 - E_3$$

$$= 2.3364 \times 10^{-18} - 1.3646 \times 10^{-18}$$

$$= 9.718 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{9.718 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.07 \text{ eV}$

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3.9

  (a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19}\,\text{J}$$

At $ka = 0$, By trial and error,

$$\alpha_o a = 0.859\pi$$

$$E_o = \frac{(0.859\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 2.5172 \times 10^{-19}\,\text{J}$$

$$\Delta E = E_1 - E_o$$

$$= 3.4114 \times 10^{-19} - 2.5172 \times 10^{-19}$$

$$= 8.942 \times 10^{-20}\,\text{J}$$

or $\Delta E = \dfrac{8.942 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.559$ eV

  (b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18}\,\text{J}$$

At $ka = \pi$. From Problem 3.5,

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.0198 \times 10^{-18}\,\text{J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646 \times 10^{-18} - 1.0198 \times 10^{-18}$$

$$= 3.4474 \times 10^{-19}\,\text{J}$$

or $\Delta E = \dfrac{3.4474 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.15$ eV

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3.10

  (a) $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19}\,\text{J}$$

From Problem 3.6, $\alpha_2 a = 1.515\pi$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 7.830 \times 10^{-19}\,\text{J}$$

$$\Delta E = E_2 - E_1$$

$$= 7.830 \times 10^{-19} - 3.4114 \times 10^{-19}$$

$$= 4.4186 \times 10^{-19}\,\text{J}$$

or $\Delta E = \dfrac{4.4186 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.76$ eV

  (b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18}\,\text{J}$$

From Problem 3.6, $\alpha_4 a = 2.375\pi$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}} \cdot a = 2.375\pi$$

$$E_4 = \frac{(2.375\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.9242 \times 10^{-18}\,\text{J}$$

$$\Delta E = E_4 - E_3$$

$$= 1.9242 \times 10^{-18} - 1.3646 \times 10^{-18}$$

$$= 5.597 \times 10^{-19}\,\text{J}$$

or $\Delta E = \dfrac{5.597 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.50$ eV

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3.11

(a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19} \, \text{J}$$

At $ka = 0$, By trial and error,

$$\alpha_o a = 0.727\pi$$

$$\sqrt{\frac{2m_o E_o}{\hbar^2}} \cdot a = 0.727\pi$$

$$E_o = \frac{(0.727\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.8030 \times 10^{-19} \, \text{J}$$

$$\Delta E = E_1 - E_o$$

$$= 3.4114 \times 10^{-19} - 1.8030 \times 10^{-19}$$

$$= 1.6084 \times 10^{-19} \, \text{J}$$

or $\Delta E = \dfrac{1.6084 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.005 \, \text{eV}$

(b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18} \, \text{J}$$

At $ka = \pi$, From Problem 3.6,

$$\alpha_2 a = 1.515\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-34})(4.2 \times 10^{-10})^2}$$

$$= 7.830 \times 10^{-19} \, \text{J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646 \times 10^{-18} - 7.830 \times 10^{-19}$$

$$= 5.816 \times 10^{-19} \, \text{J}$$

or $\Delta E = \dfrac{5.816 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.635 \, \text{eV}$

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3.12

For $T = 100 \, \text{K}$,

$$E_g = 1.170 - \frac{(4.73 \times 10^{-4})(100)^2}{636 + 100} \Rightarrow$$

$$E_g = 1.164 \, \text{eV}$$

$$T = 200 \, \text{K}, \quad E_g = 1.147 \, \text{eV}$$

$$T = 300 \, \text{K}, \quad E_g = 1.125 \, \text{eV}$$

$$T = 400 \, \text{K}, \quad E_g = 1.097 \, \text{eV}$$

$$T = 500 \, \text{K}, \quad E_g = 1.066 \, \text{eV}$$

$$T = 600 \, \text{K}, \quad E_g = 1.032 \, \text{eV}$$

_____

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2} \right)^{-1}$$

We have

$$\frac{d^2 E}{dk^2} (curve \, A) > \frac{d^2 E}{dk^2} (curve \, B)$$

so that $m^* (curve \, A) < m^* (curve \, B)$

_____

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2} \cdot \left| \frac{d^2 E}{dk^2} \right| \right)^{-1}$$

We have that

$$\left| \frac{d^2 E}{dk^2} \right| (curve \, A) > \left| \frac{d^2 E}{dk^2} \right| (curve \, B)$$

so that $m_p^* (curve \, A) < m_p^* (curve \, B)$

_____

3.15

Points A,B: $\dfrac{dE}{dk} < 0 \Rightarrow$ velocity in -x direction

Points C,D: $\dfrac{dE}{dk} > 0 \Rightarrow$ velocity in +x direction

Points A,D: $\dfrac{d^2 E}{dk^2} < 0 \Rightarrow$

negative effective mass

Points B,C: $\dfrac{d^2 E}{dk^2} > 0 \Rightarrow$

positive effective mass

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3.16

For A: $E = C_i k^2$

At $k = 0.08 \times 10^{+10} \text{ m}^{-1}$, $E = 0.05 \text{ eV}$

Or $E = (0.05)(1.6 \times 10^{-19}) = 8 \times 10^{-21} \text{ J}$

So $8 \times 10^{-21} = C_1 (0.08 \times 10^{10})^2$

$\Rightarrow C_1 = 1.25 \times 10^{-38}$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{(1.054 \times 10^{-34})^2}{2(1.25 \times 10^{-38})}$

$= 4.44 \times 10^{-31} \text{ kg}$

or $m^* = \dfrac{4.4437 \times 10^{-31}}{9.11 \times 10^{-31}} \cdot m_o$

$m^* = 0.488 \, m_o$

For B: $E = C_i k^2$

At $k = 0.08 \times 10^{+10} \text{ m}^{-1}$, $E = 0.5 \text{ eV}$

Or $E = (0.5)(1.6 \times 10^{-19}) = 8 \times 10^{-20} \text{ J}$

So $8 \times 10^{-20} = C_1 (0.08 \times 10^{10})^2$

$\Rightarrow C_1 = 1.25 \times 10^{-37}$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{(1.054 \times 10^{-34})^2}{2(1.25 \times 10^{-37})}$

$= 4.44 \times 10^{-32} \text{ kg}$

or $m^* = \dfrac{4.4437 \times 10^{-32}}{9.11 \times 10^{-31}} \cdot m_o$

$m^* = 0.0488 \, m_o$

_____

3.17

For A: $E - E_\upsilon = -C_2 k^2$

$-(0.025)(1.6 \times 10^{-19}) = -C_2 (0.08 \times 10^{10})^2$

$\Rightarrow C_2 = 6.25 \times 10^{-39}$

$m^* = \dfrac{-\hbar^2}{2C_2} = \dfrac{-(1.054 \times 10^{-34})^2}{2(6.25 \times 10^{-39})}$

$= -8.8873 \times 10^{-31} \text{ kg}$

or $m^* = \dfrac{-8.8873 \times 10^{-31}}{9.11 \times 10^{-31}} \cdot m_o$

$m^* = --0.976 \, m_o$

For B: $E - E_\upsilon = -C_2 k^2$

$-(0.3)(1.6 \times 10^{-19}) = -C_2 (0.08 \times 10^{10})^2$

$\Rightarrow C_2 = 7.5 \times 10^{-38}$

$m^* = \dfrac{-\hbar^2}{2C_2} = \dfrac{-(1.054 \times 10^{-34})^2}{2(7.5 \times 10^{-38})}$

$= -7.406 \times 10^{-32} \text{ kg}$

or $m^* = \dfrac{-7.406 \times 10^{-32}}{9.11 \times 10^{-31}} \cdot m_o$

$m^* = -0.0813 \, m_o$

_____

3.18

(a) (i) $E = h\nu$

or $\nu = \dfrac{E}{h} = \dfrac{(1.42)(1.6 \times 10^{-19})}{6.625 \times 10^{-34}}$

$= 3.429 \times 10^{14} \text{ Hz}$

(ii) $\lambda = \dfrac{hc}{E} = \dfrac{c}{\nu} = \dfrac{3 \times 10^{10}}{3.429 \times 10^{14}}$

$= 8.75 \times 10^{-5} \text{ cm} = 875 \text{ nm}$

(b) (i) $\nu = \dfrac{E}{h} = \dfrac{(1.12)(1.6 \times 10^{-19})}{6.625 \times 10^{-34}}$

$= 2.705 \times 10^{14} \text{ Hz}$

(ii) $\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{10}}{2.705 \times 10^{14}}$

$= 1.109 \times 10^{-4} \text{ cm} = 1109 \text{ nm}$

_____

3.19

(c) Curve A: Effective mass is a constant

Curve B: Effective mass is positive around $k = 0$, and is negative around $k = \pm \dfrac{\pi}{2}$.

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3.20

$E = E_O - E_1 \cos[\alpha(k - k_O)]$

Then

$\dfrac{dE}{dk} = (-E_1)(-\alpha)\sin[\alpha(k - k_O)]$

$= +E_1 \alpha \sin[\alpha(k - k_O)]$

and

$\dfrac{d^2 E}{dk^2} = E_1 \alpha^2 \cos[\alpha(k - k_O)]$

_____

Then
$$\frac{1}{m^*} = \frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2}\bigg|_{k=k_o} = \frac{E_1 \alpha^2}{\hbar^2}$$
or
$$m^* = \frac{\hbar^2}{E_1 \alpha^2}$$

_____

3.21

(a) $\quad m_{dn}^* = 4^{2/3}\left[(m_t)^2 m_l\right]^{1/3}$
$$= 4^{2/3}\left[(0.082 m_o)^2 (1.64 m_o)\right]^{1/3}$$
$$m_{dn}^* = 0.56 m_o$$

(b) $\quad \dfrac{3}{m_{cn}^*} = \dfrac{2}{m_t} + \dfrac{1}{m_l} = \dfrac{2}{0.082 m_o} + \dfrac{1}{1.64 m_o}$
$$= \frac{24.39}{m_o} + \frac{0.6098}{m_o}$$
$$m_{cn}^* = 0.12 m_o$$

_____

3.22

(a) $\quad m_{dp}^* = \left[(m_{hh})^{3/2} + (m_{lh})^{3/2}\right]^{2/3}$
$$= \left[(0.45 m_o)^{3/2} + (0.082 m_o)^{3/2}\right]^{2/3}$$
$$= [0.30187 + 0.02348]^{2/3} \cdot m_o$$
$$m_{dp}^* = 0.473 m_o$$

(b) $\quad m_{cp}^* = \dfrac{(m_{hh})^{3/2} + (m_{lh})^{3/2}}{(m_{hh})^{1/2} + (m_{lh})^{1/2}}$
$$= \frac{(0.45)^{3/2} + (0.082)^{3/2}}{(0.45)^{1/2} + (0.082)^{1/2}} \cdot m_o$$
$$m_{cp}^* = 0.34 m_o$$

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3.23

For the 3-dimensional infinite potential well,
$V(x) = 0$ when $0 < x < a$, $0 < y < a$, and
$0 < z < a$. In this region, the wave equation
is:
$$\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2}$$
$$+ \frac{2mE}{\hbar^2} \psi(x, y, z) = 0$$
Use separation of variables technique, so let
$$\psi(x, y, z) = X(x)Y(y)Z(z)$$
Substituting into the wave equation, we have

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2}$$
$$+ \frac{2mE}{\hbar^2} \cdot XYZ = 0$$
Dividing by XYZ, we obtain
$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0$$
Let
$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$
The solution is of the form:
$$X(x) = A \sin k_x x + B \cos k_x x$$
Since $\psi(x, y, z) = 0$ at $x = 0$, then $X(0) = 0$
so that $B = 0$.
Also, $\psi(x, y, z) = 0$ at $x = a$, so that
$X(a) = 0$. Then $k_x a = n_x \pi$ where
$n_x = 1, 2, 3, \ldots$
Similarly, we have
$$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -k_y^2 \quad \text{and} \quad \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} = -k_z^2$$
From the boundary conditions, we find
$$k_y a = n_y \pi \quad \text{and} \quad k_z a = n_z \pi$$
where
$$n_y = 1, 2, 3, \ldots \quad \text{and} \quad n_z = 1, 2, 3, \ldots$$
From the wave equation, we can write
$$-k_x^2 - k_y^2 - k_z^2 + \frac{2mE}{\hbar^2} = 0$$
The energy can be written as
$$E = E_{n_x n_y n_z} = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

_____

3.24

The total number of quantum states in the
3-dimensional potential well is given
(in k-space) by
$$g_T(k)dk = \frac{\pi k^2 dk}{\pi^3} \cdot a^3$$
where
$$k^2 = \frac{2mE}{\hbar^2}$$
We can then write
$$k = \frac{\sqrt{2mE}}{\hbar}$$
Taking the differential, we obtain

_____

$$dk = \frac{1}{\hbar} \cdot \sqrt{2m} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{E}} \cdot dE = \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Substituting these expressions into the density of states function, we have

$$g_T(E)dE = \frac{\pi a^3}{\pi^3}\left(\frac{2mE}{\hbar^2}\right) \cdot \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Noting that

$$\hbar = \frac{h}{2\pi}$$

this density of states function can be simplified and written as

$$g_T(E)dE = \frac{4\pi a^3}{h^3}(2m)^{3/2} \cdot \sqrt{E} \cdot dE$$

Dividing by a^3 will yield the density of states so that

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \sqrt{E}$$

_____

3.25

For a one-dimensional infinite potential well,

$$\frac{2m_n^*E}{\hbar^2} = \frac{n^2\pi^2}{a^2} = k^2$$

Distance between quantum states

$$k_{n+1} - k_n = (n+1)\left(\frac{\pi}{a}\right) = (n)\left(\frac{\pi}{a}\right) = \frac{\pi}{a}$$

Now

$$g_T(k)dk = \frac{2 \cdot dk}{\left(\frac{\pi}{a}\right)}$$

Now

$$k = \frac{1}{\hbar} \cdot \sqrt{2m_n^*E}$$

$$dk = \frac{1}{\hbar} \cdot \frac{1}{2} \cdot \sqrt{\frac{2m_n^*}{E}} \cdot dE$$

Then

$$g_T(E)dE = \frac{2a}{\pi} \cdot \frac{1}{2\hbar} \cdot \sqrt{\frac{2m_n^*}{E}} \cdot dE$$

Divide by the "volume" a, so

$$g(E) = \frac{1}{\hbar\pi} \cdot \sqrt{\frac{2m_n^*}{E}}$$

So

$$g(E) = \frac{1}{(1.054\times10^{-34})(\pi)} \cdot \frac{\sqrt{2(0.067)(9.11\times10^{-31})}}{\sqrt{E}}$$

$$g(E) = \frac{1.055\times10^{18}}{\sqrt{E}} \quad \text{m}^{-3}\,\text{J}^{-1}$$

_____

3.26

(a) Silicon, $m_n^* = 1.08m_o$

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

$$g_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2kT}\sqrt{E - E_c} \cdot dE$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot (E - E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot (2kT)^{3/2}$$

$$= \frac{4\pi[2(1.08)(9.11\times10^{-31})]^{3/2}}{(6.625\times10^{-34})^3} \cdot \frac{2}{3} \cdot (2kT)^{3/2}$$

$$= (7.953\times10^{55})(2kT)^{3/2}$$

(i) At $T = 300$ K, $kT = 0.0259$ eV

$$= (0.0259)(1.6\times10^{-19})$$
$$= 4.144\times10^{-21}\,\text{J}$$

Then $g_c = (7.953\times10^{55})[2(4.144\times10^{-21})]^{3/2}$

$$= 6.0\times10^{25}\,\text{m}^{-3}$$

or $\quad g_c = 6.0\times10^{19}\,\text{cm}^{-3}$

(ii) At $T = 400$ K, $kT = (0.0259)\left(\frac{400}{300}\right)$

$$= 0.034533\,\text{eV}$$
$$= (0.034533)(1.6\times10^{-19})$$
$$= 5.5253\times10^{-21}\,\text{J}$$

Then

$$g_c = (7.953\times10^{55})[2(5.5253\times10^{-21})]^{3/2}$$
$$= 9.239\times10^{25}\,\text{m}^{-3}$$

or $\quad g_c = 9.24\times10^{19}\,\text{cm}^{-3}$

(b) GaAs, $m_n^* = 0.067m_o$

$$g_c = \frac{4\pi[2(0.067)(9.11\times10^{-31})]^{3/2}}{(6.625\times10^{-34})^3} \cdot \frac{2}{3} \cdot (2kT)^{3/2}$$

$$= (1.2288\times10^{54})(2kT)^{3/2}$$

_____

(i) At $T = 300\,\text{K}$, $kT = 4.144 \times 10^{-21}\,\text{J}$

$$g_c = \left(1.2288 \times 10^{54}\right)\left[2\left(4.144 \times 10^{-21}\right)\right]^{3/2}$$
$$= 9.272 \times 10^{23}\,\text{m}^{-3}$$

or $g_c = 9.27 \times 10^{17}\,\text{cm}^{-3}$

(ii) At $T = 400\,\text{K}$, $kT = 5.5253 \times 10^{-21}\,\text{J}$

$$g_c = \left(1.2288 \times 10^{54}\right)\left[2\left(5.5253 \times 10^{-21}\right)\right]^{3/2}$$
$$= 1.427 \times 10^{24}\,\text{m}^{-3}$$
$$g_c = 1.43 \times 10^{18}\,\text{cm}^{-3}$$

_____

3.27

(a) Silicon, $m_p^* = 0.56 m_o$

$$g_v(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v - E}$$

$$g_v = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_v - 3kT}^{E_v}\sqrt{E_v - E}\cdot dE$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_v - E\right)^{3/2}\bigg|_{E_v - 3kT}^{E_v}$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[-\left(3kT\right)^{3/2}\right]$$

$$= \frac{4\pi\left[2(0.56)\left(9.11 \times 10^{-31}\right)\right]^{3/2}}{\left(6.625 \times 10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= \left(2.969 \times 10^{55}\right)\left(3kT\right)^{3/2}$$

(i) At $T = 300\,\text{K}$, $kT = 4.144 \times 10^{-21}\,\text{J}$

$$g_v = \left(2.969 \times 10^{55}\right)\left[3\left(4.144 \times 10^{-21}\right)\right]^{3/2}$$
$$= 4.116 \times 10^{25}\,\text{m}^{-3}$$

or $g_v = 4.12 \times 10^{19}\,\text{cm}^{-3}$

(ii) At $T = 400\,\text{K}$, $kT = 5.5253 \times 10^{-21}\,\text{J}$

$$g_v = \left(2.969 \times 10^{55}\right)\left[3\left(5.5253 \times 10^{-21}\right)\right]^{3/2}$$
$$= 6.337 \times 10^{25}\,\text{m}^{-3}$$

or $g_v = 6.34 \times 10^{19}\,\text{cm}^{-3}$

(b) GaAs, $m_p^* = 0.48 m_o$

$$g_v = \frac{4\pi\left[2(0.48)\left(9.11 \times 10^{-31}\right)\right]^{3/2}}{\left(6.625 \times 10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= \left(2.3564 \times 10^{55}\right)\left(3kT\right)^{3/2}$$

(i) At $T = 300\,\text{K}$, $kT = 4.144 \times 10^{-21}\,\text{J}$

$$g_v = \left(2.3564 \times 10^{55}\right)\left[3\left(4.144 \times 10^{-21}\right)\right]^{3/2}$$
$$= 3.266 \times 10^{25}\,\text{m}^{-3}$$

or $g_v = 3.27 \times 10^{19}\,\text{cm}^{-3}$

(ii) At $T = 400\,\text{K}$, $kT = 5.5253 \times 10^{-21}\,\text{J}$

$$g_v = \left(2.3564 \times 10^{55}\right)\left[3\left(5.5253 \times 10^{-21}\right)\right]^{3/2}$$
$$= 5.029 \times 10^{25}\,\text{m}^{-3}$$

or $g_v = 5.03 \times 10^{19}\,\text{cm}^{-3}$

_____

3.28

(a) $g_c(E) = \dfrac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_c}$

$$= \frac{4\pi\left[2(1.08)\left(9.11 \times 10^{-31}\right)\right]^{3/2}}{\left(6.625 \times 10^{-34}\right)^3}\sqrt{E - E_c}$$

$$= 1.1929 \times 10^{56}\sqrt{E - E_c}$$

For $E = E_c$; $\quad g_c = 0$

$E = E_c + 0.1\,\text{eV}$; $\quad g_c = 1.509 \times 10^{46}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_c + 0.2\,\text{eV}$; $\quad = 2.134 \times 10^{46}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_c + 0.3\,\text{eV}$; $\quad = 2.614 \times 10^{46}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_c + 0.4\,\text{eV}$; $\quad = 3.018 \times 10^{46}\,\text{m}^{-3}\,\text{J}^{-1}$

(b) $g_v = \dfrac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v - E}$

$$= \frac{4\pi\left[2(0.56)\left(9.11 \times 10^{-31}\right)\right]^{3/2}}{\left(6.625 \times 10^{-34}\right)^3}\sqrt{E_v - E}$$

$$= 4.4541 \times 10^{55}\sqrt{E_v - E}$$

For $E = E_v$; $\quad g_v = 0$

$E = E_v - 0.1\,\text{eV}$; $\quad g_v = 5.634 \times 10^{45}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_v - 0.2\,\text{eV}$; $\quad = 7.968 \times 10^{45}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_v - 0.3\,\text{eV}$; $\quad = 9.758 \times 10^{45}\,\text{m}^{-3}\,\text{J}^{-1}$

$E = E_v - 0.4\,\text{eV}$; $\quad = 1.127 \times 10^{46}\,\text{m}^{-3}\,\text{J}^{-1}$

_____

3.29

(a) $\dfrac{g_c}{g_v} = \dfrac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\dfrac{1.08}{0.56}\right)^{3/2} = 2.68$

(b) $\dfrac{g_c}{g_v} = \dfrac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\dfrac{0.067}{0.48}\right)^{3/2} = 0.0521$

_____

3.30

Plot

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3.31

(a) $W_i = \dfrac{g_i!}{N_i!(g_i - N_i)!} = \dfrac{10!}{(7!)(10-7)!}$

$= \dfrac{(10)(9)(8)(7!)}{(7!)(3!)} = \dfrac{(10)(9)(8)}{(3)(2)(1)} = 120$

(b) (i) $W_i = \dfrac{12!}{(10!)(12-10)!} = \dfrac{(12)(11)(10!)}{(10!)(2)(1)}$

$= 66$

(ii) $W_i = \dfrac{12!}{(8!)(12-8)!} = \dfrac{(12)(11)(10)(9)(8!)}{(8!)(4)(3)(2)(1)}$

$= 495$

_____

3.32

$f(E) = \dfrac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$

(a) $E - E_F = kT$, $f(E) = \dfrac{1}{1+\exp(1)} \Rightarrow$

$f(E) = 0.269$

(b) $E - E_F = 5kT$, $f(E) = \dfrac{1}{1+\exp(5)} \Rightarrow$

$f(E) = 6.69 \times 10^{-3}$

(c) $E - E_F = 10kT$, $f(E) = \dfrac{1}{1+\exp(10)} \Rightarrow$

$f(E) = 4.54 \times 10^{-5}$

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3.33

$1 - f(E) = 1 - \dfrac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$

or

$1 - f(E) = \dfrac{1}{1 + \exp\left(\dfrac{E_F - E}{kT}\right)}$

(a) $E_F - E = kT$, $1 - f(E) = 0.269$

(b) $E_F - E = 5kT$, $1 - f(E) = 6.69 \times 10^{-3}$

(c) $E_F - E = 10kT$, $1 - f(E) = 4.54 \times 10^{-5}$

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3.34

(a) $f_F \cong \exp\left[\dfrac{-(E - E_F)}{kT}\right]$

$E = E_c$; $f_F = \exp\left[\dfrac{-0.30}{0.0259}\right] = 9.32 \times 10^{-6}$

$E_c + \dfrac{kT}{2}$; $f_F = \exp\left[\dfrac{-(0.30 + 0.0259/2)}{0.0259}\right]$

$= 5.66 \times 10^{-6}$

$E_c + kT$; $f_F = \exp\left[\dfrac{-(0.30 + 0.0259)}{0.0259}\right]$

$= 3.43 \times 10^{-6}$

$E_c + \dfrac{3kT}{2}$; $f_F = \exp\left[\dfrac{-(0.30 + 3(0.0259/2))}{0.0259}\right]$

$= 2.08 \times 10^{-6}$

$E_c + 2kT$; $f_F = \exp\left[\dfrac{-(0.30 + 2(0.0259))}{0.0259}\right]$

$= 1.26 \times 10^{-6}$

(b) $1 - f_F = 1 - \dfrac{1}{1 + \exp\left[\dfrac{E - E_F}{kT}\right]}$

$\cong \exp\left[\dfrac{-(E_F - E)}{kT}\right]$

$E = E_v$; $1 - f_F = \exp\left[\dfrac{-0.25}{0.0259}\right] = 6.43 \times 10^{-5}$

$E_v - \dfrac{kT}{2}$; $1 - f_F = \exp\left[\dfrac{-(0.25 + 0.0259/2)}{0.0259}\right]$

$= 3.90 \times 10^{-5}$

$E_v - kT$; $1 - f_F = \exp\left[\dfrac{-(0.25 + 0.0259)}{0.0259}\right]$

$= 2.36 \times 10^{-5}$

$E_v - \dfrac{3kT}{2}$;

$1 - f_F = \exp\left[\dfrac{-(0.25 + 3(0.0259/2))}{0.0259}\right]$

$= 1.43 \times 10^{-5}$

$E_v - 2kT$;

$1 - f_F = \exp\left[\dfrac{-(0.25 + 2(0.0259))}{0.0259}\right]$

$= 8.70 \times 10^{-6}$

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3.35

$$f_F = \exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_c + kT - E_F)}{kT}\right]$$

and

$$1 - f_F = \exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$= \exp\left[\frac{-(E_F - (E_v - kT))}{kT}\right]$$

So $\exp\left[\frac{-(E_c + kT - E_F)}{kT}\right]$

$$= \exp\left[\frac{-(E_F - E_v + kT)}{kT}\right]$$

Then $E_c + kT - E_F = E_F - E_v + kT$

Or $E_F = \dfrac{E_c + E_v}{2} = E_{midgap}$

_____

3.36

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

For $n = 6$, Filled state

$$E_6 = \frac{(1.054 \times 10^{-34})^2 (6)^2 (\pi)^2}{2(9.11 \times 10^{-31})(12 \times 10^{-10})^2}$$

$$= 1.5044 \times 10^{-18}\,\text{J}$$

or $E_6 = \dfrac{1.5044 \times 10^{-18}}{1.6 \times 10^{-19}} = 9.40\,\text{eV}$

For $n = 7$, Empty state

$$E_7 = \frac{(1.054 \times 10^{-34})^2 (7)^2 (\pi)^2}{2(9.11 \times 10^{-31})(12 \times 10^{-10})^2}$$

$$= 2.048 \times 10^{-18}\,\text{J}$$

or $E_7 = \dfrac{2.048 \times 10^{-18}}{1.6 \times 10^{-19}} = 12.8\,\text{eV}$

Therefore $9.40 < E_F < 12.8\,\text{eV}$

_____

3.37

(a) For a 3-D infinite potential well

$$\frac{2mE}{\hbar^2} = \left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

For 5 electrons, the 5th electron occupies
the quantum state $n_x = 2, n_y = 2, n_z = 1$; so

$$E_5 = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

$$= \frac{(1.054 \times 10^{-34})^2 (\pi)^2 (2^2 + 2^2 + 1^2)}{2(9.11 \times 10^{-31})(12 \times 10^{-10})^2}$$

$$= 3.761 \times 10^{-19}\,\text{J}$$

or $E_5 = \dfrac{3.761 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.35\,\text{eV}$

For the next quantum state, which is empty,
the quantum state is $n_x = 1, n_y = 2, n_z = 2$.
This quantum state is at the same energy, so
$E_F = 2.35\,\text{eV}$

(b) For 13 electrons, the 13th electron
occupies the quantum state
$n_x = 3, n_y = 2, n_z = 3$; so

$$E_{13} = \frac{(1.054 \times 10^{-34})^2 (\pi)^2 (3^2 + 2^2 + 3^2)}{2(9.11 \times 10^{-31})(12 \times 10^{-10})^2}$$

$$= 9.194 \times 10^{-19}\,\text{J}$$

or $E_{13} = \dfrac{9.194 \times 10^{-19}}{1.6 \times 10^{-19}} = 5.746\,\text{eV}$

The 14th electron would occupy the quantum
state $n_x = 2, n_y = 3, n_z = 3$. This state is at
the same energy, so
$E_F = 5.746\,\text{eV}$

_____

3.38

The probability of a state at $E_1 = E_F + \Delta E$
being occupied is

$$f_1(E_1) = \frac{1}{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\dfrac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2 = E_F - \Delta E$
being empty is

$$1 - f_2(E_2) = 1 - \frac{1}{1 + \exp\left(\dfrac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1 + \exp\left(\dfrac{-\Delta E}{kT}\right)} = \frac{\exp\left(\dfrac{-\Delta E}{kT}\right)}{1 + \exp\left(\dfrac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1 + \exp\left(\dfrac{\Delta E}{kT}\right)}$$

so $f_1(E_1) = 1 - f_2(E_2)$ Q.E.D.

_____

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3.39

(a) At energy E_1, we want

$$\frac{\dfrac{1}{\exp\left(\dfrac{E_1 - E_F}{kT}\right)} - \dfrac{1}{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)}}{\dfrac{1}{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)}} = 0.01$$

This expression can be written as

$$\frac{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)}{\exp\left(\dfrac{E_1 - E_F}{kT}\right)} - 1 = 0.01$$

or

$$1 = (0.01)\exp\left(\frac{E_1 - E_F}{kT}\right)$$

Then

$$E_1 = E_F + kT\ln(100)$$

or

$$E_1 = E_F + 4.6kT$$

(b)

At $E = E_F + 4.6kT$,

$$f(E_1) = \frac{1}{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp(4.6)}$$

which yields

$$f(E_1) = 0.00990 \cong 0.01$$

_____

3.40

(a)

$$f_F = \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(5.80 - 5.50)}{0.0259}\right]$$

$$= 9.32 \times 10^{-6}$$

(b) $kT = (0.0259)\left(\dfrac{700}{300}\right) = 0.060433 \text{ eV}$

$$f_F = \exp\left[\frac{-0.30}{0.060433}\right] = 6.98 \times 10^{-3}$$

(c) $1 - f_F \cong \exp\left[\dfrac{-(E_F - E)}{kT}\right]$

$$0.02 = \exp\left[\frac{-0.25}{kT}\right]$$

or $\exp\left[\dfrac{+0.25}{kT}\right] = \dfrac{1}{0.02} = 50$

$$\frac{0.25}{kT} = \ln(50)$$

or

$$kT = \frac{0.25}{\ln(50)} = 0.063906 = (0.0259)\left(\frac{T}{300}\right)$$

which yields $T = 740 \text{ K}$

_____

3.41

(a)

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{7.15 - 7.0}{0.0259}\right)} = 0.00304$$

or 0.304%

(b) At $T = 1000 \text{ K}$, $kT = 0.08633 \text{ eV}$

Then

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{7.15 - 7.0}{0.08633}\right)} = 0.1496$$

or 14.96%

(c) $f(E) = \dfrac{1}{1 + \exp\left(\dfrac{6.85 - 7.0}{0.0259}\right)} = 0.997$

or 99.7%

(d)

At $E = E_F$, $f(E) = \dfrac{1}{2}$ for all temperatures

_____

3.42

(a) For $E = E_1$

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{E_1 - E_F}{kT}\right)} \cong \exp\left[\frac{-(E_1 - E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) = 9.32 \times 10^{-6}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.30 = 0.82 \text{ eV}$

Then

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\dfrac{-0.82}{0.0259}\right)}$$

or

_____

$$1 - f(E) \cong 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right) \right]$$

$$= \exp\left(\frac{-0.82}{0.0259}\right) = 1.78 \times 10^{-14}$$

(b) For $E_F - E_2 = 0.4$ eV,

$$E_1 - E_F = 0.72 \text{ eV}$$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

or

$$f(E) = 8.45 \times 10^{-13}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right]$$

$$= \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$1 - f(E) = 1.96 \times 10^{-7}$$

_____

3.43

(a) At $E = E_1$

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$f(E) = 9.32 \times 10^{-6}$$

At $E = E_2$, $E_F - E_2 = 1.42 - 0.3 = 1.12$ eV

So

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right]$$

$$= \exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$1 - f(E) = 1.66 \times 10^{-19}$$

(b) For $E_F - E_2 = 0.4$,

$$E_1 - E_F = 1.02 \text{ eV}$$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-1.02}{0.0259}\right)$$

or

$$f(E) = 7.88 \times 10^{-18}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right]$$

$$= \exp\left(\frac{-0.4}{0.0259}\right)$$

or $1 - f(E) = 1.96 \times 10^{-7}$

_____

3.44

$$f(E) = \left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-1}$$

so

$$\frac{df(E)}{dE} = (-1)\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-2}$$

$$\times \left(\frac{1}{kT}\right)\exp\left(\frac{E - E_F}{kT}\right)$$

or

$$\frac{df(E)}{dE} = \frac{\left(\frac{-1}{kT}\right)\exp\left(\frac{E - E_F}{kT}\right)}{\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^2}$$

(a) At $T = 0$ K, For

$$E < E_F \Rightarrow \exp(-\infty) = 0 \Rightarrow \frac{df}{dE} = 0$$

$$E > E_F \Rightarrow \exp(+\infty) = +\infty \Rightarrow \frac{df}{dE} = 0$$

At $E = E_F \Rightarrow \frac{df}{dE} = -\infty$

(b) At $T = 300$ K, $kT = 0.0259$ eV

For $E << E_F$, $\frac{df}{dE} = 0$

For $E >> E_F$, $\frac{df}{dE} = 0$

At $E = E_F$,

$$\frac{df}{dE} = \frac{\left(\frac{-1}{0.0259}\right)(1)}{(1+1)^2} = -9.65 \text{ (eV)}^{-1}$$

_____

(c) At $T = 500$ K, $kT = 0.04317$ eV

For $E << E_F$, $\dfrac{df}{dE} = 0$

For $E >> E_F$, $\dfrac{df}{dE} = 0$

At $E = E_F$,

$$\frac{df}{dE} = \frac{\left(\dfrac{-1}{0.04317}\right)(1)}{(1+1)^2} = -5.79\,(\text{eV})^{-1}$$

_____

3.45

(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12$ eV,

$$f(E) = \frac{1}{1+\exp\left[\dfrac{1.12}{2(0.0259)}\right]}$$

or

$$f(E) = 4.07 \times 10^{-10}$$

Ge: $E_g = 0.66$ eV

$$f(E) = \frac{1}{1+\exp\left[\dfrac{0.66}{2(0.0259)}\right]}$$

or

$$f(E) = 2.93 \times 10^{-6}$$

GaAs: $E_g = 1.42$ eV

$$f(E) = \frac{1}{1+\exp\left[\dfrac{1.42}{2(0.0259)}\right]}$$

or

$$f(E) = 1.24 \times 10^{-12}$$

(b) Using the results of Problem 3.38, the answers to part (b) are exactly the same as those given in part (a).

_____

3.46

(a) $f_F = \exp\left[\dfrac{-(E-E_F)}{kT}\right]$

$10^{-8} = \exp\left[\dfrac{-0.60}{kT}\right]$

or $\dfrac{0.60}{kT} = \ln\left(10^{+8}\right)$

$kT = \dfrac{0.60}{\ln\left(10^8\right)} = 0.032572$ eV

$0.032572 = (0.0259)\left(\dfrac{T}{300}\right)$

so $T = 377$ K

(b) $10^{-6} = \exp\left[\dfrac{-0.60}{kT}\right]$

$\dfrac{0.60}{kT} = \ln\left(10^{+6}\right)$

$kT = \dfrac{0.60}{\ln\left(10^6\right)} = 0.043429$

$0.043429 = (0.0259)\left(\dfrac{T}{300}\right)$

or $T = 503$ K

_____

3.47

(a) At $T = 200$ K,

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267 \text{ eV}$$

$$f_F = 0.05 = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

$$\exp\left(\frac{E-E_F}{kT}\right) = \frac{1}{0.05} - 1 = 19$$

$$E - E_F = kT\ln(19) = (0.017267)\ln(19)$$
$$= 0.05084 \text{ eV}$$

By symmetry, for $f_F = 0.95$,

$$E - E_F = -0.05084 \text{ eV}$$

Then $\Delta E = 2(0.05084) = 0.1017$ eV

(b) $T = 400$ K, $kT = 0.034533$ eV

For $f_F = 0.05$, from part (a),

$$E - E_F = kT\ln(19) = (0.034533)\ln(19)$$
$$= 0.10168 \text{ eV}$$

Then $\Delta E = 2(0.10168) = 0.2034$ eV

_____

_____

Chapter 4

4.1

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

$$= N_{cO} N_{\upsilon O}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

where N_{cO} and $N_{\upsilon O}$ are the values at 300 K.

		(a) Silicon
T (K)	kT (eV)	n_i (cm^{-3})
200	0.01727	7.68×10^4
400	0.03453	2.38×10^{12}
600	0.0518	9.74×10^{14}

	(b) Germanium	(c) GaAs
T (K)	n_i (cm^{-3})	n_i (cm^{-3})
200	2.16×10^{10}	1.38
400	8.60×10^{14}	3.28×10^9
600	3.82×10^{16}	5.72×10^{12}

_____

4.2

Plot

_____

4.3

(a) $n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$

$$\left(5\times10^{11}\right)^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$2.5\times10^{23} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3$$

$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 367.5\,\text{K}$

(b)

$$n_i^2 = \left(5\times10^{12}\right)^2 = 2.5\times10^{25}$$

$$= \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 417.5\,\text{K}$

_____

4.4

At $T = 200\,\text{K}$, $kT = (0.0259)\left(\frac{200}{300}\right)$

$$= 0.017267\,\text{eV}$$

At $T = 400\,\text{K}$, $kT = (0.0259)\left(\frac{400}{300}\right)$

$$= 0.034533\,\text{eV}$$

$$\frac{n_i^2(400)}{n_i^2(200)} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^2\right)^2} = 3.025\times10^{17}$$

$$= \frac{\left(\frac{400}{300}\right)^3 \exp\left[\frac{-E_g}{0.034533}\right]}{\left(\frac{200}{300}\right)^3 \exp\left[\frac{-E_g}{0.017267}\right]}$$

$$= 8\exp\left[\frac{E_g}{0.017267} - \frac{E_g}{0.034533}\right]$$

$$3.025\times10^{17} = 8\exp\left[E_g\left(57.9139 - 28.9578\right)\right]$$

or

$$E_g(28.9561) = \ln\left(\frac{3.025\times10^{17}}{8}\right) = 38.1714$$

or $E_g = 1.318\,\text{eV}$

Now

$$\left(7.70\times10^{10}\right)^2 = N_{co}N_{\upsilon o}\left(\frac{400}{300}\right)^3$$

$$\times\exp\left(\frac{-1.318}{0.034533}\right)$$

$$5.929\times10^{21} = N_{co}N_{\upsilon o}(2.370)\left(2.658\times10^{-17}\right)$$

so $N_{co}N_{\upsilon o} = 9.41\times10^{37}\,\text{cm}^{-6}$

_____

_____

4.5

$$\frac{n_i(B)}{n_i(A)} = \frac{\exp\left(\dfrac{-1.10}{kT}\right)}{\exp\left(\dfrac{-0.90}{kT}\right)} = \exp\left(\frac{-0.20}{kT}\right)$$

For $T = 200\,\text{K}$, $kT = 0.017267$ eV
For $T = 300\,\text{K}$, $kT = 0.0259$ eV
For $T = 400\,\text{K}$, $kT = 0.034533$ eV

(a) For $T = 200\,\text{K}$,
$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.017267}\right) = 9.325\times10^{-6}$$

(b) For $T = 300\,\text{K}$,
$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.0259}\right) = 4.43\times10^{-4}$$

(c) For $T = 400\,\text{K}$,
$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.034533}\right) = 3.05\times10^{-3}$$

_____

4.6

(a) $g_c f_F \propto \sqrt{E - E_c}\ \exp\left[\dfrac{-(E - E_F)}{kT}\right]$

$$\propto \sqrt{E - E_c}\ \exp\left[\frac{-(E - E_c)}{kT}\right]$$

$$\times \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Let $E - E_c = x$

Then $g_c f_F \propto \sqrt{x}\ \exp\left(\dfrac{-x}{kT}\right)$

To find the maximum value:
$$\frac{d(g_c f_F)}{dx} \propto \frac{1}{2} x^{-1/2}\ \exp\left(\frac{-x}{kT}\right)$$

$$-\frac{1}{kT}\cdot x^{1/2}\ \exp\left(\frac{-x}{kT}\right) = 0$$

which yields
$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

The maximum value occurs at
$$E = E_c + \frac{kT}{2}$$

_____

(b)

$$g_v(1 - f_F) \propto \sqrt{E_v - E}\ \exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$\propto \sqrt{E_v - E}\ \exp\left[\frac{-(E_v - E)}{kT}\right]$$

$$\times \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

Let $E_v - E = x$

Then $g_v(1 - f_F) \propto \sqrt{x}\ \exp\left(\dfrac{-x}{kT}\right)$

To find the maximum value
$$\frac{d\left[g_v(1 - f_F)\right]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\ \exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at
$$x = \frac{kT}{2}$$

or

$$E = E_v - \frac{kT}{2}$$

_____

4.7

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_c}\ \exp\left[\dfrac{-(E_1 - E_c)}{kT}\right]}{\sqrt{E_2 - E_c}\ \exp\left[\dfrac{-(E_2 - E_c)}{kT}\right]}$$

where

$$E_1 = E_c + 4kT \quad \text{and} \quad E_2 = E_c + \frac{kT}{2}$$

Then
$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\dfrac{kT}{2}}}\ \exp\left[\frac{-(E_1 - E_2)}{kT}\right]$$

$$= 2\sqrt{2}\ \exp\left[-\left(4 - \frac{1}{2}\right)\right] = 2\sqrt{2}\ \exp(-3.5)$$

or

$$\frac{n(E_1)}{n(E_2)} = 0.0854$$

_____

4.8

Plot

_____

_____

4.9

Plot

_____

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_o$, $m_n^* = 1.08m_o$

$E_{Fi} - E_{midgap} = -0.0128\,\text{eV}$

Germanium: $m_p^* = 0.37m_o$, $m_n^* = 0.55m_o$

$E_{Fi} - E_{midgap} = -0.0077\,\text{eV}$

Gallium Arsenide: $m_p^* = 0.48m_o$,

$$m_n^* = 0.067m_o$$

$E_{Fi} - E_{midgap} = +0.0382\,\text{eV}$

_____

4.11

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_v}{N_c}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04\times10^{19}}{2.8\times10^{19}}\right) = -0.4952(kT)$$

T (K)	kT (eV)	($E_{Fi} - E_{midgap}$)(eV)
200	0.01727	-0.0086
400	0.03453	-0.0171
600	0.0518	-0.0257

_____

4.12

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{0.70}{1.21}\right)$$

$$\Rightarrow -10.63\,\text{meV}$$

(b) $$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.75}{0.080}\right)$$

$$\Rightarrow +43.47\,\text{meV}$$

_____

4.13

Let $g_c(E) = K = \text{constant}$

Then

$$n_o = \int_{E_c}^{\infty} g_c(E)f_F(E)dE$$

$$= K\int_{E_c}^{\infty}\frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}dE$$

$$\cong K\int_{E_c}^{\infty}\exp\left[\frac{-(E-E_F)}{kT}\right]dE$$

Let

$$\eta = \frac{E-E_c}{kT}\ \text{so that}\ dE = kT\cdot d\eta$$

We can write

$$E-E_F = (E_c - E_F) + (E - E_c)$$

so that

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_c-E_F)}{kT}\right]\cdot\exp(-\eta)$$

The integral can then be written as

$$n_o = K\cdot kT\cdot\exp\left[\frac{-(E_c-E_F)}{kT}\right]\int_0^{\infty}\exp(-\eta)d\eta$$

which becomes

$$n_o = K\cdot kT\cdot\exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

_____

4.14

Let $g_c(E) = C_1(E - E_c)$ for $E \ge E_c$

Then

$$n_o = \int_{E_c}^{\infty} g_c(E)f_F(E)dE$$

$$= C_1\int_{E_c}^{\infty}\frac{(E-E_c)}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}dE$$

$$\cong C_1\int_{E_c}^{\infty}(E-E_c)\exp\left[\frac{-(E-E_F)}{kT}\right]dE$$

Let

$$\eta = \frac{E-E_c}{kT}\ \text{so that}\ dE = kT\cdot d\eta$$

We can write

$$E-E_F = (E-E_c) + (E_c - E_F)$$

_____

Then

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
$$\times \int_{E_c}^{\infty} (E - E_c) \exp\left[\frac{-(E - E_c)}{kT}\right] dE$$

or

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
$$\times \int_{0}^{\infty} (kT)(\eta)[\exp(-\eta)](kT) d\eta$$

We find that

$$\int_{0}^{\infty} \eta \exp(-\eta) d\eta = \exp(-\eta)(-\eta - 1)\Big|_{0}^{\infty} = +1$$

So

$$n_o = C_1 (kT)^2 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

_____

4.15

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For germanium, $\in_r = 16$, $m^* = 0.55 m_o$
Then

$$r_1 = (16)\left(\frac{1}{0.55}\right) a_o = (29)(0.53)$$

or

$$r_1 = 15.4 \overset{o}{A}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) \, \text{eV}$$
$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow E = 0.029 \text{ eV}$$

_____

4.16

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For gallium arsenide, $\in_r = 13.1$,
$$m^* = 0.067 m_o$$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53) = 104 \overset{o}{A}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or
$$E = 0.0053 \, \text{eV}$$

_____

4.17

(a) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$
$$= (0.0259) \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right)$$
$$= 0.2148 \, \text{eV}$$

(b) $E_F - E_v = E_g - (E_c - E_F)$
$$= 1.12 - 0.2148 = 0.90518 \, \text{eV}$$

(c) $p_o = N_v \exp\left[\dfrac{-(E_F - E_v)}{kT}\right]$
$$= (1.04 \times 10^{19}) \exp\left[\frac{-0.90518}{0.0259}\right]$$
$$= 6.90 \times 10^3 \, \text{cm}^{-3}$$

(d) Holes

(e) $E_F - E_{Fi} = kT \ln\left(\dfrac{n_o}{n_i}\right)$
$$= (0.0259) \ln\left(\frac{7 \times 10^{15}}{1.5 \times 10^{10}}\right)$$
$$= 0.338 \, \text{eV}$$

_____

4.18

(a) $E_F - E_v = kT \ln\left(\dfrac{N_v}{p_o}\right)$
$$= (0.0259) \ln\left(\frac{1.04 \times 10^{19}}{2 \times 10^{16}}\right)$$
$$= 0.162 \, \text{eV}$$

(b) $E_c - E_F = E_g - (E_F - E_v)$
$$= 1.12 - 0.162 = 0.958 \, \text{eV}$$

(c) $n_o = (2.8 \times 10^{19}) \exp\left(\dfrac{-0.958}{0.0259}\right)$
$$= 2.41 \times 10^3 \, \text{cm}^{-3}$$

_____

(d) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{2\times10^{16}}{1.5\times10^{10}}\right)$

$= 0.365\ \text{eV}$

_____

4.19

(a) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$= (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{2\times10^{5}}\right)$

$= 0.8436\ \text{eV}$

$E_F - E_v = E_g - (E_c - E_F)$

$= 1.12 - 0.8436$

$E_F - E_v = 0.2764\ \text{eV}$

(b) $p_o = (1.04\times10^{19})\exp\left(\dfrac{-0.27637}{0.0259}\right)$

$= 2.414\times10^{14}\ \text{cm}^{-3}$

(c) p-type

_____

4.20

(a) $kT = (0.0259)\left(\dfrac{375}{300}\right) = 0.032375\ \text{eV}$

$n_o = (4.7\times10^{17})\left(\dfrac{375}{300}\right)^{3/2}\exp\left[\dfrac{-0.28}{0.032375}\right]$

$= 1.15\times10^{14}\ \text{cm}^{-3}$

$E_F - E_v = E_g - (E_c - E_F) = 1.42 - 0.28$

$= 1.14\ \text{eV}$

$p_o = (7\times10^{18})\left(\dfrac{375}{300}\right)^{3/2}\exp\left[\dfrac{-1.14}{0.032375}\right]$

$= 4.99\times10^{3}\ \text{cm}^{-3}$

(b) $E_c - E_F = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{1.15\times10^{14}}\right)$

$= 0.2154\ \text{eV}$

$E_F - E_v = E_g - (E_c - E_F) = 1.42 - 0.2154$

$= 1.2046\ \text{eV}$

$p_o = (7\times10^{18})\exp\left[\dfrac{-1.2046}{0.0259}\right]$

$= 4.42\times10^{-2}\ \text{cm}^{-3}$

_____

4.21

(a) $kT = (0.0259)\left(\dfrac{375}{300}\right) = 0.032375\ \text{eV}$

$n_o = (2.8\times10^{19})\left(\dfrac{375}{300}\right)^{3/2}\exp\left[\dfrac{-0.28}{0.032375}\right]$

$= 6.86\times10^{15}\ \text{cm}^{-3}$

$E_F - E_v = E_g - (E_c - E_F) = 1.12 - 0.28$

$= 0.840\ \text{eV}$

$p_o = (1.04\times10^{19})\left(\dfrac{375}{300}\right)^{3/2}\exp\left[\dfrac{-0.840}{0.032375}\right]$

$= 7.84\times10^{7}\ \text{cm}^{-3}$

(b) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$= (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{6.862\times10^{15}}\right)$

$= 0.2153\ \text{eV}$

$E_F - E_v = 1.12 - 0.2153 = 0.9047\ \text{eV}$

$p_o = (1.04\times10^{19})\exp\left[\dfrac{-0.904668}{0.0259}\right]$

$= 7.04\times10^{3}\ \text{cm}^{-3}$

_____

4.22

(a) p-type

(b) $E_F - E_v = \dfrac{E_g}{4} = \dfrac{1.12}{4} = 0.28\ \text{eV}$

$p_o = N_v \exp\left[\dfrac{-(E_F - E_v)}{kT}\right]$

$= (1.04\times10^{19})\exp\left[\dfrac{-0.28}{0.0259}\right]$

$= 2.10\times10^{14}\ \text{cm}^{-3}$

$E_c - E_F = E_g - (E_F - E_v)$

$= 1.12 - 0.28 = 0.84\ \text{eV}$

$n_o = N_c \exp\left[\dfrac{-(E_c - E_F)}{kT}\right]$

$= (2.8\times10^{19})\exp\left[\dfrac{-0.84}{0.0259}\right]$

$= 2.30\times10^{5}\ \text{cm}^{-3}$

_____

_____

4.23

(a) $n_o = n_i \exp\left[\dfrac{E_F - E_{Fi}}{kT}\right]$

$= \left(1.5 \times 10^{10}\right) \exp\left[\dfrac{0.22}{0.0259}\right]$

$= 7.33 \times 10^{13} \text{ cm}^{-3}$

$p_o = n_i \exp\left[\dfrac{E_{Fi} - E_F}{kT}\right]$

$= \left(1.5 \times 10^{10}\right) \exp\left[\dfrac{-0.22}{0.0259}\right]$

$= 3.07 \times 10^{6} \text{ cm}^{-3}$

(b) $n_o = n_i \exp\left[\dfrac{E_F - E_{Fi}}{kT}\right]$

$= \left(1.8 \times 10^{6}\right) \exp\left[\dfrac{0.22}{0.0259}\right]$

$= 8.80 \times 10^{9} \text{ cm}^{-3}$

$p_o = n_i \exp\left[\dfrac{E_{Fi} - E_F}{kT}\right]$

$= \left(1.8 \times 10^{6}\right) \exp\left[\dfrac{-0.22}{0.0259}\right]$

$= 3.68 \times 10^{2} \text{ cm}^{-3}$

_____

4.24

(a) $E_F - E_{\upsilon} = kT \ln\left(\dfrac{N_{\upsilon}}{p_o}\right)$

$= \left(0.0259\right) \ln\left(\dfrac{1.04 \times 10^{19}}{5 \times 10^{15}}\right)$

$= 0.1979 \text{ eV}$

(b) $E_c - E_F = E_g - \left(E_F - E_{\upsilon}\right)$
$= 1.12 - 0.19788 = 0.92212 \text{ eV}$

(c) $n_o = \left(2.8 \times 10^{19}\right) \exp\left[\dfrac{-0.92212}{0.0259}\right]$

$= 9.66 \times 10^{3} \text{ cm}^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$= \left(0.0259\right) \ln\left(\dfrac{5 \times 10^{15}}{1.5 \times 10^{10}}\right)$

$= 0.3294 \text{ eV}$

_____

4.25

$kT = \left(0.0259\right)\left(\dfrac{400}{300}\right) = 0.034533 \text{ eV}$

$N_{\upsilon} = \left(1.04 \times 10^{19}\right)\left(\dfrac{400}{300}\right)^{3/2}$

$= 1.601 \times 10^{19} \text{ cm}^{-3}$

$N_c = \left(2.8 \times 10^{19}\right)\left(\dfrac{400}{300}\right)^{3/2}$

$= 4.3109 \times 10^{19} \text{ cm}^{-3}$

$n_i^2 = \left(4.3109 \times 10^{19}\right)\left(1.601 \times 10^{19}\right)$

$\times \exp\left[\dfrac{-1.12}{0.034533}\right]$

$= 5.6702 \times 10^{24}$

$\Rightarrow n_i = 2.381 \times 10^{12} \text{ cm}^{-3}$

(a) $E_F - E_{\upsilon} = kT \ln\left(\dfrac{N_{\upsilon}}{p_o}\right)$

$= \left(0.034533\right) \ln\left(\dfrac{1.601 \times 10^{19}}{5 \times 10^{15}}\right)$

$= 0.2787 \text{ eV}$

(b) $E_c - E_F = 1.12 - 0.27873 = 0.84127 \text{ eV}$

(c) $n_o = \left(4.3109 \times 10^{19}\right) \exp\left[\dfrac{-0.84127}{0.034533}\right]$

$= 1.134 \times 10^{9} \text{ cm}^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$= \left(0.034533\right) \ln\left(\dfrac{5 \times 10^{15}}{2.381 \times 10^{12}}\right)$

$= 0.2642 \text{ eV}$

_____

4.26

(a) $p_o = \left(7 \times 10^{18}\right) \exp\left[\dfrac{-0.25}{0.0259}\right]$

$= 4.50 \times 10^{14} \text{ cm}^{-3}$

$E_c - E_F = 1.42 - 0.25 = 1.17 \text{ eV}$

$n_o = \left(4.7 \times 10^{17}\right) \exp\left[\dfrac{-1.17}{0.0259}\right]$

$= 1.13 \times 10^{-2} \text{ cm}^{-3}$

_____

(b) $kT = 0.034533$ eV

$$N_v = (7 \times 10^{18}) \left(\frac{400}{300} \right)^{3/2}$$

$$= 1.078 \times 10^{19} \text{ cm}^{-3}$$

$$N_c = (4.7 \times 10^{17}) \left(\frac{400}{300} \right)^{3/2}$$

$$= 7.236 \times 10^{17} \text{ cm}^{-3}$$

$$E_F - E_v = kT \ln \left(\frac{N_v}{p_o} \right)$$

$$= (0.034533) \ln \left(\frac{1.078 \times 10^{19}}{4.50 \times 10^{14}} \right)$$

$$= 0.3482 \text{ eV}$$

$$E_c - E_F = 1.42 - 0.3482 = 1.072 \text{ eV}$$

$$n_o = (7.236 \times 10^{17}) \exp \left[\frac{-1.07177}{0.034533} \right]$$

$$= 2.40 \times 10^4 \text{ cm}^{-3}$$

_____

4.27

(a) $$p_o = (1.04 \times 10^{19}) \exp \left[\frac{-0.25}{0.0259} \right]$$

$$= 6.68 \times 10^{14} \text{ cm}^{-3}$$

$$E_c - E_F = 1.12 - 0.25 = 0.870 \text{ eV}$$

$$n_o = (2.8 \times 10^{19}) \exp \left[\frac{-0.870}{0.0259} \right]$$

$$n_o = 7.23 \times 10^4 \text{ cm}^{-3}$$

(b) $kT = 0.034533$ eV

$$N_v = (1.04 \times 10^{19}) \left(\frac{400}{300} \right)^{3/2}$$

$$= 1.601 \times 10^{19} \text{ cm}^{-3}$$

$$N_c = (2.8 \times 10^{19}) \left(\frac{400}{300} \right)^{3/2}$$

$$= 4.311 \times 10^{19} \text{ cm}^{-3}$$

$$E_F - E_v = kT \ln \left(\frac{N_v}{p_o} \right)$$

$$= (0.034533) \ln \left(\frac{1.601 \times 10^{19}}{6.68 \times 10^{14}} \right)$$

$$= 0.3482 \text{ eV}$$

$$E_c - E_F = 1.12 - 0.3482 = 0.7718 \text{ eV}$$

$$n_o = (4.311 \times 10^{19}) \exp \left[\frac{-0.77175}{0.034533} \right]$$

$$= 8.49 \times 10^9 \text{ cm}^{-3}$$

_____

4.28

(a) $$n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

For $E_F = E_c + kT/2$,

$$\eta_F = \frac{E_F - E_c}{kT} = \frac{kT/2}{kT} = 0.5$$

Then $F_{1/2}(\eta_F) \cong 1.0$

$$n_o = \frac{2}{\sqrt{\pi}} (2.8 \times 10^{19})(1.0)$$

$$= 3.16 \times 10^{19} \text{ cm}^{-3}$$

(b) $$n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

$$= \frac{2}{\sqrt{\pi}} (4.7 \times 10^{17})(1.0)$$

$$= 5.30 \times 10^{17} \text{ cm}^{-3}$$

_____

4.29

$$p_o = \frac{2}{\sqrt{\pi}} N_v F_{1/2}(\eta_F')$$

$$5 \times 10^{19} = \frac{2}{\sqrt{\pi}} (1.04 \times 10^{19}) F_{1/2}(\eta_F')$$

So $F_{1/2}(\eta_F') = 4.26$

We find $\eta_F' \cong 3.0 = \dfrac{E_v - E_F}{kT}$

$$E_v - E_F = (3.0)(0.0259) = 0.0777 \text{ eV}$$

_____

4.30

(a) $$\eta_F = \frac{E_F - E_c}{kT} = \frac{4kT}{kT} = 4$$

Then $F_{1/2}(\eta_F) \cong 6.0$

$$n_o = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

$$= \frac{2}{\sqrt{\pi}} (2.8 \times 10^{19})(6.0)$$

$$= 1.90 \times 10^{20} \text{ cm}^{-3}$$

_____

(b) $n_o = \dfrac{2}{\sqrt{\pi}}\left(4.7\times10^{17}\right)\left(6.0\right)$

$\qquad = 3.18\times10^{18}\ \text{cm}^{-3}$

_____

4.31

For the electron concentration
$$n(E) = g_c(E)f_F(E)$$
The Boltzmann approximation applies, so
$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_c}$$
$$\times \exp\left[\frac{-(E - E_F)}{kT}\right]$$
or
$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
$$\times \sqrt{kT}\sqrt{\frac{E - E_c}{kT}}\exp\left[\frac{-(E - E_c)}{kT}\right]$$
Define
$$x = \frac{E - E_c}{kT}$$
Then
$$n(E) \to n(x) = K\sqrt{x}\exp(-x)$$
To find maximum $n(E) \to n(x)$, set
$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x)\right.$$
$$\left. + x^{1/2}(-1)\exp(-x)\right]$$
or
$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2} - x\right]$$
which yields
$$x = \frac{1}{2} = \frac{E - E_c}{kT} \Rightarrow E = E_c + \frac{1}{2}kT$$
For the hole concentration
$$p(E) = g_v(E)\left[1 - f_F(E)\right]$$
Using the Boltzmann approximation
$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v - E}$$
$$\times \exp\left[\frac{-(E_F - E)}{kT}\right]$$
or

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_F - E_v)}{kT}\right]$$
$$\times \sqrt{kT}\sqrt{\frac{E_v - E}{kT}}\exp\left[\frac{-(E_v - E)}{kT}\right]$$
Define
$$x' = \frac{E_v - E}{kT}$$
Then
$$p(x') = K'\sqrt{x'}\exp(-x')$$
To find maximum value of $p(E) \to p(x')$, set
$$\frac{dp(x')}{dx'} = 0 \quad \text{Using the results from above,}$$
we find the maximum at
$$E = E_v - \frac{1}{2}kT$$

_____

4.32

(a) Silicon: We have
$$n_o = N_c\exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
We can write
$$E_c - E_F = (E_c - E_d) + (E_d - E_F)$$
For
$$E_c - E_d = 0.045\ \text{eV and } E_d - E_F = 3kT\ \text{eV}$$
we can write
$$n_o = \left(2.8\times10^{19}\right)\exp\left[\frac{-0.045}{0.0259} - 3\right]$$
$$= \left(2.8\times10^{19}\right)\exp(-4.737)$$
or
$$n_o = 2.45\times10^{17}\ \text{cm}^{-3}$$
We also have
$$p_o = N_v\exp\left[\frac{-(E_F - E_v)}{kT}\right]$$
Again, we can write
$$E_F - E_v = (E_F - E_a) + (E_a - E_v)$$
For
$$E_F - E_a = 3kT \quad \text{and } E_a - E_v = 0.045\ \text{eV}$$
Then
$$p_o = \left(1.04\times10^{19}\right)\exp\left[-3 - \frac{0.045}{0.0259}\right]$$
$$= \left(1.04\times10^{19}\right)\exp(-4.737)$$
or
$$p_o = 9.12\times10^{16}\ \text{cm}^{-3}$$

_____

(b) GaAs: assume $E_c - E_d = 0.0058\,\text{eV}$

Then

$$n_o = \left(4.7\times10^{17}\right)\exp\left[\frac{-0.0058}{0.0259} - 3\right]$$

$$= \left(4.7\times10^{17}\right)\exp(-3.224)$$

or

$$n_o = 1.87\times10^{16}\,\text{cm}^{-3}$$

Assume $E_a - E_v = 0.0345\,\text{eV}$

Then

$$p_o = \left(7\times10^{18}\right)\exp\left[\frac{-0.0345}{0.0259} - 3\right]$$

$$= \left(7\times10^{18}\right)\exp(-4.332)$$

or

$$p_o = 9.20\times10^{16}\,\text{cm}^{-3}$$

_____

4.33

Plot

_____

4.34

(a) $p_o = 4\times15 - 10^{15} = 3\times10^{15}\,\text{cm}^{-3}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{15}} = 7.5\times10^4\,\text{cm}^{-3}$$

(b) $n_o = N_d = 3\times10^{16}\,\text{cm}^{-3}$

$$p_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5\times10^3\,\text{cm}^{-3}$$

(c) $n_o = p_o = n_i = 1.5\times10^{10}\,\text{cm}^{-3}$

(d) $n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{375}{300}\right)^3$

$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(375)}\right]$$

$$\Rightarrow n_i = 7.334\times10^{11}\,\text{cm}^{-3}$$

$$p_o = N_a = 4\times10^{15}\,\text{cm}^{-3}$$

$$n_o = \frac{\left(7.334\times10^{11}\right)^2}{4\times10^{15}} = 1.34\times10^8\,\text{cm}^{-3}$$

(e) $n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{450}{300}\right)^3$

$$\times\exp\left[\frac{-(1.12)(300)}{(0.0259)(450)}\right]$$

$$\Rightarrow n_i = 1.722\times10^{13}\,\text{cm}^{-3}$$

$$n_o = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + \left(1.722\times10^{13}\right)^2}$$

$$= 1.029\times10^{14}\,\text{cm}^{-3}$$

$$p_o = \frac{\left(1.722\times10^{13}\right)^2}{1.029\times10^{14}} = 2.88\times10^{12}\,\text{cm}^{-3}$$

_____

4.35

(a) $p_o = N_a - N_d = 4\times10^{15} - 10^{15}$

$$= 3\times10^{15}\,\text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^6\right)^2}{3\times10^{15}} = 1.08\times10^{-3}\,\text{cm}^{-3}$$

(b) $n_o = N_d = 3\times10^{16}\,\text{cm}^{-3}$

$$p_o = \frac{\left(1.8\times10^6\right)^2}{3\times10^{16}} = 1.08\times10^{-4}\,\text{cm}^{-3}$$

(c) $n_o = p_o = n_i = 1.8\times10^6\,\text{cm}^{-3}$

(d) $n_i^2 = \left(4.7\times10^{17}\right)\left(7.0\times10^{18}\right)\left(\frac{375}{300}\right)^3$

$$\times\exp\left[\frac{-(1.42)(300)}{(0.0259)(375)}\right]$$

$$\Rightarrow n_i = 7.580\times10^8\,\text{cm}^{-3}$$

$$p_o = N_a = 4\times10^{15}\,\text{cm}^{-3}$$

$$n_o = \frac{\left(7.580\times10^8\right)^2}{4\times10^{15}} = 1.44\times10^2\,\text{cm}^{-3}$$

(e) $n_i^2 = \left(4.7\times10^{17}\right)\left(7.0\times10^{18}\right)\left(\frac{450}{300}\right)^3$

$$\times\exp\left[\frac{-(1.42)(300)}{(0.0259)(450)}\right]$$

$$\Rightarrow n_i = 3.853\times10^{10}\,\text{cm}^{-3}$$

$$n_o = N_d = 10^{14}\,\text{cm}^{-3}$$

$$p_o = \frac{\left(3.853\times10^{10}\right)^2}{10^{14}} = 1.48\times10^7\,\text{cm}^{-3}$$

_____

4.36

(a) Ge: $n_i = 2.4\times10^{13}\,\text{cm}^{-3}$

(i) $n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$

$$= \frac{2\times10^{15}}{2} + \sqrt{\left(\frac{2\times10^{15}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$$

_____

or

$$n_o \cong N_d = 2\times10^{15}\,\text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(2.4\times10^{13}\right)^2}{2\times10^{15}}$$

$$= 2.88\times10^{11}\,\text{cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 10^{16} - 7\times10^{15}$

$$= 3\times10^{15}\,\text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.4\times10^{13}\right)^2}{3\times10^{15}}$$

$$= 1.92\times10^{11}\,\text{cm}^{-3}$$

(b) GaAs: $n_i = 1.8\times10^6\,\text{cm}^{-3}$

(i) $n_o \cong N_d = 2\times10^{15}\,\text{cm}$

$$p_o = \frac{\left(1.8\times10^6\right)^2}{2\times10^{15}} = 1.62\times10^{-3}\,\text{cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 3\times10^{15}\,\text{cm}^{-3}$

$$n_o = \frac{\left(1.8\times10^6\right)^2}{3\times10^{15}} = 1.08\times10^{-3}\,\text{cm}^{-3}$$

(c) The result implies that there is only one minority carrier in a volume of $10^3\,\text{cm}^3$.

_____

4.37

(a) For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\frac{n_d}{N_d} = 8.85\times10^{-4}$$

(b) We have

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = \left(E - E_c\right) + \left(E_c - E_F\right)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)}$$

or

$$f_F(E) = 2.87\times10^{-5}$$

_____

4.38

(a) $N_a > N_d \Rightarrow$ p-type

(b) Silicon:

$$p_o = N_a - N_d = 2.5\times10^{13} - 1\times10^{13}$$

or

$$p_o = 1.5\times10^{13}\,\text{cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{13}} = 1.5\times10^7\,\text{cm}^{-3}$$

Germanium:

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5\times10^{13}}{2}\right) + \sqrt{\left(\frac{1.5\times10^{13}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$$

or

$$p_o = 3.26\times10^{13}\,\text{cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.4\times10^{13}\right)^2}{3.264\times10^{13}} = 1.76\times10^{13}\,\text{cm}^{-3}$$

Gallium Arsenide:

$$p_o = N_a - N_d = 1.5\times10^{13}\,\text{cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^6\right)^2}{1.5\times10^{13}} = 0.216\,\text{cm}^{-3}$$

_____

4.39

(a) $N_d > N_a \Rightarrow$ n-type

(b) $n_o \cong N_d - N_a = 2\times10^{15} - 1.2\times10^{15}$

$$= 8\times10^{14}\,\text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{8\times10^{14}} = 2.81\times10^5\,\text{cm}^{-3}$$

(c) $p_o \cong (N_a' + N_a) - N_d$

$4 \times 10^{15} = N_a' + 1.2 \times 10^{15} - 2 \times 10^{15}$

$\Rightarrow N_a' = 4.8 \times 10^{15} \, cm^{-3}$

$n_o = \dfrac{(1.5 \times 10^{10})^2}{4 \times 10^{15}} = 5.625 \times 10^4 \, cm^{-3}$

_____

4.40

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.5 \times 10^{10})^2}{2 \times 10^5} = 1.125 \times 10^{15} \, cm^{-3}$

$n_o > p_o \Rightarrow \text{n-type}$

_____

4.41

$n_i^2 = (1.04 \times 10^{19})(6.0 \times 10^{18})\left(\dfrac{250}{300}\right)^3$

$\times \exp\left[\dfrac{-0.66}{(0.0259)(250/300)}\right]$

$= 1.8936 \times 10^{24}$

$\Rightarrow n_i = 1.376 \times 10^{12} \, cm^{-3}$

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{n_i^2}{4n_o} \Rightarrow n_o^2 = \dfrac{1}{4} n_i^2$

$\Rightarrow n_o = \dfrac{1}{2} n_i$

So $n_o = 6.88 \times 10^{11} \, cm^{-3}$,

Then $p_o = 2.75 \times 10^{12} \, cm^{-3}$

$p_o = \dfrac{N_a}{2} + \sqrt{\left(\dfrac{N_a}{2}\right)^2 + n_i^2}$

$\left(2.752 \times 10^{12} - \dfrac{N_a}{2}\right)^2$

$= \left(\dfrac{N_a}{2}\right)^2 + 1.8936 \times 10^{24}$

$7.5735 \times 10^{24} - (2.752 \times 10^{12})N_a + \left(\dfrac{N_a}{2}\right)^2$

$= \left(\dfrac{N_a}{2}\right)^2 + 1.8936 \times 10^{24}$

so that $N_a = 2.064 \times 10^{12} \, cm^{-3}$

_____

4.42

Plot

_____

4.43

Plot

_____

4.44

Plot

_____

4.45

$n_o = \dfrac{N_d - N_a}{2} + \sqrt{\left(\dfrac{N_d - N_a}{2}\right)^2 + n_i^2}$

$1.1 \times 10^{14} = \dfrac{2 \times 10^{14} - 1.2 \times 10^{14}}{2}$

$+ \sqrt{\left(\dfrac{2 \times 10^{14} - 1.2 \times 10^{14}}{2}\right)^2 + n_i^2}$

$(1.1 \times 10^{14} - 4 \times 10^{13})^2 = (4 \times 10^{13})^2 + n_i^2$

$4.9 \times 10^{27} = 1.6 \times 10^{27} + n_i^2$

so $n_i = 5.74 \times 10^{13} \, cm^{-3}$

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{3.3 \times 10^{27}}{1.1 \times 10^{14}} = 3 \times 10^{13} \, cm^{-3}$

_____

4.46

(a) $N_a > N_d \Rightarrow \text{p-type}$

Majority carriers are holes

$p_o = N_a - N_d = 3 \times 10^{16} - 1.5 \times 10^{16}$

$= 1.5 \times 10^{16} \, cm^{-3}$

Minority carriers are electrons

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.5 \times 10^{10})^2}{1.5 \times 10^{16}} = 1.5 \times 10^4 \, cm^{-3}$

(b) Boron atoms must be added

$p_o = N_a' + N_a - N_d$

$5 \times 10^{16} = N_a' + 3 \times 10^{16} - 1.5 \times 10^{16}$

So $N_a' = 3.5 \times 10^{16} \, cm^{-3}$

$n_o = \dfrac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \, cm^{-3}$

_____

_____

4.47

(a) $p_o << n_i \Rightarrow$ n-type

(b) $p_o = \dfrac{n_i^2}{n_o} \Rightarrow n_o = \dfrac{n_i^2}{p_o}$

$n_o = \dfrac{\left(1.5\times10^{10}\right)^2}{2\times10^4} = 1.125\times10^{16}\,\text{cm}^{-3}$

$\Rightarrow$ electrons are majority carriers

$p_o = 2\times10^4\,\text{cm}^{-3}$

$\Rightarrow$ holes are minority carriers

(c) $n_o = N_d - N_a$

$1.125\times10^{16} = N_d - 7\times10^{15}$

so $N_d = 1.825\times10^{16}\,\text{cm}^{-3}$

_____

4.48

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

For Germanium

T (K)	kT (eV)	$n_i\,(\text{cm}^{-3})$
200	0.01727	2.16×10^{10}
400	0.03453	8.60×10^{14}
600	0.0518	3.82×10^{16}

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}\quad\text{and}$$

$$N_a = 10^{15}\,\text{cm}^{-3}$$

T (K)	$p_o\,(\text{cm}^{-3})$	$\left(E_{Fi} - E_F\right)(\text{eV})$
200	1.0×10^{15}	0.1855
400	1.49×10^{15}	0.01898
600	3.87×10^{16}	0.000674

_____

4.49

(a) $E_c - E_F = kT \ln\left(\dfrac{N_c}{N_d}\right)$

$= (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{N_d}\right)$

For $10^{14}\,\text{cm}^{-3}$, $E_c - E_F = 0.3249\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_c - E_F = 0.2652\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_c - E_F = 0.2056\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_c - E_F = 0.1459\,\text{eV}$

(b) $E_F - E_{Fi} = kT \ln\left(\dfrac{N_d}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{N_d}{1.5\times10^{10}}\right)$

For $10^{14}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2280\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2877\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.3473\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.4070\,\text{eV}$

_____

4.50

(a) $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$n_o = 1.05 N_d = 1.05\times10^{15}\,\text{cm}^{-3}$

$\left(1.05\times10^{15} - 0.5\times10^{15}\right)^2$

$= \left(0.5\times10^{15}\right)^2 + n_i^2$

so $n_i^2 = 5.25\times10^{28}$

Now

$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\dfrac{T}{300}\right)^3$

$\times\exp\left[\dfrac{-1.12}{(0.0259)(T/300)}\right]$

$5.25\times10^{28} = \left(2.912\times10^{38}\right)\left(\dfrac{T}{300}\right)^3$

$\times\exp\left[\dfrac{-12972.973}{T}\right]$

By trial and error, $T = 536.5\,\text{K}$

(b) At $T = 300\,\text{K}$,

$E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$E_c - E_F = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{10^{15}}\right)$

$= 0.2652\,\text{eV}$

At $T = 536.5\,\text{K}$,

$kT = (0.0259)\left(\dfrac{536.5}{300}\right) = 0.046318\,\text{eV}$

$N_c = \left(2.8\times10^{19}\right)\left(\dfrac{536.5}{300}\right)^{3/2}$

$= 6.696\times10^{19}\,\text{cm}^{-3}$

_____

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.046318)\ln\left(\frac{6.696 \times 10^{19}}{1.05 \times 10^{15}}\right)$$

$$= 0.5124 \text{ eV}$$

then $\Delta(E_c - E_F) = 0.2472 \text{ eV}$

(c) Closer to the intrinsic energy level.

_____

4.51

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

At $T = 200 \text{ K}, \ kT = 0.017267 \text{ eV}$
$T = 400 \text{ K}, \ kT = 0.034533 \text{ eV}$
$T = 600 \text{ K}, \ kT = 0.0518 \text{ eV}$

At $T = 200 \text{ K}$,

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{200}{300}\right)^3$$

$$\times \exp\left[\frac{-1.12}{0.017267}\right]$$

$$\Rightarrow n_i = 7.638 \times 10^4 \text{ cm}^{-3}$$

At $T = 400 \text{ K}$,

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3$$

$$\times \exp\left[\frac{-1.12}{0.034533}\right]$$

$$\Rightarrow n_i = 2.381 \times 10^{12} \text{ cm}^{-3}$$

At $T = 600 \text{ K}$,

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{600}{300}\right)^3$$

$$\times \exp\left[\frac{-1.12}{0.0518}\right]$$

$$\Rightarrow n_i = 9.740 \times 10^{14} \text{ cm}^{-3}$$

At $T = 200 \text{ K}$ and $T = 400 \text{ K}$,

$$p_o = N_a = 3 \times 10^{15} \text{ cm}^{-3}$$

At $T = 600 \text{ K}$,

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= \frac{3 \times 10^{15}}{2} + \sqrt{\left(\frac{3 \times 10^{15}}{2}\right)^2 + (9.740 \times 10^{14})^2}$$

$$= 3.288 \times 10^{15} \text{ cm}^{-3}$$

Then, $T = 200 \text{ K}, \ E_{Fi} - E_F = 0.4212 \text{ eV}$
$T = 400 \text{ K}, \ E_{Fi} - E_F = 0.2465 \text{ eV}$
$T = 600 \text{ K}, \ E_{Fi} - E_F = 0.0630 \text{ eV}$

_____

4.52

(a)

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{N_a}{1.8 \times 10^6}\right)$$

For $N_a = 10^{14} \text{ cm}^{-3}, \ E_{Fi} - E_F = 0.4619 \text{ eV}$
$N_a = 10^{15} \text{ cm}^{-3}, \ E_{Fi} - E_F = 0.5215 \text{ eV}$
$N_a = 10^{16} \text{ cm}^{-3}, \ E_{Fi} - E_F = 0.5811 \text{ eV}$
$N_a = 10^{17} \text{ cm}^{-3}, \ E_{Fi} - E_F = 0.6408 \text{ eV}$

(b)

$$E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{N_a}\right) = (0.0259)\ln\left(\frac{7.0 \times 10^{18}}{N_a}\right)$$

For $N_a = 10^{14} \text{ cm}^{-3}, \ E_F - E_\upsilon = 0.2889 \text{ eV}$
$N_a = 10^{15} \text{ cm}^{-3}, \ E_F - E_\upsilon = 0.2293 \text{ eV}$
$N_a = 10^{16} \text{ cm}^{-3}, \ E_F - E_\upsilon = 0.1697 \text{ eV}$
$N_a = 10^{17} \text{ cm}^{-3}, \ E_F - E_\upsilon = 0.1100 \text{ eV}$

_____

4.53

(a) $E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$

$$= \frac{3}{4}(0.0259)\ln(10)$$

or

$E_{Fi} - E_{midgap} = +0.0447 \text{ eV}$

(b) Impurity atoms to be added so
$E_{midgap} - E_F = 0.45 \text{ eV}$

(i) p-type, so add acceptor atoms
(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947 \text{ eV}$

Then

$$p_o = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= (10^5)\exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_o = N_a = 1.97 \times 10^{13} \text{ cm}^{-3}$$

_____

_____

4.54

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5 \times 10^{15} + \left(2.8 \times 10^{19}\right)\exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5 \times 10^{15} + 6.95 \times 10^{15}$$

or

$$N_d = 1.2 \times 10^{16}\ \text{cm}^{-3}$$

_____

4.55

(a) Silicon

(i) $E_c - E_F = kT \ln\left(\dfrac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8 \times 10^{19}}{6 \times 10^{15}}\right) = 0.2188\ \text{eV}$$

(ii) $E_c - E_F = 0.2188 - 0.0259 = 0.1929\ \text{eV}$

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= \left(2.8 \times 10^{19}\right)\exp\left[\frac{-0.1929}{0.0259}\right]$$

$$N_d = 1.631 \times 10^{16}\ \text{cm}^{-3} = N'_d + 6 \times 10^{15}$$

$$\Rightarrow N'_d = 1.031 \times 10^{16}\ \text{cm}^{-3}\ \text{Additional}$$
$$\text{donor atoms}$$

(b) GaAs

(i) $E_c - E_F = (0.0259)\ln\left(\dfrac{4.7 \times 10^{17}}{10^{15}}\right)$

$$= 0.15936\ \text{eV}$$

(ii) $E_c - E_F = 0.15936 - 0.0259 = 0.13346\ \text{eV}$

$$N_d = \left(4.7 \times 10^{17}\right)\exp\left[\frac{-0.13346}{0.0259}\right]$$

$$= 2.718 \times 10^{15}\ \text{cm}^{-3} = N'_d + 10^{15}$$

$$\Rightarrow N'_d = 1.718 \times 10^{15}\ \text{cm}^{-3}\ \text{Additional}$$
$$\text{donor atoms}$$

_____

4.56

(a) $E_{Fi} - E_F = kT \ln\left(\dfrac{N_v}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04 \times 10^{19}}{2 \times 10^{16}}\right) = 0.1620\ \text{eV}$$

(b) $E_F - E_{Fi} = kT \ln\left(\dfrac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8 \times 10^{19}}{2 \times 10^{16}}\right) = 0.1876\ \text{eV}$$

(c) For part (a);

$$p_o = 2 \times 10^{16}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5 \times 10^{10}\right)^2}{2 \times 10^{16}}$$

$$= 1.125 \times 10^4\ \text{cm}^{-3}$$

For part (b):

$$n_o = 2 \times 10^{16}\ \text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5 \times 10^{10}\right)^2}{2 \times 10^{16}}$$

$$= 1.125 \times 10^4\ \text{cm}^{-3}$$

_____

4.57

$$n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$= \left(1.8 \times 10^6\right)\exp\left[\frac{0.55}{0.0259}\right]$$

$$= 3.0 \times 10^{15}\ \text{cm}^{-3}$$

Add additional acceptor impurities

$$n_o = N_d - N_a$$

$$3 \times 10^{15} = 7 \times 10^{15} - N_a$$

$$\Rightarrow N_a = 4 \times 10^{15}\ \text{cm}^{-3}$$

_____

4.58

(a) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.3161\ \text{eV}$$

(b) $E_F - E_{Fi} = kT \ln\left(\dfrac{n_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3758\ \text{eV}$$

(c) $E_F = E_{Fi}$

_____

(d) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$= (0.0259)\left(\dfrac{375}{300}\right)\ln\left(\dfrac{4\times10^{15}}{7.334\times10^{11}}\right)$

$= 0.2786\,\text{eV}$

(e) $E_F - E_{Fi} = kT \ln\left(\dfrac{n_o}{n_i}\right)$

$= (0.0259)\left(\dfrac{450}{300}\right)\ln\left(\dfrac{1.029\times10^{14}}{1.722\times10^{13}}\right)$

$= 0.06945\,\text{eV}$

_____

4.59

(a) $E_F - E_\upsilon = kT \ln\left(\dfrac{N_\upsilon}{p_o}\right)$

$= (0.0259)\ln\left(\dfrac{7.0\times10^{18}}{3\times10^{15}}\right) = 0.2009\,\text{eV}$

(b) $E_F - E_\upsilon = (0.0259)\ln\left(\dfrac{7.0\times10^{18}}{1.08\times10^{-4}}\right)$

$= 1.360\,\text{eV}$

(c) $E_F - E_\upsilon = (0.0259)\ln\left(\dfrac{7.0\times10^{18}}{1.8\times10^{6}}\right)$

$= 0.7508\,\text{eV}$

(d) $E_F - E_\upsilon = (0.0259)\left(\dfrac{375}{300}\right)$

$\times\ln\left[\dfrac{(7.0\times10^{18})(375/300)^{3/2}}{4\times10^{15}}\right]$

$= 0.2526\,\text{eV}$

(e) $E_F - E_\upsilon = (0.0259)\left(\dfrac{450}{300}\right)$

$\times\ln\left[\dfrac{(7.0\times10^{18})(450/300)^{3/2}}{1.48\times10^{7}}\right]$

$= 1.068\,\text{eV}$

_____

4.60

n-type

$E_F - E_{Fi} = kT \ln\left(\dfrac{n_o}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{1.125\times10^{16}}{1.5\times10^{10}}\right) = 0.3504\,\text{eV}$

_____

4.61

$p_o = \dfrac{N_a}{2} + \sqrt{\left(\dfrac{N_a}{2}\right)^2 + n_i^2}$

$5.08\times10^{15} = \dfrac{5\times10^{15}}{2}$

$+ \sqrt{\left(\dfrac{5\times10^{15}}{2}\right)^2 + n_i^2}$

$(5.08\times10^{15} - 2.5\times10^{15})^2$

$= (2.5\times10^{15})^2 + n_i^2$

$6.6564\times10^{30} = 6.25\times10^{30} + n_i^2$

$\Rightarrow n_i^2 = 4.064\times10^{29}$

$n_i^2 = N_c N_\upsilon \exp\left[\dfrac{-E_g}{kT}\right]$

$kT = (0.0259)\left(\dfrac{350}{300}\right) = 0.030217\,\text{eV}$

$N_c = (1.2\times10^{19})\left(\dfrac{350}{300}\right)^2 = 1.633\times10^{19}\,\text{cm}^{-3}$

$N_\upsilon = (1.8\times10^{19})\left(\dfrac{350}{300}\right)^2 = 2.45\times10^{19}\,\text{cm}^{-3}$

Now

$4.064\times10^{29} = (1.633\times10^{19})(2.45\times10^{19})$

$\times\exp\left[\dfrac{-E_g}{0.030217}\right]$

So

$E_g = (0.030217)\ln\left[\dfrac{(1.633\times10^{19})(2.45\times10^{19})}{4.064\times10^{29}}\right]$

$\Rightarrow E_g = 0.6257\,\text{eV}$

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4.62

(a) Replace Ga atoms $\Rightarrow$ Silicon acts as a
donor

$$N_d = (0.05)(7 \times 10^{15}) = 3.5 \times 10^{14} \text{ cm}^{-3}$$

Replace As atoms $\Rightarrow$ Silicon acts as an
acceptor

$$N_a = (0.95)(7 \times 10^{15}) = 6.65 \times 10^{15} \text{ cm}^{-3}$$

(b) $N_a > N_d \Rightarrow$ p-type

(c) $p_o = N_a - N_d = 6.65 \times 10^{15} - 3.5 \times 10^{14}$

$$= 6.3 \times 10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{6.3 \times 10^{15}} = 5.14 \times 10^{-4} \text{ cm}^{-3}$$

(d) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{6.3 \times 10^{15}}{1.8 \times 10^6}\right) = 0.5692 \text{ eV}$$

_____

_____

Chapter 5

5.1

(a) $\rho = \dfrac{1}{e\mu_n N_d} = \dfrac{1}{(1.6\times10^{-19})(1300)(10^{15})}$

$= 4.808\,\Omega$-cm

(b) $\sigma = \dfrac{1}{\rho} = \dfrac{1}{4.8077} = 0.208\,(\Omega\text{ -cm})^{-1}$

_____

5.2

$\sigma = e\mu_p N_a$

or $N_a = \dfrac{\sigma}{e\mu_p} = \dfrac{1.80}{(1.6\times10^{-19})(380)}$

$= 2.96\times10^{16}\text{ cm}^{-3}$

_____

5.3

(a) $\sigma = e\mu_n N_d$

$10 = (1.6\times10^{-19})\mu_n N_d$

From Figure 5.3, for $N_d = 6\times10^{16}\text{ cm}^{-3}$ we

find $\mu_n \cong 1050\text{ cm}^2/\text{V-s}$ which gives

$\sigma = (1.6\times10^{-19})(1050)(6\times10^{16})$

$= 10.08\,(\Omega\text{ -cm})^{-1}$

(b) $\rho = \dfrac{1}{e\mu_p N_a}$

$0.20 = \dfrac{1}{(1.6\times10^{-19})\mu_p N_a}$

From Figure 5.3, for $N_a = 10^{17}\text{ cm}^{-3}$ we

find $\mu_p \cong 320\text{ cm}^2/\text{V-s}$ which gives

$\rho = \dfrac{1}{(1.6\times10^{-19})(320)(10^{17})} = 0.195\ \Omega\text{ -cm}$

_____

5.4

(a) $\rho = \dfrac{1}{e\mu_p N_a}$

$0.35 = \dfrac{1}{(1.6\times10^{-19})\mu_p N_a}$

From Figure 5.3, for $N_a = 8\times10^{16}\text{ cm}^{-3}$ we

find $\mu_p \cong 220\text{ cm}^2/\text{V-s}$ which gives

$\rho = \dfrac{1}{(1.6\times10^{-19})(220)(8\times10^{16})}$

$= 0.355\ \Omega\text{ -cm}$

(b) $\sigma = e\mu_n N_d$

$120 = (1.6\times10^{-19})\mu_n N_d$

From Figure 5.3, for $N_d = 2\times10^{17}\text{ cm}^{-3}$,

then $\mu_n \cong 3800\text{ cm}^2/\text{V-s}$ which gives

$\sigma = (1.6\times10^{-19})(3800)(2\times10^{17})$

$= 121.6\,(\Omega\text{ -cm})^{-1}$

_____

5.5

$R = \dfrac{\rho L}{A} = \dfrac{L}{\sigma A} = \dfrac{L}{(e\mu_n N_d)A}$

or $\mu_n = \dfrac{L}{(eN_d)RA}$

$= \dfrac{2.5}{(1.6\times10^{-19})(2\times10^{15})(70)(0.1)}$

$= 1116\text{ cm}^2/\text{V-s}$

_____

5.6

(a) $n_o = N_d = 10^{16}\text{ cm}^{-3}$

and

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4}\text{ cm}^{-3}$

(b)

$J = e\mu_n n_o E$

For GaAs doped at $N_d = 10^{16}\text{ cm}^{-3}$,

$\mu_n \cong 7500\text{ cm}^2/\text{V-s}$

Then

$J = (1.6\times10^{-19})(7500)(10^{16})(10)$

or

$J = 120\text{ A/cm}^2$

(b) (i) $p_o = N_a = 10^{16}\text{ cm}^{-3}$

$n_o = \dfrac{n_i^2}{p_o} = 3.24\times10^{-4}\text{ cm}^{-3}$

(ii) For GaAs doped at $N_a = 10^{16}\text{ cm}^{-3}$,

$\mu_p \cong 310\text{ cm}^2/\text{V-s}$

_____

$$J = e\mu_p p_o E$$
$$= \left(1.6\times10^{-19}\right)(310)\left(10^{16}\right)(10)$$

or

$$J = 4.96 \text{ A/cm}^2$$

_____

5.7

(a) $V = IR \Rightarrow 10 = (0.1)R$

or

$$R = 100\,\Omega$$

(b)

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{RA}$$

or

$$\sigma = \frac{10^{-3}}{(100)(10^{-3})} = 0.01\ (\Omega\text{-cm})^{-1}$$

(c) $\sigma \cong e\mu_n N_d$

or

$$0.01 = \left(1.6\times10^{-19}\right)(1350)N_d$$

Then

$$N_d = 4.63\times10^{13} \text{ cm}^{-3}$$

(d) $\sigma \cong e\mu_p p_o$

or

$$0.01 = \left(1.6\times10^{-19}\right)(480)p_o$$

Then

$$p_o = 1.30\times10^{14} \text{ cm}^{-3} = N_a - N_d$$

So

$$N_a = 1.30\times10^{14} + 10^{15} = 1.13\times10^{15} \text{ cm}^{-3}$$

Note: For the doping concentrations obtained, the assumed mobility values are valid.

_____

5.8

(a) $R = \dfrac{L}{\sigma A} = \dfrac{L}{(e\mu_p N_a)A}$

For $N_a = 2\times10^{16} \text{ cm}^{-3}$, then

$$\mu_p \cong 400 \text{ cm}^2/\text{V-s}$$

$$R = \frac{(0.075)}{\left(1.6\times10^{-19}\right)(400)\left(2\times10^{16}\right)\left(8.5\times10^{-4}\right)}$$
$$= 68.93\,\Omega$$

$$I = \frac{V}{R} = \frac{2}{68.93} = 0.0290 \text{ A}$$

or $\quad I = 29.0 \text{ mA}$

(b) $R \propto L \Rightarrow R = (68.93)(3) = 206.79\,\Omega$

$$I = \frac{V}{R} = \frac{2}{206.79} = 0.00967 \text{ A}$$

or $\quad I = 9.67 \text{ mA}$

(c) $J = ep_o \upsilon_d$

For (a), $J = \dfrac{29.0\times10^{-3}}{8.5\times10^{-4}} = 34.12 \text{ A/cm}^2$

Then $\upsilon_d = \dfrac{J}{ep_o} = \dfrac{34.12}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}$

$$= 1.066\times10^4 \text{ cm/s}$$

For (b), $J = \dfrac{9.67\times10^{-3}}{8.5\times10^{-4}} = 11.38 \text{ A/cm}^2$

$$\upsilon_d = \frac{11.38}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}$$

$$= 3.55\times10^3 \text{ cm/s}$$

_____

5.9

(a) For $N_d = 2\times10^{15} \text{ cm}^{-3}$, then

$$\mu_n \cong 8000 \text{ cm}^2/\text{V-s}$$

$$R = \frac{V}{I} = \frac{5}{25\times10^{-3}} = 200\,\Omega$$

$$R = \frac{L}{(e\mu_n N_d)A}$$

or $L = (e\mu_n N_d)RA$

$$= \left(1.6\times10^{-19}\right)(8000)\left(2\times10^{15}\right)(200)\left(5\times10^{-5}\right)$$
$$= 0.0256 \text{ cm}$$

(b) $J = \dfrac{I}{A} = en_o\upsilon_d$

or $\upsilon_d = \dfrac{I}{A(en_o)}$

$$= \frac{25\times10^{-3}}{\left(5\times10^{-5}\right)\left(1.6\times10^{-19}\right)\left(2\times10^{15}\right)}$$

$$= 1.56\times10^6 \text{ cm/s}$$

(c) $I = (en_o\upsilon_d)A$

$$= \left(1.6\times10^{-19}\right)\left(2\times10^{15}\right)\left(5\times10^6\right)\left(5\times10^{-5}\right)$$
$$= 0.080 \text{ A}$$

or $\quad I = 80 \text{ mA}$

_____

_____

5.10

  (a) $E = \dfrac{V}{L} = \dfrac{3}{1} = 3$ V/cm

     $\upsilon_d = \mu_n E \Rightarrow \mu_n = \dfrac{\upsilon_d}{E} = \dfrac{10^4}{3}$

  or

     $\mu_n = 3333$ cm^2/V-s

  (b)

     $\upsilon_d = \mu_n E = (800)(3)$

  or

     $\upsilon_d = 2.4 \times 10^3$ cm/s

_____

5.11

  (a)   Silicon: For $E = 1$ kV/cm,

     $\upsilon_d = 1.2 \times 10^6$ cm/s

  Then

     $t_t = \dfrac{d}{\upsilon_d} = \dfrac{10^{-4}}{1.2 \times 10^6} = 8.33 \times 10^{-11}$ s

  For GaAs: $\upsilon_d = 7.5 \times 10^6$ cm/s

  Then

     $t_t = \dfrac{d}{\upsilon_d} = \dfrac{10^{-4}}{7.5 \times 10^6} = 1.33 \times 10^{-11}$ s

  (b)   Silicon: For $E = 50$ kV/cm,

     $\upsilon_d = 9.5 \times 10^6$ cm/s

  Then

     $t_t = \dfrac{10^{-4}}{9.5 \times 10^6} = 1.05 \times 10^{-11}$ s

  For GaAs: $\upsilon_d = 7 \times 10^6$ cm/s

  Then

     $t_t = \dfrac{10^{-4}}{7 \times 10^6} = 1.43 \times 10^{-11}$ s

_____

5.12

     $\rho = \dfrac{1}{e\mu_n n_o + e\mu_p p_o} = \dfrac{1}{e(\mu_n + \mu_p)n_i}$

  (a) $N_a = N_d = 10^{14}$ cm^{-3}

     $\Rightarrow \mu_n \cong 1350$ cm^2/V-s

     $\mu_p \cong 480$ cm^2/V-s

     $\rho = \dfrac{1}{(1.6 \times 10^{-19})(1350 + 480)(1.5 \times 10^{10})}$

     $= 2.28 \times 10^5$ Ω-cm

  (b) $N_a = N_d = 10^{16}$ cm^{-3}

     $\Rightarrow \mu_n \cong 1250$ cm^2/V-s

     $\mu_p \cong 410$ cm^2/V-s

     $\rho = \dfrac{1}{(1.6 \times 10^{-19})(1250 + 410)(1.5 \times 10^{10})}$

     $= 2.51 \times 10^5$ Ω-cm

  (c) $N_a = N_d = 10^{18}$ cm^{-3}

     $\Rightarrow \mu_n \cong 290$ cm^2/V-s

     $\mu_p \cong 130$ cm^2/V-s

     $\rho = \dfrac{1}{(1.6 \times 10^{-19})(290 + 130)(1.5 \times 10^{10})}$

     $= 9.92 \times 10^5$ Ω-cm

_____

5.13

  (a)   GaAs:

     $\sigma \cong e\mu_p p_o \Rightarrow 5 = (1.6 \times 10^{-19})\mu_p p_o$

  From Figure 5.3, and using trial and error, we find

     $p_o \cong 1.3 \times 10^{17}$ cm^{-3} and

     $\mu_p \cong 240$ cm^2/V-s

  Then

     $n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.8 \times 10^6)^2}{1.3 \times 10^{17}} = 2.49 \times 10^{-5}$ cm^{-3}

  (b)   Silicon:

     $\sigma = \dfrac{1}{\rho} \cong e\mu_n n_o$

  or

     $n_o = \dfrac{1}{\rho e \mu_n} = \dfrac{1}{(8)(1.6 \times 10^{-19})(1350)}$

  which gives

     $n_o = 5.79 \times 10^{14}$ cm^{-3}

  and

     $p_o = \dfrac{n_i^2}{n_o} = \dfrac{(1.5 \times 10^{10})^2}{5.79 \times 10^{14}} = 3.89 \times 10^5$ cm^{-3}

  Note: For the doping concentrations obtained in part (b), the assumed mobility values are valid.

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5.14

$$\sigma_i = en_i\left(\mu_n + \mu_p\right)$$

Then

$$10^{-6} = \left(1.6\times10^{-19}\right)\left(1000+600\right)n_i$$

or

$$n_i\,(300\text{ K}) = 3.91\times10^9\text{ cm}^{-3}$$

Now

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

or

$$E_g = kT\ln\left(\frac{N_c N_\upsilon}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(10^{19}\right)^2}{\left(3.91\times10^9\right)^2}\right]$$

which gives

$$E_g = 1.122\,\text{eV}$$

Now

$$n_i^2\,(500\text{K}) = \left(10^{19}\right)^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right]$$

$$= 5.15\times10^{26}$$

or

$$n_i\,(500\text{ K}) = 2.27\times10^{13}\text{ cm}^{-3}$$

Then

$$\sigma_i = \left(1.6\times10^{-19}\right)\left(2.27\times10^{13}\right)\left(1000+600\right)$$

which gives

$$\sigma_i\,(500\text{ K}) = 5.81\times10^{-3}\ \left(\Omega\text{-cm}\right)^{-1}$$

_____

5.15

(a) (i) Silicon: $\sigma_i = en_i\left(\mu_n + \mu_p\right)$

$$\sigma_i = \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)\left(1350+480\right)$$

or

$$\sigma_i = 4.39\times10^{-6}\,\left(\Omega\text{-cm}\right)^{-1}$$

(ii) Ge:

$$\sigma_i = \left(1.6\times10^{-19}\right)\left(2.4\times10^{13}\right)\left(3900+1900\right)$$

or

$$\sigma_i = 2.23\times10^{-2}\,\left(\Omega\text{-cm}\right)^{-1}$$

(iii) GaAs:

$$\sigma_i = \left(1.6\times10^{-19}\right)\left(1.8\times10^6\right)\left(8500+400\right)$$

or

$$\sigma_i = 2.56\times10^{-9}\,\left(\Omega\text{-cm}\right)^{-1}$$

(b) $R = \dfrac{L}{\sigma A}$

(i) Si:

$$R = \frac{200\times10^{-4}}{\left(4.39\times10^{-6}\right)\left(85\times10^{-8}\right)} = 5.36\times10^9\ \Omega$$

(ii) Ge:

$$R = \frac{200\times10^{-4}}{\left(2.23\times10^{-2}\right)\left(85\times10^{-8}\right)} = 1.06\times10^6\ \Omega$$

(iii) GaAs:

$$R = \frac{200\times10^{-4}}{\left(2.56\times10^{-9}\right)\left(85\times10^{-8}\right)} = 9.19\times10^{12}\ \Omega$$

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5.16

(a) $\sigma = e\mu_n N_d$

$$0.25 = \left(1.6\times10^{-19}\right)\mu_n N_d$$

From Figure 5.3, for $N_d = 1.2\times10^{15}\text{ cm}^{-3}$,

then $\mu_n \cong 1300\text{ cm}^2/\text{V-s}$

So $\sigma = \left(1.6\times10^{-19}\right)(1300)\left(1.2\times10^{15}\right)$

$$= 0.2496\,\left(\Omega\text{-cm}\right)^{-1}$$

(b) Using Figure 5.2,
(i) For $T = 250\text{ K}\,(-23°\text{C})$,

$$\Rightarrow \mu_n \cong 1800\text{ cm}^2/\text{V-s}$$

$$\sigma = \left(1.6\times10^{-19}\right)(1800)\left(1.2\times10^{15}\right)$$

$$= 0.346\,\left(\Omega\text{-cm}\right)^{-1}$$

(ii) For $T = 400\text{ K}\,(127°\text{C})$,

$$\Rightarrow \mu_n \cong 670\text{ cm}^2/\text{V-s}$$

$$\sigma = \left(1.6\times10^{-19}\right)(670)\left(1.2\times10^{15}\right)$$

$$= 0.129\,\left(\Omega\text{-cm}\right)^{-1}$$

_____

5.17

$$\sigma_{avg} = \frac{1}{t}\int_0^t \sigma(x)dx = \frac{1}{t}\int_0^t \sigma_o \exp\left(\frac{-x}{d}\right)dx$$

$$= \frac{\sigma_o}{t}(-d)\exp\left(\frac{-x}{d}\right)\Big|_0^t$$

$$= \frac{-\sigma_o d}{t}\left[\exp\left(\frac{-t}{d}\right)-1\right]$$

$$= \frac{(20)(0.3)}{(1.5)}\left[1-\exp\left(\frac{-1.5}{0.3}\right)\right]$$

$$= 3.97\,\left(\Omega\text{-cm}\right)^{-1}$$

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5.18

  (a) $E = \dfrac{V}{L} = \dfrac{2}{150 \times 10^{-4}} = 133.3 \text{ V/cm}$

  (b) $\sigma(x) = e\mu_n N_d(x)$

$$\sigma_{avg} = e\mu_n \cdot \frac{1}{T}\int_0^T \left(2\times10^{16}\right)\left(1 - \frac{x}{1.111T}\right)dx$$

$$= \frac{e\mu_n\left(2\times10^{16}\right)}{T}\left[x - \frac{x^2}{2(1.111T)}\right]_0^T$$

$$= \frac{e\mu_n\left(2\times10^{16}\right)}{T}\left[T - \frac{T^2}{2(1.111T)}\right]$$

$$= e\mu_n\left(2\times10^{16}\right)(0.55)$$

$$= \left(1.6\times10^{-19}\right)(750)\left(2\times10^{16}\right)(0.55)$$

$$\sigma_{avg} = 1.32\,(\Omega\text{-cm})^{-1}$$

  (c) $I = \dfrac{\sigma_{avg}A}{L}\cdot V = \dfrac{(1.32)\left(7.5\times10^{-4}\right)\left(10^{-4}\right)}{150\times10^{-4}}\cdot 2$

$$= 1.32\times10^{-5}\text{ A}$$

  or $I = 13.2\,\mu\text{A}$

  (d) Top surface;

$$\sigma = \left(1.6\times10^{-19}\right)(750)\left(2\times10^{16}\right)$$

$$= 2.4\,(\Omega\text{-cm})^{-1}$$

$$J = \sigma E = (2.4)(133.3) = 320\text{ A/cm}^2$$

  Bottom surface:

$$\sigma = \left(1.6\times10^{-19}\right)(750)\left(2\times10^{15}\right)$$

$$= 0.24\,(\Omega\text{-cm})^{-1}$$

$$J = \sigma E = (0.24)(133.3 = 32)\text{A/cm}^2$$

_____

5.19

  Plot

_____

5.20

  (a) $E = 10\text{ V/cm}$

  so

$$\upsilon_d = \mu_n E = (1350)(10) = 1.35\times10^4\text{ cm/s}$$

  or

$$\upsilon_d = 1.35\times10^2\text{ m/s}$$

  Then

$$T = \frac{1}{2}m_n^*\upsilon_d^2$$

$$= \frac{1}{2}(1.08)\left(9.11\times10^{-31}\right)\left(1.35\times10^2\right)^2$$

  or

$$T = 8.97\times10^{-27}\text{ J} \Rightarrow 5.60\times10^{-8}\text{ eV}$$

  (b) $E = 1\text{ kV/cm}$

$$\upsilon_d = (1350)(1000) = 1.35\times10^6\text{ cm/s}$$

  or

$$\upsilon_d = 1.35\times10^4\text{ m/s}$$

  Then

$$T = \frac{1}{2}(1.08)\left(9.11\times10^{-31}\right)\left(1.35\times10^4\right)^2$$

  or

$$T = 8.97\times10^{-23}\text{ J} \Rightarrow 5.60\times10^{-4}\text{ eV}$$

_____

5.21

  (a) $n_i^2 = N_c N_\upsilon \exp\left(\dfrac{-E_g}{kT}\right)$

$$= \left(2\times10^{19}\right)\left(1\times10^{19}\right)\exp\left(\frac{-1.10}{0.0259}\right)$$

$$= 7.18\times10^{19}$$

  or

$$n_i = 8.47\times10^9\text{ cm}^{-3}$$

For $N_d = 10^{14}\text{ cm}^{-3} >> n_i \Rightarrow n_o = 10^{14}\text{ cm}^{-3}$

Then

$$J = \sigma E = e\mu_n n_o E$$

$$= \left(1.6\times10^{-19}\right)(1000)\left(10^{14}\right)(100)$$

  or

$$J = 1.60\text{ A/cm}^2$$

  (b) A 5% increase is due to a 5% increase in electron concentration, so

$$n_o = 1.05\times10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

which becomes

$$\left(1.05\times10^{14} - 5\times10^{13}\right)^2 = \left(5\times10^{13}\right)^2 + n_i^2$$

and yields

$$n_i^2 = 5.25\times10^{26}$$

$$= \left(2\times10^{19}\right)\left(1\times10^{19}\right)\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

  or

$$2.625\times10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left[\frac{-1.10}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$T = 456\text{ K}$$

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5.22

(a) $\sigma = e\mu_n n_o + e\mu_p p_o$ and $n_o = \dfrac{n_i^2}{p_o}$

Then

$$\sigma = \frac{e\mu_n n_i^2}{p_o} + e\mu_p p_o$$

To find the minimum conductivity, set

$$\frac{d\sigma}{dp_o} = 0 = \frac{(-1)e\mu_n n_i^2}{p_o^2} + e\mu_p$$

which yields

$$p_o = n_i\left(\frac{\mu_n}{\mu_p}\right)^{1/2} \quad \text{(Answer to part (b))}$$

Substituting into the conductivity expression

$$\sigma = \sigma_{\min} = \frac{e\mu_n n_i^2}{\left[n_i\left(\mu_n/\mu_p\right)^{1/2}\right]}$$
$$+ e\mu_p\left[n_i\left(\mu_n/\mu_p\right)^{1/2}\right]$$

which simplifies to

$$\sigma_{\min} = 2en_i\sqrt{\mu_n\mu_p}$$

The intrinsic conductivity is defined as

$$\sigma_i = en_i\left(\mu_n + \mu_p\right) \Rightarrow en_i = \frac{\sigma_i}{\mu_n + \mu_p}$$

The minimum conductivity can then be written as

$$\sigma_{\min} = \frac{2\sigma_i\sqrt{\mu_n\mu_p}}{\mu_n + \mu_p}$$

_____

5.23

(a) n-type: $n_o = N_d = 5\times10^{16}\,\text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}}$$
$$= 4.5\times10^3\,\text{cm}^{-3}$$

p-type: $p_o = N_a = 2\times10^{16}\,\text{cm}^{-3}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}} = 1.125\times10^4\,\text{cm}^{-3}$$

compensated: $n_o = N_d - N_a$
$$= 5\times10^{16} - 2\times10^{16}$$
$$= 3\times10^{16}\,\text{cm}^{-3}$$

$$p_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5\times10^3\,\text{cm}^{-3}$$

(b) From Figure 5.3,

n-type: $\mu_n \cong 1100\,\text{cm}^2/\text{V-s}$

p-type: $\mu_p \cong 400\,\text{cm}^2/\text{V-s}$

compensated: $\mu_n \cong 1000\,\text{cm}^2/\text{V-s}$

(c) n-type: $\sigma = e\mu_n n_o$
$$= \left(1.6\times10^{-19}\right)\left(1100\right)\left(5\times10^{16}\right)$$
$$= 8.8\,(\Omega\text{-cm})^{-1}$$

p-type: $\sigma = e\mu_p p_o$
$$= \left(1.6\times10^{-19}\right)\left(400\right)\left(2\times10^{16}\right)$$
$$= 1.28\,(\Omega\text{-cm})^{-1}$$

compensated: $\sigma = e\mu_n n_o$
$$= \left(1.6\times10^{-19}\right)\left(1000\right)\left(3\times10^{16}\right)$$
$$= 4.8\,(\Omega\text{-cm})^{-1}$$

(d) $J = \sigma\text{E} \Rightarrow \text{E} = \dfrac{J}{\sigma}$

n-type: $\text{E} = \dfrac{120}{8.8} = 13.6\,\text{V/cm}$

p-type: $\text{E} = \dfrac{120}{1.28} = 93.75\,\text{V/cm}$

compensated: $\text{E} = \dfrac{120}{4.8} = 25\,\text{V/cm}$

_____

5.24

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}$$
$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{500}$$
$$= 0.00050 + 0.000667 + 0.0020$$

or

$$\frac{1}{\mu} = 0.003167$$

Then

$$\mu = 316\,\text{cm}^2/\text{V-s}$$

_____

5.25

$$\mu_n = \left(1300\right)\left(\frac{T}{300}\right)^{-3/2} = \left(1300\right)\left(\frac{300}{T}\right)^{+3/2}$$

(a) At $T = 200\,\text{K}$,

$$\mu_n = \left(1300\right)\left(\frac{300}{200}\right)^{3/2} = 2388\,\text{cm}^2/\text{V-s}$$

(b) At $T = 400\,\text{K}$, $\mu_n = 844\,\text{cm}^2/\text{V-s}$

_____

5.26

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{250} + \frac{1}{500} = 0.006$$

Then

$$\mu = 167 \text{ cm}^2/\text{V-s}$$

5.27

Plot

5.28

Plot

5.29

$$J_n = eD_n\frac{dn}{dx} = eD_n\left(\frac{5\times10^{14}-n(0)}{0.01-0}\right)$$

$$0.19 = \left(1.6\times10^{-19}\right)(25)\left(\frac{5\times10^{14}-n(0)}{0.010}\right)$$

Then

$$\frac{(0.19)(0.010)}{\left(1.6\times10^{-19}\right)(25)} = 5\times10^{14}-n(0)$$

which yields

$$n(0) = 0.25\times10^{14} \text{ cm}^{-3}$$

5.30

$$J_n = eD_n\frac{dn}{dx} = eD_n\frac{\Delta n}{\Delta x}$$

$$J_n = \left(1.6\times10^{-19}\right)(27)\left[\frac{2\times10^{16}-5\times10^{15}}{0-0.012}\right]$$

$$J_n = -5.4 \text{ A/cm}^2$$

5.31

(a) $$J_n = eD_n\frac{dn}{dx} = eD_n\frac{\Delta n}{\Delta x}$$

$$-2 = \left(1.6\times10^{-19}\right)(30)\left[\frac{10^{15}-n(x_1)}{0-20\times10^{-4}}\right]$$

$$4\times10^{-3} = 4.8\times10^{-3} - 4.8\times10^{-18}\,n(x_1)$$

which yields

$$n(x_1) = 1.67\times10^{14} \text{ cm}^{-3}$$

(b) $$-2 = \left(1.6\times10^{-19}\right)(230)\left[\frac{10^{15}-n(x_1)}{0-20\times10^{-4}}\right]$$

$$4\times10^{-3} = 3.68\times10^{-2} - 3.68\times10^{-17}\,n(x_1)$$

$$n(x_1) = 8.91\times10^{14} \text{ cm}^{-3}$$

5.32

$$J_p = -eD_p\frac{dp}{dx} = -eD_p\frac{d}{dx}\left[10^{16}\left(1+\frac{x}{L}\right)^2\right]$$

$$= -eD_p\cdot\frac{10^{16}}{L}\cdot2\left(1+\frac{x}{L}\right)$$

(a)   For $x = 0$,

$$J_p = \frac{-\left(1.6\times10^{-19}\right)(10)\left(10^{16}\right)(2)}{12\times10^{-4}}$$

$$= -26.7 \text{ A/cm}^2$$

(b)   For $x = -6\,\mu\text{m}$,

$$J_p = \frac{-\left(1.6\times10^{-19}\right)(10)\left(10^{16}\right)(2)\left(1-\frac{6}{12}\right)}{12\times10^{-4}}$$

$$= -13.3 \text{ A/cm}^2$$

(c)   For $x = -12\,\mu\text{m}$,

$$J_p = 0$$

5.33

For electrons:

$$J_n = eD_n\frac{dn}{dx} = eD_n\frac{d}{dx}\left[10^{15}\,e^{-x/L_n}\right]$$

$$= \frac{-eD_n\left(10^{15}\right)e^{-x/L_n}}{L_n}$$

At $x = 0$,

$$J_n = \frac{-\left(1.6\times10^{-19}\right)(25)\left(10^{15}\right)}{2\times10^{-3}} = -2 \text{ A/cm}^2$$

For holes:

$$J_p = -eD_p\frac{dp}{dx} = -eD_p\frac{d}{dx}\left[5\times10^{15}\,e^{+x/L_p}\right]$$

$$= \frac{-eD_p\left(5\times10^{15}\right)e^{+x/L_p}}{L_p}$$

For $x = 0$,

$$J_p = \frac{-\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)}{5\times10^{-4}} = -16 \text{ A/cm}^2$$

$$J_{Total} = J_n(x=0) + J_p(x=0)$$

$$= -2 + (-16) = -18 \text{ A/cm}^2$$

_____

5.34

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dx}\left[5\times10^{15}\, e^{-x/L_p}\right]$$

$$= \frac{eD_p\left(5\times10^{15}\right)e^{-x/L_p}}{L_p}$$

(a) (i) $\quad J_p = \frac{\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)}{50\times10^{-4}}$

$$= 1.6\,\text{A/cm}^2$$

(ii) $\quad J_p = \frac{\left(1.6\times10^{-19}\right)(48)\left(5\times10^{15}\right)}{22.5\times10^{-4}}$

$$= 17.07\,\text{A/cm}^2$$

(b) (i) $\quad J_p = \frac{\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)e^{-1}}{50\times10^{-4}}$

$$= 0.589\,\text{A/cm}^2$$

(ii) $\quad J_p = \frac{\left(1.6\times10^{-19}\right)(48)\left(5\times10^{15}\right)e^{-1}}{22.5\times10^{-4}}$

$$= 6.28\,\text{A/cm}^2$$

_____

5.35

$$J_n = e\mu_n n\text{E} + eD_n \frac{dn}{dx}$$

or

$$-40 = \left(1.6\times10^{-19}\right)(960)\left[10^{16}\exp\left(\frac{-x}{18}\right)\right]\text{E}$$
$$+ \left(1.6\times10^{-19}\right)(25)\left(10^{16}\right)$$
$$\times\left(\frac{-1}{18\times10^{-4}}\right)\exp\left(\frac{-x}{18}\right)$$

Then

$$-40 = (1.536)\left[\exp\left(\frac{-x}{18}\right)\right]\text{E} - 22.22\exp\left(\frac{-x}{18}\right)$$

We find

$$\text{E} = \frac{(22.22)\exp\left(\frac{-x}{18}\right) - 40}{(1.536)\exp\left(\frac{-x}{18}\right)}$$

or

$$\text{E} = 14.5 - (26.0)\exp\left(\frac{+x}{18}\right)$$

_____

5.36

(a) $J_n = eD_n \frac{dn}{dx} = eD_n \frac{d}{dx}\left[2\times10^{15}\, e^{-x/L}\right]$

$$= \frac{-eD_n\left(2\times10^{15}\right)e^{-x/L}}{L}$$

$$= \frac{-\left(1.6\times10^{-19}\right)(27)\left(2\times10^{15}\right)e^{-x/L}}{15\times10^{-4}}$$

$$= -5.76e^{-x/L}$$

(b) $J_p = J_{Total} - J_n = -10 - \left(-5.76e^{-x/L}\right)$

$$= \left[5.76e^{-x/L} - 10\right]\text{A/cm}^2$$

(c) We have $\quad J_p = \sigma\text{E} = \left(e\mu_p p_o\right)\text{E}$

$$5.76e^{-x/L} - 10 = \left(1.6\times10^{-19}\right)(420)\left(10^{16}\right)\text{E}$$

So $\quad \text{E} = \left[8.57e^{-x/L} - 14.88\right]\text{V/cm}$

_____

5.37

(a) $\quad J = e\mu_n n(x)\text{E} + eD_n \frac{dn(x)}{dx}$

We have $\quad \mu_n = 8000\,\text{cm}^2/\text{V-s}$, so that

$$D_n = (0.0259)(8000) = 207\,\text{cm}^2/\text{s}$$

Then

$$100 = \left(1.6\times10^{-19}\right)(8000)(12)n(x)$$
$$+ \left(1.6\times10^{-19}\right)(207)\frac{dn(x)}{dx}$$

which yields

$$100 = \left(1.536\times10^{-14}\right)n(x) + \left(3.312\times10^{-17}\right)\frac{dn(x)}{dx}$$

Solution is of the form

$$n(x) = A + B\exp\left(\frac{-x}{d}\right)$$

so that

$$\frac{dn(x)}{dx} = \frac{-B}{d}\exp\left(\frac{-x}{d}\right)$$

Substituting into the differential equation, we have

$$100 = \left(1.536\times10^{-14}\right)\left[A + B\exp\left(\frac{-x}{d}\right)\right]$$
$$- \frac{\left(3.312\times10^{-17}\right)}{d}B\exp\left(\frac{-x}{d}\right)$$

This equation is valid for all x, so

$$100 = \left(1.536\times10^{-14}\right)A$$

or

$$A = 6.51\times10^{15}$$

Also

$$1.536 \times 10^{-14} B \exp\left(\frac{-x}{d}\right)$$

$$-\frac{\left(3.312 \times 10^{-17}\right)}{d} B \exp\left(\frac{-x}{d}\right) = 0$$

which yields

$$d = 2.156 \times 10^{-3} \text{ cm}$$

At $x = 0$, $e\mu_n n(0)E = 50$

so that

$$50 = \left(1.6 \times 10^{-19}\right)(8000)(12)(A+B)$$

which yields

$$B = -3.255 \times 10^{15}$$

Then

$$n(x) = 6.51 \times 10^{15} - 3.255 \times 10^{15} \exp\left(\frac{-x}{d}\right) \text{cm}^{-3}$$

(b)

At $x = 0$, $n(0) = 6.51 \times 10^{15} - 3.255 \times 10^{15}$

Or

$$n(0) = 3.26 \times 10^{15} \text{ cm}^{-3}$$

At $x = 50\,\mu\text{m}$,

$$n(50) = 6.51 \times 10^{15} - 3.255 \times 10^{15} \exp\left(\frac{-50}{21.56}\right)$$

or

$$n(50) = 6.19 \times 10^{15} \text{ cm}^{-3}$$

(c)

At $x = 50\,\mu\text{m}$,

$$J_{drf} = e\mu_n n(50)E$$

$$= \left(1.6 \times 10^{-19}\right)(8000)\left(6.19 \times 10^{15}\right)(12)$$

or

$$J_{drf}(x = 50) = 95.08 \text{ A/cm}^2$$

Then

$$J_{diff}(x = 50) = 100 - 95.08$$

or

$$J_{diff}(x = 50) = 4.92 \text{ A/cm}^2$$

_____

5.38

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

(a) $E_F - E_{Fi} = ax + b$, $b = 0.4$

$$0.15 = a\left(10^{-3}\right) + 0.4$$

which yields

$$a = -2.5 \times 10^2$$

Then

$$E_F - E_{Fi} = 0.4 - 2.5 \times 10^2 x$$

so

$$n = n_i \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{kT}\right)$$

(b) $J_n = eD_n \dfrac{dn}{dx}$

$$= eD_n n_i \left(\frac{-2.5 \times 10^2}{kT}\right) \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{kT}\right)$$

Assume $T = 300$ K, so $kT = 0.0259$ eV and

$$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Then

$$J_n = \frac{-\left(1.6 \times 10^{-19}\right)(25)\left(1.5 \times 10^{10}\right)\left(2.5 \times 10^2\right)}{(0.0259)}$$

$$\times \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{0.0259}\right)$$

or

$$J_n = -5.79 \times 10^{-4} \exp\left(\frac{0.4 - 2.5 \times 10^2 x}{0.0259}\right)$$

(i)   At $x = 0$, $J_n = -2.95 \times 10^3$ A/cm^2

(ii)   At $x = 5\,\mu\text{m}$, $J_n = -23.7$ A/cm^2

_____

5.39

(a) $J_n = e\mu_n n E + eD_n \dfrac{dn}{dx}$

$$-80 = \left(1.6 \times 10^{-19}\right)(1000)\left(10^{16}\right)\left(1 - \frac{x}{L}\right)E$$

$$+ \left(1.6 \times 10^{-19}\right)(25.9)\left(\frac{-10^{16}}{L}\right)$$

where $L = 10 \times 10^{-4} = 10^{-3}$ cm

We find

$$-80 = (1.6)E - (1.6)\left(\frac{x}{10^{-3}}\right)E - 41.44$$

or

$$80 = (1.6)\left(\frac{x}{L} - 1\right)E + 41.44$$

Solving for the electric field, we find

$$E = \frac{24.1}{\left(\dfrac{x}{L} - 1\right)} \text{ V/cm}$$

_____

(b) For $J_n = -20$ A/cm^2

$$20 = (1.6)\left(\frac{x}{L} - 1\right)E + 41.44$$

Then

$$E = \frac{13.3}{\left(1 - \dfrac{x}{L}\right)} \text{ V/cm}$$

_____

5.40

(a) $E_X = -\left(\dfrac{kT}{e}\right) \cdot \dfrac{1}{N_d(x)} \cdot \dfrac{dN_d(x)}{dx}$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}} \cdot \frac{d}{dx}\left[N_{do}e^{-x/L}\right]$$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}} \cdot \left(\frac{-1}{L}\right)N_{do}e^{-x/L}$$

$$= \frac{0.0259}{L} = \frac{0.0259}{10\times10^{-4}}$$

  or $E_X = 25.9$ V/cm

(b) $\phi = -\displaystyle\int_0^L E_X\, dx = -(25.9)(L-0)$

$$= -(25.9)\left(10\times10^{-4}\right) = -0.0259 \text{ V}$$

  or $\phi = -25.9$ mV

_____

5.41

From Example 5.6

$$E_x = \frac{(0.0259)(10^{19})}{(10^{16} - 10^{19}x)} = \frac{(0.0259)(10^3)}{(1 - 10^3 x)}$$

$$V = -\int_0^{10^{-4}} E_x\, dx$$

$$= -(0.0259)(10^3)\int_0^{10^{-4}} \frac{dx}{(1 - 10^3 x)}$$

$$= -(0.0259)(10^3)\left(\frac{-1}{10^3}\right)\ln\left[1 - 10^3 x\right]\Big|_0^{10^{-4}}$$

$$= (0.0259)\left[\ln(1 - 0.1) - \ln(1)\right]$$

or

$$V = -2.73 \text{ mV}$$

_____

5.42

$$E_x = -\left(\frac{kT}{e}\right) \cdot \frac{1}{N_d(x)} \cdot \frac{dN_d(x)}{dx}$$

For $N_d(x) = N_{do}e^{-x/L}$

So $E_X = \dfrac{0.0259}{L} = 500$ V/cm

Which yields $L = 5.18\times10^{-5}$ cm

_____

5.43

(a) We have

$$J_{diff} = eD_n\frac{dn}{dx} = eD_n\frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)}\cdot N_{do}\exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (6000)(0.0259)$$

or

$$D_n = 155.4\, \text{cm}^2/\text{s}$$

Then

$$J_{diff} = \frac{-\left(1.6\times10^{-19}\right)(155.4)\left(5\times10^{16}\right)}{\left(0.1\times10^{-4}\right)}\exp\left(\frac{-x}{L}\right)$$

or

$$J_{diff} = -1.243\times10^5 \exp\left(\frac{-x}{L}\right) \text{ A/cm}^2$$

(b)

$$0 = J_{drf} + J_{diff}$$

Now

$$J_{drf} = e\mu_n nE$$

$$= \left(1.6\times10^{-19}\right)(6000)\left(5\times10^{16}\right)\left[\exp\left(\frac{-x}{L}\right)\right]E$$

or

$$J_{drf} = (48)\left[\exp\left(\frac{-x}{L}\right)\right]E$$

We have

$$J_{drf} = -J_{diff}$$

so

$$(48)\left[\exp\left(\frac{-x}{L}\right)\right]E = 1.243\times10^5 \exp\left(\frac{-x}{L}\right)$$

which yields

$$E = 2.59\times10^3 \text{ V/cm}$$

_____

_____

5.44

Plot

_____

5.45

(a) (i) $D_n = (0.0259)(1150) = 29.8 \, \text{cm}^2/\text{s}$

   (ii) $D_n = (0.0259)(6200) = 160.6 \, \text{cm}^2/\text{s}$

(b) (i) $\mu_p = \dfrac{8}{0.0259} = 308.9 \, \text{cm}^2/\text{V-s}$

   (ii) $\mu_p = \dfrac{35}{0.0259} = 1351 \, \text{cm}^2/\text{V-s}$

_____

5.46

$L = 10^{-1} \, \text{cm}, \quad W = 10^{-2} \, \text{cm}, \quad d = 10^{-3} \, \text{cm}$

(a)

$$V_H = \frac{-I_X B_Z}{ned} = \frac{-(1.2 \times 10^{-3})(5 \times 10^{-2})}{(2 \times 10^{22})(1.6 \times 10^{-19})(10^{-5})}$$

$$= -1.875 \times 10^{-3} \, \text{V}$$

or $V_H = -1.875 \, \text{mV}$

(b)

$$\text{E}_H = \frac{V_H}{W} = \frac{-1.875 \times 10^{-3}}{10^{-2}} = -0.1875 \, \text{V/cm}$$

_____

5.47

(a) $V_H = \dfrac{-I_x B_z}{ned}$

$$= \frac{-(250 \times 10^{-6})(5 \times 10^{-2})}{(5 \times 10^{21})(1.6 \times 10^{-19})(5 \times 10^{-5})}$$

or

$V_H = -0.3125 \, \text{mV}$

(b)

$$\text{E}_H = \frac{V_H}{W} = \frac{-0.3125 \times 10^{-3}}{2 \times 10^{-2}}$$

or

$\text{E}_H = -1.56 \times 10^{-2} \, \text{V/cm}$

(c)

$\mu_n = \dfrac{I_x L}{enV_x Wd}$

$$= \frac{(250 \times 10^{-6})(10^{-3})}{(1.6 \times 10^{-19})(5 \times 10^{21})(0.1)(2 \times 10^{-4})(5 \times 10^{-5})}$$

or

$\mu_n = 0.3125 \, \text{m}^2/\text{V-s} = 3125 \, \text{cm}^2/\text{V-s}$

_____

5.48

(a) $V_H < 0 \Rightarrow$ n-type

(b) $n = \dfrac{-I_X B_Z}{edV_H} = \dfrac{-(0.50 \times 10^{-3})(0.10)}{(1.6 \times 10^{-19})(10^{-5})(-5.2 \times 10^{-3})}$

$$= 6.01 \times 10^{21} \, \text{m}^{-3}$$

or $n = 6.01 \times 10^{15} \, \text{cm}^{-3}$

(c) $\mu_n = \dfrac{I_X L}{enV_X Wd}$

$$= \frac{(0.5 \times 10^{-3})(10^{-3})}{(1.6 \times 10^{-19})(6.01 \times 10^{21})(15)(10^{-4})(10^{-5})}$$

$$= 0.03466 \, \text{m}^2/\text{V-s}$$

or $\mu_n = 346.6 \, \text{cm}^2/\text{V-s}$

_____

5.49

(a) $V_H = \text{E}_H W = -(16.5 \times 10^{-3})(5 \times 10^{-2})$

or

$V_H = -0.825 \, \text{mV}$

(b) $V_H = \text{negative} \Rightarrow$ n-type

(c) $n = \dfrac{-I_x B_z}{edV_H}$

$$= \frac{-(0.5 \times 10^{-3})(6.5 \times 10^{-2})}{(1.6 \times 10^{-19})(5 \times 10^{-5})(-0.825 \times 10^{-3})}$$

or

$n = 4.924 \times 10^{21} \, \text{m}^{-3} = 4.924 \times 10^{15} \, \text{cm}^{-3}$

(d)

$\mu_n = \dfrac{I_x L}{enV_x Wd}$

$$= \frac{(0.5 \times 10^{-3})(0.5 \times 10^{-2})}{(1.6 \times 10^{-19})(4.924 \times 10^{21})(1.25)(5 \times 10^{-4})(5 \times 10^{-5})}$$

or

$\mu_n = 0.1015 \, \text{m}^2/\text{V-s} = 1015 \, \text{cm}^2/\text{V-s}$

_____

_____

5.50

(a) $V_H = $ negative $\Rightarrow$ n-type

(b) $n = \dfrac{-I_x B_z}{ed V_H}$

$\qquad = \dfrac{-\left(2.5\times10^{-3}\right)\left(2.5\times10^{-2}\right)}{\left(1.6\times10^{-19}\right)\left(0.01\times10^{-2}\right)\left(-4.5\times10^{-3}\right)}$

or

$\quad n = 8.68\times10^{20}\,\mathrm{m}^{-3} = 8.68\times10^{14}\,\mathrm{cm}^{-3}$

(c) $\mu_n = \dfrac{I_x L}{en V_x W d}$

$\qquad = \left[\dfrac{\left(2.5\times10^{-3}\right)\left(0.5\times10^{-2}\right)}{\left(1.6\times10^{-19}\right)\left(8.68\times10^{20}\right)\left(2.2\right)} \right]$

$\qquad\qquad \times \left[\dfrac{1}{\left(0.05\times10^{-2}\right)\left(0.01\times10^{-2}\right)} \right]$

or

$\quad \mu_n = 0.8182\,\mathrm{m}^2/\mathrm{V\text{-}s} = 8182\,\mathrm{cm}^2/\mathrm{V\text{-}s}$

(d) $\sigma = \dfrac{1}{\rho} = e\mu_n n$

$\qquad = \left(1.6\times10^{-19}\right)\left(8182\right)\left(8.68\times10^{14}\right)$

or

$\quad \rho = 0.88\,(\Omega\text{-cm})$

_____

Chapter 6

6.1

$$n_o = N_d = 5 \times 10^{15} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{N_d} = \frac{\left(1.5 \times 10^{10}\right)^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

(a) Minority carrier hole lifetime is a
constant.

$$\tau_{pt} = \tau_{p0} = 2 \times 10^{-7} \text{ s}$$

$$R_{po} = \frac{p_o}{\tau_{p0}} = \frac{4.5 \times 10^4}{2 \times 10^{-7}} = 2.25 \times 10^{11} \text{ cm}^{-3} \text{ s}^{-1}$$

(b) $R'_{po} = \dfrac{p_o + \delta p}{\tau_{p0}} = \dfrac{4.5 \times 10^4 + 10^{14}}{2 \times 10^{-7}}$

$$= 5 \times 10^{20} \text{ cm}^{-3} \text{ s}^{-1}$$

_____

6.2

$$p_o = N_a = 2 \times 10^{16} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8 \times 10^6\right)^2}{2 \times 10^{16}} = 1.62 \times 10^{-4} \text{ cm}^{-3}$$

(a) $R' = \dfrac{\delta n}{\tau_{n0}} = \dfrac{5 \times 10^{14}}{5 \times 10^{-7}} = 10^{21} \text{ cm}^{-3} \text{ s}^{-1}$

(b) $R_p = \dfrac{p_o}{\tau_{pt}} = \dfrac{n_o}{\tau_{nt}} = \dfrac{n_o}{\tau_{n0}}$

$$\tau_{pt} = \frac{p_o}{n_o} \cdot \tau_{n0} = \frac{\left(2 \times 10^{16}\right)}{\left(1.62 \times 10^{-4}\right)} \cdot \left(5 \times 10^{-7}\right)$$

$$= 6.17 \times 10^{13} \text{ s}$$

_____

6.3

(a) Recombination rates are equal

$$\frac{n_o}{\tau_{nO}} = \frac{p_o}{\tau_{pO}}$$

$$n_o = N_d = 10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Then

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25 \times 10^4}{20 \times 10^{-6}}$$

which yields

$$\tau_{nO} = 8.89 \times 10^{+6} \text{ s}$$

(b) Generation rate = recombination rate
Then

$$G = \frac{2.25 \times 10^4}{20 \times 10^{-6}} = 1.125 \times 10^9 \text{ cm}^{-3} \text{ s}^{-1}$$

(c)

$$R = G = 1.125 \times 10^9 \text{ cm}^{-3} \text{ s}^{-1}$$

_____

6.4

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{\left(6.625 \times 10^{-34}\right)\left(3 \times 10^8\right)}{6300 \times 10^{-10}}$

or

$E = 3.15 \times 10^{-19}$ J; energy of one photon

Now

$1 \text{ W} = 1 \text{ J/s} \Rightarrow 3.17 \times 10^{18}$ photons/s

Volume $= (1)(0.1) = 0.1 \text{ cm}^3$

Then

$$g = \frac{3.17 \times 10^{18}}{0.1}$$

$$= 3.17 \times 10^{19} \quad \text{e-h pairs/cm}^3 \text{-s}$$

(b)

$$\delta n = \delta p = g\tau = \left(3.17 \times 10^{19}\right)\left(10 \times 10^{-6}\right)$$

or

$$\delta n = \delta p = 3.17 \times 10^{14} \text{ cm}^{-3}$$

_____

6.5

We have

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p p\text{E} - eD_p \nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p p\text{E} - D_p \nabla p$$

Now

$$\nabla \bullet F_p^+ = \mu_p \nabla \bullet (p\text{E}) - D_p \nabla \bullet \nabla p$$

We can write

$$\nabla \bullet (p\text{E}) = \text{E} \bullet \nabla p + p\nabla \bullet \text{E}$$

and

$$\nabla \bullet \nabla p = \nabla^2 p$$

so

$$\nabla \bullet F_p^+ = \mu_p \left(\text{E} \bullet \nabla p + p\nabla \bullet \text{E}\right) - D_p \nabla^2 p$$

_____

Then

$$\frac{\partial p}{\partial t} = -\mu_p \left(\text{E} \bullet \nabla p + p \nabla \bullet \text{E} \right)$$

$$+ D_p \nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p \nabla^2 p - \mu_p \left(\text{E} \bullet \nabla p + p \nabla \bullet \text{E} \right)$$

$$+ g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

_____

6.6

From Equation (6.18),

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \bullet F_p^+ + g_p - R_p$$

For a one-dimensional case,

$$\frac{dF_p^+}{dx} = g_p - R_p = 10^{20} - 2 \times 10^{19}$$

or

$$\frac{dF_p^+}{dx} = 8 \times 10^{19}\,\text{cm}^{-3}\,\text{s}^{-1}$$

_____

6.7

From Equation (6.18),

$$0 = -\frac{dF_p^+}{dx} + 0 - 2 \times 10^{19}$$

or

$$\frac{dF_p^+}{dx} = -2 \times 10^{19}\,\text{cm}^{-3}\,\text{s}^{-1}$$

_____

6.8

We have the continuity equations

(1) $\quad D_p \nabla^2 (\delta p) - \mu_p \left[\text{E} \bullet \nabla(\delta p) + p \nabla \bullet \text{E} \right]$

$$+ g_p - \frac{p}{\tau_p} = \frac{\partial(\delta p)}{\partial t}$$

and

(2) $\quad D_n \nabla^2 (\delta n) + \mu_n \left[\text{E} \bullet \nabla(\delta n) + n \nabla \bullet \text{E} \right]$

$$+ g_n - \frac{n}{\tau_n} = \frac{\partial(\delta n)}{\partial t}$$

By charge neutrality,

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla(\delta n) = \nabla(\delta p)$$

and

$$\nabla^2 (\delta n) = \nabla^2 (\delta p) \ \text{ and } \ \frac{\partial(\delta n)}{\partial t} = \frac{\partial(\delta p)}{\partial t}$$

Also

$$g_n = g_p \equiv g \ , \ \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we have

(1) $\quad D_p \nabla^2 (\delta n) - \mu_p \left[\text{E} \bullet \nabla(\delta n) + p \nabla \bullet \text{E} \right]$

$$+ g - R = \frac{\partial(\delta n)}{\partial t}$$

and

(2) $\quad D_n \nabla^2 (\delta n) + \mu_n \left[\text{E} \bullet \nabla(\delta n) + n \nabla \bullet \text{E} \right]$

$$+ g - R = \frac{\partial(\delta n)}{\partial t}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and add the two equations.
We find

$$\left(\mu_n n D_p + \mu_p p D_n \right) \nabla^2 (\delta n)$$

$$+ \mu_n \mu_p (p - n) \text{E} \bullet \nabla(\delta n)$$

$$+ \left(\mu_n n + \mu_p p \right)(g - R)$$

$$= \left(\mu_n n + \mu_p p \right) \frac{\partial(\delta n)}{\partial t}$$

Divide by $\left(\mu_n n + \mu_p p \right)$, then

$$\left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \right) \nabla^2 (\delta n)$$

$$+ \left[\frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \right] \text{E} \bullet \nabla(\delta n)$$

$$+ (g - R) = \frac{\partial(\delta n)}{\partial t}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

and

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Then we have

$$D' \nabla^2 (\delta n) + \mu' \text{E} \bullet \nabla(\delta n) + (g - R) = \frac{\partial(\delta n)}{\partial t}$$

Q.E.D.

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_____

6.9

p-type material;

   minority carriers are electrons

(a) $\mu' = \mu_n$

   From Figure 5.3, $\mu_n \cong 1300 \text{ cm}^2/\text{V-s}$

(b) $D' = D_n = \left(\dfrac{kT}{e}\right)\cdot\mu_n = (0.0259)(1300)$

       $= 33.67 \text{ cm}^2/\text{s}$

(c) $\tau_{nt} = \tau_{n0} = 10^{-7} \text{ s}$

      $p_o = N_a = 7 \times 10^{15} \text{ cm}^{-3}$

      $n_o = \dfrac{n_i^2}{N_a} = \dfrac{\left(1.5\times10^{10}\right)^2}{7\times10^{15}}$

         $= 3.21 \times 10^4 \text{ cm}^{-3}$

      $\dfrac{n_o}{\tau_{nt}} = \dfrac{p_o}{\tau_{pt}}$

      $\dfrac{3.214\times10^4}{10^{-7}} = \dfrac{7\times10^{15}}{\tau_{pt}}$

  so $\tau_{pt} = 2.18\times10^4 \text{ s}$

_____

6.10

  For Ge: $n_i = 2.4\times10^{13} \text{ cm}^{-3}$

    $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

      $= \dfrac{4\times10^{13}}{2} + \sqrt{\left(\dfrac{4\times10^{13}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$

      $= 5.124\times10^{13} \text{ cm}^{-3}$

    $p_o = \dfrac{n_i^2}{n_o} = \dfrac{\left(2.4\times10^{13}\right)^2}{5.124\times10^{13}} = 1.124\times10^{13} \text{ cm}^{-3}$

(a) We have:

    $\mu_n = 3900 \text{ cm}^2/\text{V-s},\ \ D_n = 101 \text{ cm}^2/\text{s}$

    $\mu_p = 1900 \text{ cm}^2/\text{V-s},\ \ D_p = 49.2 \text{ cm}^2/\text{s}$

  For very, very low injection,

  $D' = \dfrac{D_n D_p (n+p)}{D_n n + D_p p}$

    $= \dfrac{(101)(49.2)\left(5.124\times10^{13} + 1.124\times10^{13}\right)}{(101)\left(5.124\times10^{13}\right) + (49.2)\left(1.124\times10^{13}\right)}$

    $= 54.2 \text{ cm}^2/\text{s}$

and

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$$

$$= \frac{(3900)(1900)\left(1.124\times10^{13} - 5.124\times10^{13}\right)}{(3900)\left(5.124\times10^{13}\right) + (1900)\left(1.124\times10^{13}\right)}$$

$$= -1340 \text{ cm}^2/\text{V-s}$$

(b) For holes, $\tau_{pt} = \tau_{p0} = 2\times10^{-6} \text{ s}$

    For electrons,

    $\dfrac{n}{\tau_{nt}} = \dfrac{p}{\tau_{p0}}$

    $\dfrac{5.124\times10^{13}}{\tau_{nt}} = \dfrac{1.124\times10^{13}}{2\times10^{-6}}$

    $\Rightarrow \tau_{nt} = 9.12\times10^{-6} \text{ s}$

_____

6.11

   $\sigma = e\mu_n n + e\mu_p p$

  With excess carriers

   $n = n_o + \delta n$ and $p = p_o + \delta p$

  For an n-type semiconductor, we can write

   $\delta n = \delta p \equiv \delta p$

  Then

   $\sigma = e\mu_n (n_o + \delta p) + e\mu_p (p_o + \delta p)$

  or

   $\sigma = e\mu_n n_o + e\mu_p p_o + e(\mu_n + \mu_p)(\delta p)$

  so

   $\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$

  In steady-state, $\delta p = g'\tau_{pO}$

  So that

   $\Delta\sigma = e(\mu_n + \mu_p)(g'\tau_{pO})$

_____

6.12

(a) $p_o = N_a = 10^{16} \text{ cm}^{-3}$

    $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$

    $\sigma = e\mu_n (n_o + \delta n) + e\mu_p (p_o + \delta p)$

    $\cong e\mu_p p_o + e(\mu_n + \mu_p)\delta n$

  Now $\delta n = \delta p = g'\tau_{n0}\left(1 - e^{-t/\tau_{n0}}\right)$

      $= \left(8\times10^{20}\right)\left(5\times10^{-7}\right)\left(1 - e^{-t/\tau_{n0}}\right)$

      $= 4\times10^{14}\left(1 - e^{-t/\tau_{n0}}\right) \text{ cm}^{-3}$

_____

Then $\sigma = \left(1.6\times10^{-19}\right)\left(380\right)\left(10^{16}\right)$

$+\left(1.6\times10^{-19}\right)\left(900+380\right)$

$\times\left(4\times10^{14}\right)\left(1-e^{-t/\tau_{n0}}\right)$

$\sigma = 0.608 + 0.0819\left(1-e^{-t/\tau_{n0}}\right)$ (Ω -cm)$^{-1}$

(b) (i) $\sigma(0) = 0.608$ (Ω -cm)$^{-1}$

(ii) $\sigma(\infty) = 0.690$ (Ω -cm)$^{-1}$

_____

6.13

(a) For $0 \le t \le 10^{-6}$ s,

$\delta n = \delta p = g'\tau_{p0}\left(1-e^{-t/\tau_{p0}}\right)$

$= \left(4\times10^{21}\right)\left(5\times10^{-8}\right)\left(1-e^{-t/\tau_{p0}}\right)$

$= \left(2\times10^{14}\right)\left(1-e^{-t/\tau_{p0}}\right)$ cm^{-3}

At $t = 10^{-6}$ s,

$\delta p\left(10^{-6}\right) = \left(2\times10^{14}\right)\left(1-e^{-10^{-6}/5\times10^{-8}}\right)$

$= 2\times10^{14}$ cm^{-3}

Then for $t \ge 10^{-6}$ s,

$\delta p = \left(2\times10^{14}\right)e^{-\left(t-10^{-6}\right)/\tau_{p0}}$ cm^{-3}

(b) $n_o = 5\times10^{15}$ cm^{-3}

$\sigma = e\mu_n n_o + e\left(\mu_n + \mu_p\right)\delta p$

For $0 \le t \le 10^{-6}$ s,

$\sigma = \left(1.6\times10^{-19}\right)\left(7500\right)\left(5\times10^{15}\right)$

$+\left(1.6\times10^{-19}\right)\left(7500+310\right)$

$\times\left(2\times10^{14}\right)\left(1-e^{-t/\tau_{p0}}\right)$

$= 6.0 + 0.250\left(1-e^{-t/\tau_{p0}}\right)$ (Ω -cm)$^{-1}$

For $t \ge 10^{-6}$ s,

$\sigma = 6.0 + 0.250e^{-\left(t-10^{-6}\right)/\tau_{p0}}$ (Ω -cm)$^{-1}$

_____

6.14

$I = \dfrac{V}{R}$; $R = \dfrac{L}{\sigma A}$

$\Rightarrow I = \dfrac{\sigma A}{L}\cdot V$

For $N_I = N_d + N_a = 8\times10^{15} + 2\times10^{15}$

$= 10^{16}$ cm^{-3}

Then, $\mu_n \cong 1300$ cm^2/V-s

$\mu_p \cong 400$ cm^2/V-s

$\sigma \cong e\mu_n n_o + e\left(\mu_n + \mu_p\right)\delta p$

where $\delta p = g'\tau_{p0}e^{-t/\tau_{p0}}$

$= \left(8\times10^{20}\right)\left(5\times10^{-7}\right)e^{-t/\tau_{p0}}$

$= 4\times10^{14}e^{-t/\tau_{p0}}$ cm^{-3}

$\sigma = \left(1.6\times10^{-19}\right)\left(1300\right)\left(8\times10^{15} - 2\times10^{15}\right)$

$+\left(1.6\times10^{-19}\right)\left(1300+400\right)$

$\times\left(4\times10^{14}\right)e^{-t/\tau_{p0}}$

$\sigma = 1.248 + 0.109e^{-t/\tau_{p0}}$

$I = \dfrac{\left[1.248 + 0.109e^{-t/\tau_{p0}}\right]\left(10^{-5}\right)\left(10\right)}{0.05}$

$= 2.496\times10^{-3} + 2.18\times10^{-4}e^{-t/\tau_{p0}}$ A

or $I = 2.496 + 0.218e^{-t/\tau_{p0}}$ mA

_____

6.15

$p_o = N_a - N_d = 2\times10^{16} - 6\times10^{15}$

$= 1.4\times10^{16}$ cm^{-3}

(a) $\delta n = \delta p = g'\tau_{n0}$

$5\times10^{14} = 2\times10^{21}\tau_{n0}$

$\Rightarrow \tau_{n0} = 2.5\times10^{-7}$ s

(b) $\delta n = \delta p = g'\tau_{n0}\left(1-e^{-t/\tau_{n0}}\right)$

$= 5\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)$

$R' = \dfrac{\delta n}{\tau_{n0}} = \dfrac{5\times10^{14}}{2.5\times10^{-7}}\left(1-e^{-t/\tau_{n0}}\right)$

$= 2\times10^{21}\left(1-e^{-t/\tau_{no}}\right)$ cm^{-3} s^{-1}

(c)

(i) $\left(\dfrac{1}{4}\right)\left(5\times10^{14}\right) = 5\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)$

$t = \tau_{n0}\ln(1.3333) = 7.19\times10^{-8}$ s

(ii) $\left(\dfrac{1}{2}\right)\left(5\times10^{14}\right) = 5\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)$

$t = \tau_{n0}\ln(2) = 1.73\times10^{-7}$ s

(iii) $\left(\dfrac{3}{4}\right)\left(5\times10^{14}\right) = 5\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)$

$t = \tau_{n0}\ln(4) = 3.47\times10^{-7}$ s

(iv) $\left(0.95\right)\left(5\times10^{14}\right) = 5\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)$

$t = \tau_{n0}\ln(20) = 7.49\times10^{-7}$ s

_____

6.16

$$n_o = N_d - N_a = 8 \times 10^{15} - 2 \times 10^{15}$$
$$= 6 \times 10^{15} \text{ cm}^{-3}$$
$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{6 \times 10^{15}} = 5.4 \times 10^{-4} \text{ cm}^{-3}$$

(a) $R_o = \frac{p_o}{\tau_{p0}} \Rightarrow 4 \times 10^4 = \frac{5.4 \times 10^{-4}}{\tau_{p0}}$

so $\tau_{p0} = 1.35 \times 10^{-8}$ s

(b) $\delta p = g' \tau_{p0} = (2 \times 10^{21})(1.35 \times 10^{-8})$
$$= 2.7 \times 10^{13} \text{ cm}^{-3}$$

(c) $\tau = \tau_{p0} = 1.35 \times 10^{-8}$ s

_____

6.17

(a) (i) For $0 \le t \le 5 \times 10^{-7}$ s
$$\delta p(t) = g' \tau_{p0} \left(1 - e^{-t/\tau_{p0}}\right)$$
$$= (5 \times 10^{20})(5 \times 10^{-7})\left(1 - e^{-t/\tau_{p0}}\right)$$
$$= 2.5 \times 10^{14}\left(1 - e^{-t/\tau_{p0}}\right) \text{ cm}^{-3}$$

At $t = 5 \times 10^{-7}$ s,
$$\delta p = 2.5 \times 10^{14}\left(1 - e^{-1/1}\right)$$
$$= 1.58 \times 10^{14} \text{ cm}^{-3}$$

For $t \ge 5 \times 10^{-7}$ s
$$\delta p(t) = 1.58 \times 10^{14} e^{-\left(t - 5 \times 10^{-7}\right)/\tau_{pO}} \text{ cm}^{-3}$$

(ii) $\delta p(5 \times 10^{-7}) = 1.58 \times 10^{14} \text{ cm}^{-3}$

(b) (i) For $0 \le t \le 2 \times 10^{-6}$ s
$$\delta p(t) = 2.5 \times 10^{14}\left(1 - e^{-t/\tau_{p0}}\right) \text{ cm}^{-3}$$

At $t = 2 \times 10^{-6}$ s,
$$\delta p = 2.5 \times 10^{14}\left(1 - e^{-\left(2 \times 10^{-6}\right)/\left(5 \times 10^{-7}\right)}\right)$$
$$= 2.454 \times 10^{14} \text{ cm}^{-3}$$

For $t \ge 2 \times 10^{-6}$ s,
$$\delta p(t) = 2.454 \times 10^{14} e^{-\left(t - 2 \times 10^{-6}\right)/\tau_{pO}} \text{ cm}^{-3}$$

(ii) $\delta p(2 \times 10^{-6}) = 2.454 \times 10^{14} \text{ cm}^{-3}$

_____

6.18

(a) For $0 \le t \le 2 \times 10^{-6}$ s
$$\delta n(t) = g' \tau_{n0} e^{-t/\tau_{n0}}$$
$$= (10^{21})(5 \times 10^{-7}) e^{-t/\tau_{n0}}$$
$$= 5 \times 10^{14} e^{-t/\tau_{n0}} \text{ cm}^{-3}$$

At $t = 2 \times 10^{-6}$ s,
$$\delta n_1 = 5 \times 10^{14} e^{-\left(2 \times 10^{-6}\right)/\left(5 \times 10^{-7}\right)}$$
$$= 9.16 \times 10^{12} \text{ cm}^{-3}$$

For $t \ge 2 \times 10^{-6}$ s
$$\delta n = (5 \times 10^{14} - 9.16 \times 10^{12})\left(1 - e^{-t/\tau_{n0}}\right)$$
$$+ 9.16 \times 10^{12}$$
$$= 4.908 \times 10^{14}\left(1 - e^{-t/\tau_{n0}}\right) + 9.16 \times 10^{12} \text{ cm}^{-3}$$

(b) (i) $\delta n(0) = 5 \times 10^{14} \text{ cm}^{-3}$

(ii) $\delta n(2 \times 10^{-6}) = 9.16 \times 10^{12} \text{ cm}^{-3}$

(iii) $\delta n(\infty) = 5 \times 10^{14} \text{ cm}^{-3}$

_____

6.19

p-type; minority carriers - electrons
$$D_n = \left(\frac{kT}{e}\right)\mu_n = (0.0259)(1200)$$
$$= 31.08 \text{ cm}^2/\text{s}$$
$$L_n = \sqrt{D_n \tau_{n0}} = \left[(31.08)(10^{-6})\right]^{1/2}$$
$$= 5.575 \times 10^{-3} \text{ cm}$$

(a) $\delta n(x) = \delta p(x) = 2 \times 10^{14} e^{-x/L_n} \text{ cm}^{-3}$

(b) $J_n = eD_n \frac{d(\delta n)}{dx} = eD_n \frac{d}{dx}\left[2 \times 10^{14} e^{-x/L_n}\right]$
$$= \frac{-eD_n}{L_n}\left(2 \times 10^{14}\right)e^{-x/L_n}$$
$$= \frac{-(1.6 \times 10^{-19})(31.08)(2 \times 10^{14})}{(5.575 \times 10^{-3})} e^{-x/L_n}$$
$$J_n = -0.1784 e^{-x/L_n} \text{ A/cm}^2$$

Holes diffuse at same rate as minority carrier electrons, so
$$J_p = +0.1784 e^{-x/L_n} \text{ A/cm}^2$$

_____

6.20

(a) p-type; $p_{pO} = 10^{14} \text{ cm}^{-3}$
and
$$n_{pO} = \frac{n_i^2}{p_{pO}} = \frac{(1.5 \times 10^{10})^2}{10^{14}} = 2.25 \times 10^6 \text{ cm}^{-3}$$

(b) Excess minority carrier concentration
$$\delta n = n_p - n_{pO}$$

At $x = 0$, $n_p = 0$ so that
$$\delta n(0) = 0 - n_{pO} = -2.25 \times 10^6 \text{ cm}^{-3}$$

_____

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where } L_n^2 = D_n \tau_{nO}$$

The general solution is of the form

$$\delta n = A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{+x}{L_n}\right)$$

For $x \to \infty$, δn remains finite, so $B = 0$.
Then the solution is

$$\delta n = -n_{pO} \exp\left(\frac{-x}{L_n}\right)$$

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6.21

$$\delta n(x) = 5 \times 10^{14} \, e^{-x/L_n} \text{ cm}^{-3}$$

where $L_n = \sqrt{D_n \tau_{n0}} = \left[(25)(10^{-6})\right]^{1/2}$

$$= 5 \times 10^{-3} \text{ cm}$$

$$J_n = eD_n \frac{d(\delta n)}{dx} = eD_n \frac{d}{dx}\left[\left(5 \times 10^{14}\right)e^{-x/L_n}\right]$$

$$= -\frac{eD_n}{L_n}\left(5 \times 10^{14}\right)e^{-x/L_n}$$

$$= -\frac{\left(1.6 \times 10^{-19}\right)(25)\left(5 \times 10^{14}\right)}{\left(5 \times 10^{-3}\right)} e^{-x/L_n}$$

$$J_n = -0.4 e^{-x/L_n} \text{ A/cm}^2$$

(a) For $x = 0$,

$$\delta n(0) = 5 \times 10^{14} \text{ cm}^{-3}$$

$$J_n(0) = -0.4 \text{ A/cm}^2$$

$$J_p(0) = +0.4 \text{ A/cm}^2$$

(b) For $x = L_n = 5 \times 10^{-3}$ cm,

$$\delta n(L_n) = \left(5 \times 10^{14}\right)e^{-1} = 1.84 \times 10^{14} \text{ cm}^{-3}$$

$$J_n(L_n) = -0.4 e^{-1} = -0.147 \text{ A/cm}^2$$

$$J_p(L_n) = +0.4 e^{-1} = +0.147 \text{ A/cm}^2$$

(c) For $x = 15 \times 10^{-3}$ cm $= 3L_n$

$$\delta n(3L_n) = \left(5 \times 10^{14}\right)e^{-3} = 2.49 \times 10^{13} \text{ cm}^{-3}$$

$$J_n(3L_n) = -0.4 e^{-3} = -0.020 \text{ A/cm}^2$$

$$J_p(3L_n) = +0.4 e^{-3} = +0.020 \text{ A/cm}^2$$

_____

6.22

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx} - \mu_p E_o \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{pO}} = 0$$

Assume the solution is of the form

$$\delta p = A \exp(sx)$$

Then

$$\frac{d(\delta p)}{dx} = As \exp(sx), \quad \frac{d^2(\delta p)}{dx^2} = As^2 \exp(sx)$$

Substituting into the differential equation

$$D_p As^2 \exp(sx) - \mu_p E_o As \exp(sx)$$

$$- \frac{A \exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p E_o s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p, we have

$$s^2 - \frac{\mu_p E_o}{D_p} s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2}\left[\frac{\mu_p}{D_p}E_o \pm \sqrt{\left(\frac{\mu_p}{D_p}E_o\right)^2 + \frac{4}{L_p^2}}\right]$$

which can be rewritten as

$$s = \frac{1}{L_p}\left[\frac{\mu_p L_p E_o}{2D_p} \pm \sqrt{\left(\frac{\mu_p L_p E_o}{2D_p}\right)^2 + 1}\right]$$

Define

$$\beta \equiv \frac{\mu_p L_p E_o}{2D_p}$$

Then

$$s = \frac{1}{L_p}\left[\beta \pm \sqrt{1 + \beta^2}\right]$$

In order that $\delta p = 0$ as $x \to +\infty$, use the minus sign for $x > 0$ and the plus sign for $x < 0$. Then the solution is

$$\delta p = A \exp(s_- x) \quad \text{for } x > 0$$

$$\delta p = A \exp(s_+ x) \quad \text{for } x < 0$$

where

$$s_\pm = \frac{1}{L_p}\left[\beta \pm \sqrt{1 + \beta^2}\right]$$

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6.23

Plot

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6.24

(a) From Equation (6.55)

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n E_o \frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n} E_o \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n} E_o = \frac{E_o}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'} \cdot \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

The solution is of the form

$$\delta n = \delta n(0) \exp(-\alpha x) \text{ where } \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \text{ and } \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we find

$$\alpha^2(\delta n) + \frac{1}{L'}[-\alpha(\delta n)] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n}\left[\frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'}\right)^2 + 1}\right]$$

We may note that if $E_o = 0$, then $L' \to \infty$

and $\alpha = \frac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n \tau_{nO}} \text{ where } D_n = \mu_n\left(\frac{kT}{e}\right)$$

so

$$D_n = (1200)(0.0259) = 31.1 \text{ cm}^2/\text{s}$$

and

$$L_n = \sqrt{(31.1)(5 \times 10^{-7})} = 39.4 \times 10^{-4} \text{ cm}$$

or

$$L_n = 39.4 \, \mu\text{m}$$

For $E_o = 12$ V/cm, then

$$L' = \frac{(kT/e)}{E_o} = \frac{0.0259}{12} = 21.6 \times 10^{-4} \text{ cm}$$

and

$$\alpha = 5.75 \times 10^2 \text{ cm}^{-1}$$

(c) Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

_____

6.25

p-type so the minority carriers are electrons and

$$D_n \nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{nO} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \text{ which gives } \delta n = g't + C_1$$

For $t \leq 0$, $\delta n = 0 \Rightarrow C_1 = 0$

Then

$$\delta n = G_o't \text{ for } 0 \leq t \leq T$$

For $t > T$, $g' = 0$ so that $\frac{d(\delta n)}{dt} = 0$

And

$$\delta n = G_o'T \text{ (no recombination)}$$

_____

6.26

n-type, so minority carriers are holes and

$$D_p \nabla^2(\delta p) - \mu_p E \bullet \nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $E = 0$, and

$\frac{\partial(\delta p)}{\partial t} = 0$ (steady-state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \text{ or } \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G_o' = $ constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G_o'}{D_p} x + C_1$$

_____

and

$$\delta p = -\frac{G'_o}{2D_p}x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_3 \text{ and}$$

$$\delta p = C_3 x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_5 \text{ and}$$

$$\delta p = C_5 x + C_6$$

The boundary conditions are:
(1) $\delta p = 0$ at $x = +3L$
(2) $\delta p = 0$ at $x = -3L$
(3) δp continuous at $x = L$
(4) δp continuous at $x = -L$
(5) $\dfrac{d(\delta p)}{dx}$ continuous at $x = L$
(6) $\dfrac{d(\delta p)}{dx}$ continuous at $x = -L$

Applying the boundary conditions, we find

$$\delta p = \frac{G'_o}{2D_p}\left(5L^2 - x^2\right) \text{ for } -L < x < +L$$

$$\delta p = \frac{G'_o L}{D_p}\left(3L - x\right) \text{ for } L < x < 3L$$

$$\delta p = \frac{G'_o L}{D_p}\left(3L + x\right) \text{ for } -3L < x < -L$$

_____

6.27

$$E_0 = \frac{V}{L} = \frac{8}{0.4} = 20 \text{ V/cm}$$

$$\mu_p = \frac{d}{E_0 t_0} = \frac{0.25}{(20)(32\times10^{-6})}$$

$$= 390.6 \text{ cm}^2/\text{V-s}$$

$$D_p = \frac{\left(\mu_p E_0\right)^2 (\Delta t)^2}{16 t_0}$$

$$= \frac{[(390.6)(20)]^2 \left(9.35\times10^{-6}\right)^2}{16\left(32\times10^{-6}\right)}$$

$$D_p = 10.42 \text{ cm}^2/\text{s}$$

We find

$$\frac{D_p}{\mu_p} = \frac{10.42}{390.6} = 0.02668 \text{ V}$$

This value is very close to 0.0259 for $T = 300$ K.

_____

6.28
(a)

Assume that $f(x,t) = (4\pi Dt)^{-1/2}\exp\left(\dfrac{-x^2}{4Dt}\right)$

is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\frac{\partial^2 f}{\partial x^2} = (4\pi Dt)^{-1/2}\left[\left(\frac{-2x}{4Dt}\right)^2\exp\left(\frac{-x^2}{4Dt}\right)\right.$$

$$\left.+\left(\frac{-2}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)\right]$$

Also

$$\frac{\partial f}{\partial t} = (4\pi Dt)^{-1/2}\left(\frac{-x^2}{4D}\right)\left(\frac{-1}{t^2}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

$$+(4\pi D)^{-1/2}\left(\frac{-1}{2}\right)t^{-3/2}\exp\left(\frac{-x^2}{4Dt}\right)$$

Substituting the expressions for $\dfrac{\partial^2 f}{\partial x^2}$ and

$\dfrac{\partial f}{\partial t}$ into the differential equation, we find

$0 = 0$.

Q.E.D.

(b)
Consider

$$\int_{-\infty}^{+\infty}\exp\left(\frac{-x^2}{4Dt}\right)dx$$

Let $u = x^2$, then $du = 2x \cdot dx$ or

$$dx = \frac{du}{2x} = \frac{du}{2\sqrt{u}}$$

Let $a = \dfrac{1}{4Dt}$

Now

_____

$$\int_{-\infty}^{+\infty} \exp\left(\frac{-x^2}{4Dt}\right)dx = 2\int_{0}^{\infty} \exp\left(\frac{-x^2}{4Dt}\right)dx$$

$$= 2\int_{0}^{\infty} \frac{1}{2\sqrt{u}} \exp(-au)du = \int_{0}^{\infty} \frac{1}{\sqrt{u}} \exp(-au)du$$

$$= \sqrt{\frac{\pi}{a}} = \sqrt{4\pi Dt}$$

Then

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)dx = \frac{\sqrt{4\pi Dt}}{\sqrt{4\pi Dt}} = 1$$

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6.29

Plot

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6.30

(a) $E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.383225 \text{ eV}$$

(b) $\delta n = \delta p = g'\tau_{p0} = (2\times10^{21})(5\times10^{-7})$

$$= 10^{15} \text{ cm}^{-3}$$

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4\times10^{16} + 10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.383865 \text{ eV}$$

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$\cong (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.28768 \text{ eV}$$

(c) $E_{Fn} - E_F = 0.383865 - 0.383225$

$$= 0.000640 \text{ eV}$$

or $= 0.640 \text{ meV}$

_____

6.31

(a) p-type

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.3294 \text{ eV}$$

(b)

$$\delta n = \delta p = 5\times10^{14} \text{ cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} = 4.5\times10^4 \text{ cm}^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4.5\times10^4 + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fn} - E_{Fi} = 0.2697 \text{ eV}$$

and

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15} + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_{Fp} = 0.3318 \text{ eV}$$

_____

6.32

(a) For n-type,

$$E_{Fn} - E_F = (E_{Fn} - E_{Fi}) - (E_F - E_{Fi})$$

$$= kT \ln\left(\frac{n_o + \delta n}{n_i}\right) - kT \ln\left(\frac{n_o}{n_i}\right)$$

$$= kT \ln\left(\frac{n_o + \delta n}{n_o}\right)$$

So $0.00102 = (0.0259)\ln\left(\frac{5\times10^{15} + \delta n}{5\times10^{15}}\right)$

$$5\times10^{15} + \delta n = 5\times10^{15} \exp\left(\frac{0.00102}{0.0259}\right)$$

Which yields $\delta n \cong 2\times10^{14} \text{ cm}^{-3}$

_____

(b) $E_{Fn} - E_{Fi} = kT \ln\left(\dfrac{n_o + \delta n}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{5\times10^{15} + 2\times10^{14}}{1.5\times10^{10}}\right)$

$= 0.33038 \text{ eV}$

(c) $E_{Fi} - E_{Fp} \cong kT \ln\left(\dfrac{\delta p}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{2\times10^{14}}{1.5\times10^{10}}\right)$

$= 0.2460 \text{ eV}$

_____

6.33

(a) $E_{Fn} - E_{Fi} \cong kT \ln\left(\dfrac{\delta n}{n_i}\right)$

or $\delta n = n_i \exp\left[\dfrac{E_{Fn} - E_{Fi}}{kT}\right]$

$= (1.5\times10^{10})\exp\left[\dfrac{0.270}{0.0259}\right]$

$= 5.05\times10^{14} \text{ cm}^{-3}$

(b) $E_{Fi} - E_{Fp} = kT \ln\left(\dfrac{p_o + \delta p}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{6\times10^{15} + 5.05\times10^{14}}{1.5\times10^{10}}\right)$

$= 0.33618 \text{ eV}$

(c) (i) $E_F - E_{Fp} = (E_{Fi} - E_{Fp}) - (E_{Fi} - E_F)$

$= kT \ln\left(\dfrac{p_o + \delta p}{n_i}\right) - kT \ln\left(\dfrac{p_o}{n_i}\right)$

$= kT \ln\left(\dfrac{p_o + \delta p}{p_o}\right)$

(ii) $E_F - E_{Fp}$

$= (0.0259)\ln\left(\dfrac{6\times10^{15} + 5.05\times10^{14}}{6\times10^{15}}\right)$

$= 2.093\times10^{-3} \text{ eV}$

or $= 2.093 \text{ meV}$

_____

6.34

(a) (i) $E_{Fn} - E_{Fi} = kT \ln\left(\dfrac{n_o + n}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{(1.02)(10^{16})}{1.8\times10^6}\right)$

$= 0.58166 \text{ eV}$

(ii) $E_{Fi} - E_{Fp} \cong kT \ln\left(\dfrac{\delta p}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{0.02\times10^{16}}{1.8\times10^6}\right)$

$= 0.47982 \text{ eV}$

(b) (i) $E_{Fn} - E_{Fi} = (0.0259)\ln\left(\dfrac{1.1\times10^{16}}{1.8\times10^6}\right)$

$= 0.58361 \text{ eV}$

(ii) $E_{Fi} - E_{Fp} = (0.0259)\ln\left(\dfrac{0.1\times10^{16}}{1.8\times10^6}\right)$

$= 0.52151 \text{ eV}$

_____

6.35

Quasi-Fermi level for minority carrier electrons:

$E_{Fn} - E_{Fi} = kT \ln\left(\dfrac{n_o + \delta n}{n_i}\right)$

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4} \text{ cm}^{-3}$

We have

$\delta n = (10^{14})\left(\dfrac{x}{50}\right)$

Then

$E_{Fn} - E_{Fi} = kT \ln\left[\dfrac{3.24\times10^{-4} + (10^{14} x/50)}{1.8\times10^6}\right]$

We find

x (μm)	($E_{Fn} - E_{Fi}$) (eV)
0	-0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

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Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

We have $p_o = 10^{16} \, \text{cm}^{-3}$ and $\delta n = \delta p$.
We find

$x \, (\mu\text{m})$	$(E_{Fi} - E_{Fp})$ (eV)
0	+0.58115
50	+0.58140

_____

6.36

(a) We can write

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

and

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

so that

$$\left(E_{Fi} - E_{Fp}\right) - \left(E_{Fi} - E_F\right) = E_F - E_{Fp}$$

$$= kT \ln\left(\frac{p_o + \delta p}{n_i}\right) - kT \ln\left(\frac{p_o}{n_i}\right)$$

or

$$E_F - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{p_o}\right) = (0.01)kT$$

Then

$$\frac{p_o + \delta p}{p_o} = \exp(0.01) = 1.010$$

or

$$\frac{\delta p}{p_o} = 0.010 \Rightarrow \text{low injection, so that}$$

$$\delta p = 5 \times 10^{12} \, \text{cm}^{-3}$$

(b)

$$E_{Fn} - E_{Fi} \cong kT \ln\left(\frac{\delta p}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5 \times 10^{12}}{1.5 \times 10^{10}}\right)$$

or

$$E_{Fn} - E_{Fi} = 0.1505 \, \text{eV}$$

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6.37

Plot

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6.38

(a) $E_{Fi} - E_{Fp} \cong kT \ln\left(\frac{\delta p}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{\delta p}{1.5 \times 10^{10}}\right)$$

$\delta p = 10^{11} \, \text{cm}^{-3}$, $E_{Fi} - E_{Fp} = 0.04914 \, \text{eV}$

10^{12}	0.10877
10^{13}	0.16841
10^{14}	0.22805
10^{15}	0.28768

(b) $E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{2 \times 10^{16} + \delta n}{1.5 \times 10^{10}}\right)$$

$\delta n = 10^{11} \, \text{cm}^{-3}$, $E_{Fn} - E_{Fi} = 0.365273 \, \text{eV}$

10^{12}	0.365274
10^{13}	0.365286
10^{14}	0.365402
10^{15}	0.366536

_____

6.39

(a)

$$R = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n(n + n') + C_p(p + p')}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}(n + n') + \tau_{nO}(p + p')}$$

Let $n' = p' = n_i$. For $n = p = 0$

$$R = \frac{-n_i^2}{\tau_{pO} n_i + \tau_{nO} n_i} = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$$

(b) We had defined the net generation rate as

$$g - R = g_o + g' - \left(R_o + R'\right)$$

where $g_o = R_o$ since these are the thermal equilibrium generation and recombination rates.

If $g' = 0$, then $g - R = -R'$ and

$$R' = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$$

so that $g - R = + \dfrac{n_i}{\tau_{pO} + \tau_{nO}}$

Thus a negative recombination rate implies a net positive generation rate.

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6.40

We have that

$$R = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n\left(n+n'\right)+C_p\left(p+p'\right)}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}\left(n+n_i\right)+\tau_{nO}\left(p+n_i\right)}$$

If $n = n_o + \delta n$ and $p = p_o + \delta n$, then

$$R = \frac{\left(n_o + \delta n\right)\left(p_o + \delta n\right)-n_i^2}{\tau_{pO}\left(n_o + \delta n + n_i\right)+\tau_{nO}\left(p_o + \delta n + n_i\right)}$$

$$= \frac{n_o p_o + \delta n\left(n_o + p_o\right)+\left(\delta n\right)^2 - n_i^2}{\tau_{pO}\left(n_o + \delta n + n_i\right)+\tau_{nO}\left(p_o + \delta n + n_i\right)}$$

If $\delta n << n_i$, we can neglect $\left(\delta n\right)^2$: also

$$n_o p_o = n_i^2$$

Then

$$R = \frac{\delta n\left(n_o + p_o\right)}{\tau_{pO}\left(n_o + n_i\right)+\tau_{nO}\left(p_o + n_i\right)}$$

(a) For n-type; $n_o >> p_O$, $n_o >> n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7} \text{ s}^{-1}$$

(b) For intrinsic, $n_o = p_o = n_i$

Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO}\left(2n_i\right)+\tau_{nO}\left(2n_i\right)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO}+\tau_{nO}} = \frac{1}{10^{-7}+5\times10^{-7}} \Rightarrow$$

$$\frac{R}{\delta n} = 1.67\times10^{+6} \text{ s}^{-1}$$

(c) For p-type; $p_o >> n_o$, $p_o >> n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5\times10^{-7}} = 2\times10^{+6} \text{ s}^{-1}$$

_____

6.41

(a) From Equation (6.56)

$$D_p \frac{d^2\left(\delta p\right)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g'\tau_{pO} + A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right)$$

At $x = +\infty$, $\delta p = g'\tau_{pO}$ so that $B = 0$,

Then

$$\delta p = g'\tau_{pO} + A\exp\left(\frac{-x}{L_p}\right)$$

We have

$$D_p \left.\frac{d\left(\delta p\right)}{dx}\right|_{x=0} = s\left(\delta p\right)\Big|_{x=0}$$

We can write

$$\left.\frac{d\left(\delta p\right)}{dx}\right|_{x=0} = \frac{-A}{L_p} \text{ and } \left(\delta p\right)\Big|_{x=0} = g'\tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s\left(g'\tau_{pO} + A\right)$$

Solving for A, we find

$$A = \frac{-sg'\tau_{pO}}{\dfrac{D_p}{L_p}+s}$$

The excess concentration is then

$$\delta p = g'\tau_{pO}\left[1 - \frac{s}{\left(D_p/L_p\right)+s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

where

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(10^{-7})} = 10^{-3} \text{ cm}$$

Now

$$\delta p = \left(10^{21}\right)\left(10^{-7}\right)$$

$$\times\left[1 - \frac{s}{\left(10/10^{-3}\right)+s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

or

$$\delta p = 10^{14}\left[1 - \frac{s}{10^4 + s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

(i) For $s = 0$,

$$\delta p = 10^{14} \text{ cm}^{-3}$$

(ii) For $s = 2000$ cm/s,

$$\delta p = 10^{14}\left[1 - 0.167\exp\left(\frac{-x}{L_p}\right)\right]$$

(iii) For $s = \infty$,

$$\delta p = 10^{14}\left[1 - \exp\left(\frac{-x}{L_p}\right)\right]$$

_____

(b)
(i) For $s = 0$,
$$\delta p(0) = 10^{14} \text{ cm}^{-3}$$
(ii) For $s = 2000$ cm/s,
$$\delta p(0) = 0.833 \times 10^{14} \text{ cm}^{-3}$$
(iii) For $s = \infty$,
$$\delta p(0) = 0$$

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6.42
$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(5 \times 10^{-7})}$$
$$= 35.4 \times 10^{-4} \text{ cm}$$
(a) At $x = 0$,
$$g' \tau_{nO} = (2 \times 10^{21})(5 \times 10^{-7}) = 10^{15} \text{ cm}^{-3}$$
or
$$\delta n(0) = g' \tau_{nO} = 10^{15} \text{ cm}^{-3}$$
For $x > 0$
$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \Rightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$
The solution is of the form
$$\delta n = A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{+x}{L_n}\right)$$
At $x = 0$,
$$\delta n = \delta n(0) = A + B$$
At $x = W$,
$$\delta n = 0 = A \exp\left(\frac{-W}{L_n}\right) + B \exp\left(\frac{+W}{L_n}\right)$$
Solving these two equations, we find
$$A = \frac{-\delta n(0) \exp(+2W/L_n)}{1 - \exp(+2W/L_n)}$$
and
$$B = \frac{\delta n(0)}{1 - \exp(+2W/L_n)}$$
Substituting into the general solution, we find
$$\delta n = \frac{\delta n(0)}{\left[\exp\left(\frac{+W}{L_n}\right) - \exp\left(\frac{-W}{L_n}\right)\right]}$$
$$\times \left\{\exp\left[\frac{+(W-x)}{L_n}\right] - \exp\left[\frac{-(W-x)}{L_n}\right]\right\}$$
which can be written as

$$\delta n = \frac{\delta n(0) \sinh\left[\frac{W-x}{L_n}\right]}{\sinh\left[\frac{W}{L_n}\right]}$$
where
$$\delta n(0) = 10^{15} \text{ cm}^{-3} \text{ and } L_n = 35.4 \mu \text{ m}$$
(b) If $\tau_{nO} = \infty$, we have
$$\frac{d^2(\delta n)}{dx^2} = 0$$
so the solution is of the form
$$\delta n = Cx + D$$
Applying the boundary conditions, we find
$$\delta n = \delta n(0)\left[1 - \frac{x}{W}\right]$$

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6.43
For $\tau_{pO} = \infty$, we have
$$\frac{d^2(\delta p)}{dx^2} = 0$$
So the solution is of the form
$$\delta p = Ax + B$$
At $x = W$
$$-D_p \frac{d(\delta p)}{dx}\bigg|_{x=W} = s(\delta p)\bigg|_{x=W}$$
or
$$-D_p A = s(AW + B)$$
which yields
$$B = \frac{-A}{s}(D_p + sW)$$
At $x = 0$, the flux of excess holes is
$$10^{19} = -D_p \frac{d(\delta p)}{dx}\bigg|_{x=0} = -D_p A$$
so that
$$A = \frac{-10^{19}}{10} = -10^{18} \text{ cm}^{-4}$$
and
$$B = \frac{10^{18}}{s}(10 + sW) = 10^{18}\left(\frac{10}{s} + W\right)$$
The solution is now
$$\delta p = 10^{18}\left(W - x + \frac{10}{s}\right)$$

_____

(a) For $s = \infty$,

$$\delta p = 10^{18}\left(20\times10^{-4} - x\right) \text{ cm}^{-3}$$

Then

$$J_p = -eD_p \frac{d(\delta p)}{dx}$$
$$= -\left(1.6\times10^{-19}\right)(10)\left(-10^{18}\right)$$

or

$$J_p = 1.6 \text{ A/cm}^2$$

(b) For $s = 2\times10^3 \text{ cm/s}$,

$$\delta p = 10^{18}\left(70\times10^{-4} - x\right) \text{ cm}^{-3}$$

Also

$$J_p = 1.6 \text{ A/cm}^2$$

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6.44

For $-W < x < 0$

$$D_n \frac{d^2(\delta n)}{dx^2} + G_o' = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G_o'}{D_n} x + C_1$$

and

$$\delta n = -\frac{G_o'}{2D_n} x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so that

$$\delta n = C_3 x + C_4$$

The boundary conditions are

(1) $s = 0$ at $x = -W$ so that

$$\left.\frac{d(\delta n)}{dx}\right|_{x=-W} = 0$$

(2) $s = \infty$ at $x = +W$ so that

$$\delta n(W) = 0$$

(3) δn continuous at $x = 0$

(4) $\dfrac{d(\delta n)}{dx}$ continuous at $x = 0$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G_o'W}{D_n} \text{ and } C_2 = C_4 = +\frac{G_o'W^2}{D_n}$$

Then for $-W < x < 0$

$$\delta n = \frac{G_o'}{2D_n}\left(-x^2 - 2Wx + 2W^2\right)$$

and for $0 < x < +W$

$$\delta n = \frac{G_o'W}{D_n}\left(W - x\right)$$

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6.45

Plot

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6.48

(a) GaAs:

$$R = \frac{V}{I} = \frac{2}{2\times10^{-6}} = 10^6 \ \Omega$$

$$R = \frac{L}{(\Delta\sigma)A} \text{ and } \Delta\sigma = e\left(\mu_n + \mu_p\right)\delta p$$

$$\delta p = g'\tau_{p0} = \left(10^{21}\right)\left(5\times10^{-8}\right) = 5\times10^{13} \text{ cm}^{-3}$$

For $N_d = 10^{16} \text{ cm}^{-3}$, from Figure 5.3,

$$\mu_n \cong 7000 \text{ cm}^2/\text{V-s}, \ \mu_p \cong 310 \text{ cm}^2/\text{V-s}$$

$$\Delta\sigma = \left(1.6\times10^{-19}\right)(7000 + 310)\left(5\times10^{13}\right)$$

$$= 0.05848 \ (\Omega\text{-cm})^{-1}$$

Let $W = 20 \ \mu\text{m}$

Then $A = Wd = \left(20\times10^{-4}\right)\left(4\times10^{-4}\right)$

$$= 80\times10^{-8} \text{ cm}^2$$

So $R = 10^6 = \dfrac{L}{(0.05848)\left(80\times10^{-8}\right)}$

Which yields $L = 4.68\times10^{-2} \text{ cm}$

(b) Silicon:

$$R = 10^6 \ \Omega, \ \delta p = 5\times10^{13} \text{ cm}^{-3}$$

For $N_d = 10^{16} \text{ cm}^{-3}$, from Figure 5.3,

$$\mu_n \cong 1300 \text{ cm}^2/\text{V-s}, \ \mu_p \cong 410 \text{ cm}^2/\text{V-s}$$

$$\Delta\sigma = \left(1.6\times10^{-19}\right)(1300 + 410)\left(5\times10^{13}\right)$$

$$= 0.01368 \ (\Omega\text{-cm})^{-1}$$

Let $W = 20 \ \mu\text{m}$

Then $A = Wd = \left(20\times10^{-4}\right)\left(4\times10^{-4}\right)$

$$= 80\times10^{-8} \text{ cm}^2$$

So $R = 10^6 = \dfrac{L}{(0.01368)\left(80\times10^{-8}\right)}$

Which yields $L = 1.09\times10^{-2} \text{ cm}$

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Chapter 7

7.1

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

(a)

$$\text{(i)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.611\,\text{V}$$

$$\text{(ii)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.671\,\text{V}$$

$$\text{(iii)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{17})}{(1.5\times10^{10})^2}\right]$$

$$= 0.731\,\text{V}$$

(b)

$$\text{(i)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.731\,\text{V}$$

$$\text{(ii)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.790\,\text{V}$$

$$\text{(iii)}\, V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{17})}{(1.5\times10^{10})^2}\right]$$

$$= 0.850\,\text{V}$$

_____

7.2

Si: $n_i = 1.5\times10^{10}\,\text{cm}^{-3}$

Ge: $n_i = 2.4\times10^{13}\,\text{cm}^{-3}$

GaAs: $n_i = 1.8\times10^{6}\,\text{cm}^{-3}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)\ \text{and}\ V_t = 0.0259\ \text{V}$$

(a) $N_d = 10^{14}\,\text{cm}^{-3}$, $N_a = 10^{17}\,\text{cm}^{-3}$,

Then   Si: $V_{bi} = 0.635\,\text{V}$

        Ge: $V_{bi} = 0.253\,\text{V}$

        GaAs: $V_{bi} = 1.10\,\text{V}$

(b) $N_d = 5\times10^{16}\,\text{cm}^{-3}$, $N_a = 5\times10^{16}\,\text{cm}^{-3}$

    Si: $V_{bi} = 0.778\,\text{V}$

    Ge: $V_{bi} = 0.396\,\text{V}$

   GaAs: $V_{bi} = 1.25\,\text{V}$

(c) $N_d = 10^{17}\,\text{cm}^{-3}$, $N_a = 10^{17}\,\text{cm}^{-3}$

    Si: $V_{bi} = 0.814\,\text{V}$

    Ge: $V_{bi} = 0.432\,\text{V}$

   GaAs: $V_{bi} = 1.28\,\text{V}$

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7.3

(a) Silicon ($T = 300\,\text{K}$)

$$V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$$

For $N_a = N_d = 10^{14}\,\text{cm}^{-3}$; $V_{bi} = 0.4561\,\text{V}$

$$= 10^{15}\quad;\quad = 0.5754\,\text{V}$$
$$= 10^{16}\quad;\quad = 0.6946\,\text{V}$$
$$= 10^{17}\quad;\quad = 0.8139\,\text{V}$$

(b) GaAs ($T = 300\,\text{K}$)

$$V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.8\times10^{6})^2}\right]$$

For $N_a = N_d = 10^{14}\,\text{cm}^{-3}$; $V_{bi} = 0.9237\,\text{V}$

$$= 10^{15}\quad;\quad = 1.043\,\text{V}$$
$$= 10^{16}\quad;\quad = 1.162\,\text{V}$$
$$= 10^{17}\quad;\quad = 1.282\,\text{V}$$

(c) Silicon (400 K), $kT = 0.034533$

$$n_i = 2.38\times10^{12}\,\text{cm}^{-3}$$

For $N_a = N_d = 10^{14}\,\text{cm}^{-3}$; $V_{bi} = 0.2582\,\text{V}$

$$= 10^{15}\quad;\quad = 0.4172\,\text{V}$$
$$= 10^{16}\quad;\quad = 0.5762\,\text{V}$$
$$= 10^{17}\quad;\quad = 0.7353\,\text{V}$$

GaAs(400 K), $n_i = 3.29\times10^{9}\,\text{cm}^{-3}$

For $N_a = N_d = 10^{14}\,\text{cm}^{-3}$; $V_{bi} = 0.7129\,\text{V}$

$$= 10^{15}\quad;\quad = 0.8719\,\text{V}$$
$$= 10^{16}\quad;\quad = 1.031\,\text{V}$$
$$= 10^{17}\quad;\quad = 1.190\,\text{V}$$

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7.4

  (a) n-side

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3294 \text{ eV}$$

p-side

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.4070 \text{ eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$V_{bi} = 0.7364 \text{ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259) \ln\left[\frac{\left(10^{17}\right)\left(5 \times 10^{15}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.7363 \text{ V}$$

(d)

$$x_n = \left[\frac{2 \in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.736)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{10^{17}}{5 \times 10^{15}}\right)\left(\frac{1}{10^{17} + 5 \times 10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 0.426 \times 10^{-4} \text{ cm} = 0.426 \, \mu\text{m}$$

Now

$$x_p = \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.736)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{10^{17}}\right)\left(\frac{1}{10^{17} + 5 \times 10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 0.0213 \times 10^{-4} \text{ cm} = 0.0213 \, \mu\text{m}$$

We have

$$\left|\text{E}_{max}\right| = \frac{e N_d x_n}{\in_s}$$

$$= \frac{\left(1.6 \times 10^{-19}\right)\left(5 \times 10^{15}\right)\left(0.426 \times 10^{-4}\right)}{(11.7)\left(8.85 \times 10^{-14}\right)}$$

or

$$\left|\text{E}_{max}\right| = 3.29 \times 10^4 \text{ V/cm}$$

_____

7.5

  (a) n-side

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3653 \text{ eV}$$

p-side

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.3653 \text{ eV}$$

(b)

$$V_{bi} = 0.3653 + 0.3653$$

or

$$V_{bi} = 0.7306 \text{ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259) \ln\left[\frac{\left(2 \times 10^{16}\right)\left(2 \times 10^{16}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.7305 \text{ V}$$

(d)

$$x_n = \left[\frac{2 \in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.7305)}{1.6\times10^{-19}} \right.$$

$$\left. \times \left(\frac{2\times10^{16}}{2\times10^{16}} \right) \left(\frac{1}{2\times10^{16}+2\times10^{16}} \right) \right]^{1/2}$$

or

$$x_n = 0.154\times10^{-4}\,\text{cm} = 0.154\,\mu\text{m}$$

By symmetry

$$x_p = 0.154\times10^{-4}\,\text{cm} = 0.154\,\mu\text{m}$$

Now

$$\left| \text{E}_{max} \right| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(2\times10^{16})(0.1537\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$$\left| \text{E}_{max} \right| = 4.75\times10^4\,\text{V/cm}$$

7.6

(b) $N_d = n_i \exp\left(\dfrac{E_F - E_{Fi}}{kT} \right)$

$$= (1.5\times10^{10}) \exp\left(\frac{0.365}{0.0259} \right)$$

or $\quad N_d = 1.98\times10^{16}\,\text{cm}^{-3}$

$$N_a = n_i \exp\left(\frac{E_{Fi} - E_F}{kT} \right)$$

$$= (1.5\times10^{10}) \exp\left(\frac{0.330}{0.0259} \right)$$

or $\quad N_a = 5.12\times10^{15}\,\text{cm}^{-3}$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{(5.12\times10^{15})(1.98\times10^{16})}{(1.5\times10^{10})^2} \right]$$

$$= 0.695\,\text{V}$$

7.7

200 K; $kT = 0.017267$; $n_i = 1.38\,\text{cm}^{-3}$

300 K; $kT = 0.0259$; $\quad n_i = 1.8\times10^6\,\text{cm}^{-3}$

400 K; $kT = 0.034533$; $n_i = 3.28\times10^9\,\text{cm}^{-3}$

For 200 K;

$$V_{bi} = (0.017267)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.38)^2} \right]$$

$$= 1.257\,\text{V}$$

For 300 K;

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.8\times10^6)^2} \right]$$

$$= 1.157\,\text{V}$$

For 400 K;

$$V_{bi} = (0.034533)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(3.28\times10^9)^2} \right]$$

$$= 1.023\,\text{V}$$

7.8

$$x_n = 0.25W = 0.25(x_n + x_p)$$

$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$

$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $\quad N_d = 3N_a$

(a) $\quad V_{bi} = (0.0259)\ln\left[\dfrac{N_a N_d}{(1.5\times10^{10})^2} \right]$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{(1.5\times10^{10})^2} \right]$$

or $\quad 3N_a^2 = (1.5\times10^{10})^2 \exp\left(\dfrac{0.710}{0.0259} \right)$

which yields $\quad N_a = 7.766\times10^{15}\,\text{cm}^{-3}$

$$N_d = 2.33\times10^{16}\,\text{cm}^{-3}$$

$$x_n = \left\{ \frac{2\in_s V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}} \right.$$

$$\left. \times \left(\frac{1}{3} \right) \left[\frac{1}{4(7.766\times10^{15})} \right] \right\}^{1/2}$$

$$\Rightarrow x_n = 9.93\times10^{-6}\,\text{cm}$$

or $\quad x_n = 0.0993\,\mu\text{m}$

$$x_p = \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}} \right.$$

$$\left. \times \left(\frac{3}{1} \right) \left[\frac{1}{4(7.766\times10^{15})} \right] \right\}^{1/2}$$

$$= 2.979\times10^{-5}\,\text{cm}$$

or $\quad x_p = 0.2979\,\mu\text{m}$

Now

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6 \times 10^{-19})(2.33 \times 10^{16})(0.0993 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

$$= 3.58 \times 10^4 \text{ V/cm}$$

(b) From part (a), we can write

$$3N_a^2 = (1.8 \times 10^6)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127 \times 10^{15} \text{ cm}^{-3}$

$$N_d = 2.438 \times 10^{16} \text{ cm}^{-3}$$

$$x_n = \left\{ \frac{2(13.1)(8.85 \times 10^{-14})(1.180)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{1}{3}\right)\left[\frac{1}{4(8.127 \times 10^{15})}\right] \right\}^{1/2}$$

$$= 1.324 \times 10^{-5} \text{ cm}$$

or $x_n = 0.1324 \, \mu\text{m}$

$$x_p = \left\{ \frac{2(13.1)(8.85 \times 10^{-14})(1.180)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{3}{1}\right)\left[\frac{1}{4(8.127 \times 10^{15})}\right] \right\}^{1/2}$$

$$= 3.973 \times 10^{-5} \text{ cm}$$

or $x_p = 0.3973 \, \mu\text{m}$

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6 \times 10^{-19})(2.438 \times 10^{16})(0.1324 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})}$$

$$= 4.45 \times 10^4 \text{ V/cm}$$

_____

7.9

(a) $V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2}\right]$

or

$V_{bi} = 0.635 \text{ V}$

(b)

$$x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right) \right\}^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.6350)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{10^{16}}{10^{15}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right) \right]^{1/2}$$

or

$x_n = 0.8644 \times 10^{-4} \text{ cm} = 0.8644 \, \mu\text{m}$

Now

$$x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right) \right\}^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.6350)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right) \right]^{1/2}$$

or

$x_p = 0.08644 \times 10^{-4} \text{ cm} = 0.08644 \, \mu\text{m}$

(c)

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6 \times 10^{-19})(10^{15})(0.8644 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{max}| = 1.34 \times 10^4 \text{ V/cm}$$

_____

7.10

(a) $V_{bi} = (0.0259)\ln\left[\frac{(2 \times 10^{17})(4 \times 10^{16})}{(1.5 \times 10^{10})^2}\right]$

$$= 0.80813 \text{ V}$$

(b) V_{bi} increases as temperature decreases

At $T = 300$ K, we can write

$$n_i^2 = (1.5 \times 10^{10})^2$$

$$= K(2.8 \times 10^{19})(1.04 \times 10^{19})\exp\left(\frac{-1.12}{0.0259}\right)$$

$$\Rightarrow K = 4.659$$

At $T = 287$ K, $kT = 0.024778$ eV

$$n_i^2 = K(2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{287}{300}\right)^3$$

$$\times \exp\left(\frac{-1.12}{0.024778}\right)$$

$$= (4.659)(2.5496 \times 10^{38})(2.3404 \times 10^{-20})$$

So $n_i^2 = 2.780 \times 10^{19}$

_____

Then

$$V_{bi} = (0.024778)\ln\left[\frac{\left(2\times10^{17}\right)\left(4\times10^{16}\right)}{2.780\times10^{19}}\right]$$

$$= 0.82494\,\text{V}$$

We find

$$\frac{V_{bi}(287)-V_{bi}(300)}{V_{bi}(300)}\times100\%$$

$$=\frac{0.82494-0.80813}{0.80813}\times100\% = 2.08\%$$

$$\cong 2\%$$

_____

7.11

$$V_{bi} = V_t\,\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$0.550 = (0.0259)\left(\frac{T}{300}\right)\ln\left[\frac{\left(4\times10^{16}\right)\left(2\times10^{15}\right)}{n_i^2}\right]$$

Using the procedure from Problem 7.10, we can write, for $T = 300\,\text{K}$,

$$n_i^2 = \left(1.5\times10^{10}\right)^2$$

$$= K\left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\exp\left(\frac{-1.12}{0.0259}\right)$$

$$\Rightarrow K = 4.659$$

At $T = 300\,\text{K}$,

$$V_{bi} = (0.0259)\ln\left[\frac{\left(4\times10^{16}\right)\left(2\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.68886\,\text{V}$$

For $V_{bi} = 0.550\,\text{V}$, $\Rightarrow T > 300\,\text{K}$

At $T = 380\,\text{K}$, $kT = 0.032807\,\text{eV}$

Also

$$n_i^2 = (4.659)\left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{380}{300}\right)^3$$

$$\times\exp\left(\frac{-1.12}{0.032807}\right)$$

$$= 4.112\times10^{24}$$

Then

$$V_{bi} = (0.032807)\ln\left[\frac{\left(4\times10^{16}\right)\left(2\times10^{15}\right)}{4.112\times10^{24}}\right]$$

$$= 0.5506\,\text{V} \cong 0.550\,\text{V}$$

_____

7.12

(b) For $N_d = 10^{16}\,\text{cm}^{-3}$,

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3473\,\text{eV}$$

For $N_d = 10^{15}\,\text{cm}^{-3}$

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.2877\,\text{eV}$$

Then

$$V_{bi} = 0.34732 - 0.28768$$

or

$$V_{bi} = 0.0596\,\text{V}$$

_____

7.13

(a) $V_{bi} = V_t\,\ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{\left(10^{12}\right)\left(10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.456\,\text{V}$$

(b)

$$x_n = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.456)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{12}}{10^{16}}\right)\left(\frac{1}{10^{12}+10^{16}}\right)\right]^{1/2}$$

or

$$x_n = 2.43\times10^{-7}\,\text{cm}$$

(c)

$$x_p = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.456)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{16}}{10^{12}}\right)\left(\frac{1}{10^{12}+10^{16}}\right)\right]^{1/2}$$

or

$$x_p = 2.43\times10^{-3}\,\text{cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6 \times 10^{-19})(10^{16})(2.43 \times 10^{-7})}{(11.7)(8.85 \times 10^{-14})}$$

or

$$|E_{max}| = 3.75 \times 10^2 \text{ V/cm}$$

_____

7.14

Assume silicon, so

$$L_D = \left(\frac{\epsilon_s kT}{e^2 N_d} \right)^{1/2}$$

$$= \left[\frac{(11.7)(8.85 \times 10^{-14})(0.0259)(1.6 \times 10^{-19})}{(1.6 \times 10^{-19})^2 N_d} \right]^{1/2}$$

or

$$L_D = \left(\frac{1.676 \times 10^5}{N_d} \right)^{1/2}$$

(a) $N_d = 8 \times 10^{14} \text{ cm}^{-3}$, $L_D = 0.1447 \, \mu m$

(b) $N_d = 2.2 \times 10^{16} \text{ cm}^{-3}$, $L_D = 0.02760 \, \mu m$

(c) $N_d = 8 \times 10^{17} \text{ cm}^{-3}$, $L_D = 0.004577 \, \mu m$

Now

(a) $V_{bi} = 0.7427$ V

(b) $V_{bi} = 0.8286$ V

(c) $V_{bi} = 0.9216$ V

Also

$$x_n = \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi})}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{8 \times 10^{17}}{N_d} \right)\left(\frac{1}{8 \times 10^{17} + N_d} \right) \right]^{1/2}$$

Then

(a) $x_n = 1.096 \, \mu m$

(b) $x_n = 0.2178 \, \mu m$

(c) $x_n = 0.02730 \, \mu m$

Now

(a) $\dfrac{L_D}{x_n} = 0.1320$

(b) $\dfrac{L_D}{x_n} = 0.1267$

(c) $\dfrac{L_D}{x_n} = 0.1677$

_____

7.15

$$|E_{max}| = \left\{ \frac{2eV_{bi}}{\epsilon_s}\left(\frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2}$$

We find

$$\frac{2e}{\epsilon_s} = \frac{2(1.6 \times 10^{-19})}{(11.7)(8.85 \times 10^{-14})} = 3.0904 \times 10^{-7}$$

(a)

(i) For $N_a = 10^{17}$, $N_d = 10^{14}$; $V_{bi} = 0.6350$ V

(ii) $\qquad\qquad = 10^{15}$; $\quad = 0.6946$ V

(iii) $\qquad\qquad = 10^{16}$; $\quad = 0.7543$ V

(iv) $\qquad\qquad = 10^{17}$; $\quad = 0.8139$ V

(i) For $N_a = 10^{17}$,

$\qquad N_d = 10^{14}$; $|E_{max}| = 0.443 \times 10^4$ V/cm

(ii) $\qquad = 10^{15}$; $\qquad = 1.46 \times 10^4$ V/cm

(iii) $\qquad = 10^{16}$; $\qquad = 4.60 \times 10^4$ V/cm

(iv) $\qquad = 10^{17}$; $\qquad = 11.2 \times 10^4$ V/cm

(b)

(i) For $N_a = 10^{14}$, $N_d = 10^{14}$; $V_{bi} = 0.4561$ V

(ii) $\qquad\qquad = 10^{15}$; $\quad = 0.5157$ V

(iii) $\qquad\qquad = 10^{16}$; $\quad = 0.5754$ V

(iv) $\qquad\qquad = 10^{17}$; $\quad = 0.6350$ V

(i) For $N_a = 10^{14}$,

$\qquad N_d = 10^{14}$; $|E_{max}| = 0.265 \times 10^4$ V/cm

(ii) $\qquad = 10^{15}$; $\qquad = 0.381 \times 10^4$ V/cm

(iii) $\qquad = 10^{16}$; $\qquad = 0.420 \times 10^4$ V/cm

(iv) $\qquad = 10^{17}$; $\qquad = 0.443 \times 10^4$ V/cm

(c) $|E_{max}|$ increases as the doping increases, and the electric field extends further into the low-doped side of the pn junction.

_____

7.16

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{(5 \times 10^{16})(10^{15})}{(1.5 \times 10^{10})^2} \right]$

$\qquad = 0.6767$ V

(b) $W = \left\{ \dfrac{2\epsilon_s (V_{bi} + V_R)}{e}\left(\dfrac{N_a + N_d}{N_a N_d} \right) \right\}^{1/2}$

_____

(i) For $V_R = 0$,

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.6767)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left[\frac{5 \times 10^{16} + 10^{15}}{(5 \times 10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 9.452 \times 10^{-5} \, \text{cm}$$

or $W = 0.9452 \, \mu\text{m}$

(ii) For $V_R = 5 \, \text{V}$,

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.6767 + 5)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left[\frac{5 \times 10^{16} + 10^{15}}{(5 \times 10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 2.738 \times 10^{-4} \, \text{cm}$$

or $W = 2.738 \, \mu\text{m}$

(c) $|E_{max}| = \dfrac{2(V_{bi} + V_R)}{W}$

(i) For $V_R = 0$,

$$|E_{max}| = \frac{2(0.6767)}{0.9452 \times 10^{-4}} = 1.43 \times 10^4 \, \text{V/cm}$$

(ii) For $V_R = 5 \, \text{V}$,

$$|E_{max}| = \frac{2(0.6767 + 5)}{2.738 \times 10^{-4}} = 4.15 \times 10^4 \, \text{V/cm}$$

_____

7.17

(a) $V_{bi} = (0.0259) \ln \left[\dfrac{(2 \times 10^{17})(4 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$

$$= 0.8081 \, \text{V}$$

(b)

$$x_n = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.8081 + 2.5)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{2 \times 10^{17}}{4 \times 10^{16}} \right) \left(\frac{1}{2 \times 10^{17} + 4 \times 10^{16}} \right) \right\}^{1/2}$$

$$= 0.2987 \times 10^{-4} \, \text{cm}$$

or $x_n = 0.2987 \, \mu\text{m}$

$$x_p = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.8081 + 2.5)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{4 \times 10^{16}}{2 \times 10^{17}} \right) \left(\frac{1}{2 \times 10^{17} + 4 \times 10^{16}} \right) \right\}^{1/2}$$

$$= 5.97 \times 10^{-6} \, \text{cm}$$

or $x_p = 0.0597 \, \mu\text{m}$

$$W = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.8081 + 2.5)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left[\frac{2 \times 10^{17} + 4 \times 10^{16}}{(2 \times 10^{17})(4 \times 10^{16})} \right] \right\}^{1/2}$$

$$= 0.3584 \times 10^{-4} \, \text{cm}$$

or $W = 0.3584 \, \mu\text{m}$

Also $W = x_n + x_p = 0.3584 \, \mu\text{m}$

(c) $|E_{max}| = \dfrac{2(V_{bi} + V_R)}{W} = \dfrac{2(0.8081 + 2.5)}{0.3584 \times 10^{-4}}$

$$= 1.85 \times 10^5 \, \text{V/cm}$$

(d) $C = A \left\{ \dfrac{e \in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$

$$= (2 \times 10^{-4}) \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(0.8081 + 2.5)} \right.$$

$$\left. \times \left[\frac{(2 \times 10^{17})(4 \times 10^{16})}{2 \times 10^{17} + 4 \times 10^{16}} \right] \right\}^{1/2}$$

$$= 5.78 \times 10^{-12} \, \text{F}$$

or $C = 5.78 \, \text{pF}$

_____

7.18

(a) $V_{bi} = V_t \ln \left(\dfrac{N_a N_d}{n_i^2} \right)$

$$= V_t \ln \left(\frac{80 N_d^2}{n_i^2} \right)$$

We find

$$80 N_d^2 = n_i^2 \exp \left(\frac{V_{bi}}{V_t} \right)$$

$$= (1.5 \times 10^{10})^2 \exp \left(\frac{0.740}{0.0259} \right)$$

$$= 5.762 \times 10^{32}$$

_____

$$\Rightarrow N_d = 2.684 \times 10^{15} \text{ cm}^{-3}$$

$$N_a = 2.147 \times 10^{17} \text{ cm}^{-3}$$

(b)

$$x_n = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.740 + 10)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{80}{1} \right) \left(\frac{1}{2.147 \times 10^{17} + 2.684 \times 10^{15}} \right) \right\}^{1/2}$$

$$= 2.262 \times 10^{-4} \text{ cm}$$

or $x_n = 2.262 \, \mu\text{m}$

$$x_p = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.740 + 10)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{1}{80} \right) \left(\frac{1}{2.147 \times 10^{17} + 2.684 \times 10^{15}} \right) \right\}^{1/2}$$

$$= 2.83 \times 10^{-6} \text{ cm}$$

or $x_p = 0.0283 \, \mu\text{m}$

(c) $|\text{E}_{max}| = \dfrac{2(V_{bi} + V_R)}{W}$

$$= \frac{2(0.740 + 10)}{(2.262 + 0.0283) \times 10^{-4}}$$

$$= 9.38 \times 10^4 \text{ V/cm}$$

(d) $C' = \left\{ \dfrac{e \in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$

$$= \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(0.740 + 10)} \right.$$

$$\left. \times \left[\frac{(2.147 \times 10^{17})(2.684 \times 10^{15})}{2.147 \times 10^{17} + 2.684 \times 10^{15}} \right] \right\}^{1/2}$$

$$C' = 4.52 \times 10^{-9} \text{ F/cm}^2$$

_____

7.19

(a) $V_{bi}(3N_a) - V_{bi}(N_a)$

$$= V_t \ln \left[\frac{N_d (3N_a)}{n_i^2} \right] - V_t \ln \left[\frac{N_d N_a}{n_i^2} \right]$$

$$= V_t \left\{ \ln(3) + \ln \left[\frac{N_d N_a}{n_i^2} \right] \right\} - V_t \ln \left[\frac{N_d N_a}{n_i^2} \right]$$

$$= V_t \ln(3) = (0.0259) \ln(3)$$

$$= 0.02845 \text{ V}$$

(b) $C' \cong \left\{ \dfrac{e \in_s N_a}{2(V_{bi} + V_R)} \right\}^{1/2}$

So $\dfrac{C'(3N_a)}{C'(N_a)} = \left\{ \dfrac{3N_a}{N_a} \right\}^{1/2} = \sqrt{3} = 1.732$

(c) For a larger doping, the space charge width narrows which results in a larger capacitance.

_____

7.20

(a) $V_{bi} = (0.0259) \ln \left[\dfrac{(4 \times 10^{15})(4 \times 10^{17})}{(1.5 \times 10^{10})^2} \right]$

or

$V_{bi} = 0.766 \text{ V}$

Now

$$|\text{E}_{max}| = \left[\frac{2e(V_{bi} + V_R)}{\in_s} \left(\frac{N_a N_d}{N_a + N_d} \right) \right]^{1/2}$$

or

$$(3 \times 10^5)^2 = \left[\frac{2(1.6 \times 10^{-19})(V_{bi} + V_R)}{(11.7)(8.85 \times 10^{-14})} \right.$$

$$\left. \times \frac{(4 \times 10^{15})(4 \times 10^{17})}{4 \times 10^{15} + 4 \times 10^{17}} \right]$$

or

$$9 \times 10^{10} = 1.224 \times 10^9 (V_{bi} + V_R)$$

so that

$(V_{bi} + V_R) = 73.53 \text{ V}$

which yields

$V_R = 72.8 \text{ V}$

(b) $V_{bi} = (0.0259) \ln \left[\dfrac{(4 \times 10^{16})(4 \times 10^{17})}{(1.5 \times 10^{10})^2} \right]$

or

$V_{bi} = 0.826 \text{ V}$

We have

$$(3 \times 10^5)^2 = \left[\frac{2(1.6 \times 10^{-19})(V_{bi} + V_R)}{(11.7)(8.85 \times 10^{-14})} \right.$$

$$\left. \times \frac{(4 \times 10^{16})(4 \times 10^{17})}{4 \times 10^{16} + 4 \times 10^{17}} \right]$$

so that

$(V_{bi} + V_R) = 8.008 \text{ V}$

which yields
$$V_R = 7.18 \text{ V}$$

(c) $V_{bi} = (0.0259)\ln\left[\dfrac{(4\times10^{17})(4\times10^{17})}{(1.5\times10^{10})^2}\right]$

or
$$V_{bi} = 0.886 \text{ V}$$

We have
$$(3\times10^5)^2 = \left[\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})}\right.$$
$$\left.\times\frac{(4\times10^{17})(4\times10^{17})}{4\times10^{17}+4\times10^{17}}\right]$$

so that
$$(V_{bi}+V_R) = 1.456 \text{ V}$$
which yields
$$V_R = 0.570 \text{ V}$$

_____

7.21

(a)
$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in_s(V_{biA}+V_R)}{e}\left(\dfrac{N_a+N_{dA}}{N_a N_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in_s(V_{biB}+V_R)}{e}\left(\dfrac{N_a+N_{dB}}{N_a N_{dB}}\right)\right]^{1/2}}$$

or
$$\frac{W(A)}{W(B)} = \left[\frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}\cdot\frac{(N_a+N_{dA})}{(N_a+N_{dB})}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find
$$V_{biA} = (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.7543 \text{ V}$$

$$V_{biB} = (0.0259)\ln\left[\frac{(10^{18})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.8139 \text{ V}$$

We find
$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right)\right.$$
$$\left.\times\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or
$$\frac{W(A)}{W(B)} = 3.13$$

(b)
$$\frac{\text{E}(A)}{\text{E}(B)} = \frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}} = \frac{W(B)}{W(A)}\cdot\frac{V_{biA}+V_R}{V_{biB}+V_R}$$

$$= \left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or
$$\frac{\text{E}(A)}{\text{E}(B)} = 0.316$$

(c)
$$\frac{C_j'(A)}{C_j'(B)} = \frac{\left[\dfrac{\in_s N_a N_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in_s N_a N_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or
$$\frac{C_j'(A)}{C_j'(B)} = 0.319$$

_____

7.22

(a) We have
$$\frac{C_j'(0)}{C_j'(10)} = \frac{\left[\dfrac{\in_s N_a N_d}{2(V_{bi})(N_a+N_d)}\right]^{1/2}}{\left[\dfrac{\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}}$$

or
$$\frac{C_j'(0)}{C_j'(10)} = 3.13 = \left(\frac{V_{bi}+V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10$ V, we find
$$(3.13)^2 V_{bi} = V_{bi} + 10$$
or
$$V_{bi} = 1.137 \text{ V}$$

(b)
$$x_p = 0.2W = 0.2(x_p+x_n)$$

_____

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

so

$$1.137 = (0.0259)\ln\left[\frac{0.25 N_a^2}{\left(1.8\times10^6\right)^2}\right]$$

We can then write

$$N_a = \frac{1.8\times10^6}{\sqrt{0.25}}\exp\left[\frac{1.137}{2(0.0259)}\right]$$

which yields

$$N_a = 1.23\times10^{16}\ \text{cm}^{-3}$$

and

$$N_d = 3.07\times10^{15}\ \text{cm}^{-3}$$

_____

7.23

$$V_{bi} = (0.0259)\ln\left[\frac{\left(2\times10^{16}\right)\left(5\times10^{15}\right)}{\left(1.8\times10^6\right)^2}\right]$$

$$= 1.162\ \text{V}$$

$$C' \propto \frac{1}{\sqrt{V_{bi}+V_R}}$$

So $\dfrac{C'(V_{R1})}{C'(V_{R2})} = \sqrt{\dfrac{V_{bi}+V_{R2}}{V_{bi}+V_{R1}}}$

$$1.50 = \sqrt{\frac{1.162+V_{R2}}{1.162+0.5}}$$

$$(1.50)^2 = \frac{1.162+V_{R2}}{1.662}$$

which yields $V_{R2} = 2.58\ \text{V}$

_____

7.24

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{\left(2\times10^{15}\right)\left(4\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$

$$= 0.6889\ \text{V}$$

$$C = AC' = A\left\{\frac{e \in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= \left(5\times10^{-4}\right)\left\{\frac{\left(1.6\times10^{-19}\right)\left(11.7\right)\left(8.85\times10^{-14}\right)}{2\left(0.6889+V_R\right)}\right.$$

$$\left.\times\frac{\left(2\times10^{15}\right)\left(4\times10^{16}\right)}{\left(2\times10^{15}+4\times10^{16}\right)}\right\}^{1/2}$$

$$C = \frac{6.2806\times10^{-12}}{\sqrt{0.6889+V_R}}$$

(i) For $V_R = 0$,

$$C = 7.567\ \text{pF}$$

(ii) For $V_R = 5\ \text{V}$,

$$C = 2.633\ \text{pF}$$

(b) $V_{bi} = (0.0259)\ln\left[\dfrac{\left(2\times10^{15}\right)\left(4\times10^{16}\right)}{\left(1.8\times10^6\right)^2}\right]$

$$= 1.157\ \text{V}$$

$$C = AC' = A\left\{\frac{e \in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= \left(5\times10^{-4}\right)\left\{\frac{\left(1.6\times10^{-19}\right)\left(13.1\right)\left(8.85\times10^{-14}\right)}{2\left(1.157+V_R\right)}\right.$$

$$\left.\times\frac{\left(2\times10^{15}\right)\left(4\times10^{16}\right)}{\left(2\times10^{15}+4\times10^{16}\right)}\right\}^{1/2}$$

$$C = \frac{6.6457\times10^{-12}}{\sqrt{1.157+V_R}}$$

(i) For $V_R = 0$,

$$C = 6.178\ \text{pF}$$

(ii) For $V_R = 5\ \text{V}$,

$$C = 2.678\ \text{pF}$$

_____

7.25

$$V_{bi} = (0.0259)\ln\left[\frac{\left(2\times10^{17}\right)\left(5\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.7543\ \text{V}$$

(a) $C = AC' = A\left\{\dfrac{e \in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$

$$= \left(8\times10^{-4}\right)\left\{\frac{\left(1.6\times10^{-19}\right)\left(11.7\right)\left(8.85\times10^{-14}\right)}{2\left(0.7543+10\right)}\right.$$

$$\left.\times\frac{\left(2\times10^{17}\right)\left(5\times10^{15}\right)}{\left(2\times10^{17}+5\times10^{15}\right)}\right\}^{1/2}$$

$$C = 4.904\times10^{-12}\ \text{F}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{C(2\pi f)^2}$$

_____

$$L = \frac{1}{\left(4.904 \times 10^{-12}\right)\left[2\pi\left(1.25 \times 10^6\right)\right]^2}$$

$$= 3.306 \times 10^{-3}\,\text{H} = 3.306\,\text{mH}$$

(b)

(i) For $V_R = 1\,\text{V}$, $C = 12.14\,\text{pF}$

$$f = \frac{1}{2\pi\left[\left(3.306 \times 10^{-3}\right)\left(12.14 \times 10^{-12}\right)\right]^{1/2}}$$

$$= 7.94 \times 10^5\,\text{Hz} = 0.794\,\text{MHz}$$

(ii) For $V_R = 5\,\text{V}$, $C = 6.704\,\text{pF}$

$$f = \frac{1}{2\pi\left[\left(3.306 \times 10^{-3}\right)\left(6.704 \times 10^{-12}\right)\right]^{1/2}}$$

$$= 1.069 \times 10^6\,\text{Hz} = 1.069\,\text{MHz}$$

_____

7.26

$$|\text{E}_{max}| \cong \left[\frac{2e(V_{bi} + V_R)N_d}{\epsilon_s}\right]^{1/2}$$

Let $V_{bi} \cong 0.75\,\text{V}$

(a) $\left(2.5 \times 10^5\right)^2$

$$= \left[\frac{2\left(1.6 \times 10^{-19}\right)(0.75 + 10)N_d}{(11.7)\left(8.85 \times 10^{-14}\right)}\right]$$

$$\Rightarrow N_d = 1.88 \times 10^{16}\,\text{cm}^{-3}$$

(b) $\left(10^5\right)^2$

$$= \left[\frac{2\left(1.6 \times 10^{-19}\right)(0.75 + 10)N_d}{(11.7)\left(8.85 \times 10^{-14}\right)}\right]$$

$$\Rightarrow N_d = 3.01 \times 10^{15}\,\text{cm}^{-3}$$

_____

7.27

$$x_p = (0.20)W = (0.20)(x_n + x_p)$$
$$(0.8)x_p = (0.2)x_n$$
$$x_n = 4x_p$$
$$N_a x_p = N_d x_n = N_d(4x_p)$$
$$\Rightarrow N_a = 4N_d$$

(a) $V_{bi} = V_t \ln\left[\dfrac{N_a N_d}{n_i^2}\right]$

$$= (0.0259)\ln\left[\frac{4N_d^2}{\left(1.8 \times 10^6\right)^2}\right]$$

$$C = AC' = A\left\{\frac{e \epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right\}^{1/2}$$

$$= \left(10^{-4}\right)\left\{\frac{\left(1.6 \times 10^{-19}\right)(13.1)\left(8.85 \times 10^{-14}\right)}{2(V_{bi} + 2)}\right.$$

$$\left. \times \frac{\left(4N_d^2\right)}{\left(5N_d\right)}\right\}^{1/2}$$

$$0.6 \times 10^{-12} = 2.724 \times 10^{-20}\sqrt{\frac{N_d}{V_{bi} + 2}}$$

By trial and error,

$$N_d = 1.504 \times 10^{15}\,\text{cm}^{-3},$$
$$N_a = 6.016 \times 10^{15}\,\text{cm}^{-3},$$
$$V_{bi} = 1.10\,\text{V}$$

(b) From part (a),

$$0.6 \times 10^{-12} = 2.724 \times 10^{-20}\sqrt{\frac{N_d}{V_{bi} + 5}}$$

By trial and error,

$$N_d = 2.976 \times 10^{15}\,\text{cm}^{-3},$$
$$N_a = 1.19 \times 10^{16}\,\text{cm}^{-3},$$
$$V_{bi} = 1.135\,\text{V}$$

_____

7.28

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{\left(5 \times 10^{15}\right)\left(10^{14}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$

or

$$V_{bi} = 0.5574\,\text{V}$$

(b)

$$x_p = \left[\frac{2 \epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.5574)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{10^{14}}{5 \times 10^{15}}\right)\left(\frac{1}{10^{14} + 5 \times 10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 5.32 \times 10^{-6}\,\text{cm}$$

Also

$$x_n = \left[\frac{2 \epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

_____

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.5574)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{10^{14}} \right) \left(\frac{1}{10^{14} + 5 \times 10^{15}} \right) \right]^{1/2}$$

or

$$x_n = 2.66 \times 10^{-4} \text{ cm}$$

(c) For $x_n = 30 \, \mu$m, we have

$$30 \times 10^{-4} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_R)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{10^{14}} \right) \left(\frac{1}{10^{14} + 5 \times 10^{15}} \right) \right]^{1/2}$$

which becomes

$$9 \times 10^{-6} = 1.269 \times 10^{-7} (V_{bi} + V_R)$$

We find

$$V_R = 70.4 \text{ V}$$

_____

7.29

An $n^+ p$ junction with $N_a = 10^{14} \text{ cm}^{-3}$,

(a) A one-sided junction and assume
$V_R \gg V_{bi}$. Then

$$x_p \cong \left[\frac{2 \in_s V_R}{e N_a} \right]^{1/2}$$

or

$$(50 \times 10^{-4})^2 = \frac{2(11.7)(8.85 \times 10^{-14}) V_R}{(1.6 \times 10^{-19})(10^{14})}$$

which yields

$$V_R = 193 \text{ V}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p \left(\frac{N_a}{N_d} \right)$$

so

$$x_n = (50 \times 10^{-4}) \left(\frac{10^{14}}{10^{16}} \right)$$

$$= 0.50 \times 10^{-4} \text{ cm} = 0.50 \, \mu\text{m}$$

(c)

$$|E_{max}| \cong \frac{2V_R}{W} = \frac{2(193.15)}{50.5 \times 10^{-4}}$$

or

$$|E_{max}| = 7.65 \times 10^4 \text{ V/cm}$$

_____

7.30

(a) $V_{bi} = (0.0259) \ln \left[\frac{(2 \times 10^{17})(2 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$

$$= 0.7305 \text{ V}$$

(b) $C = AC' \cong A \cdot \left[\frac{e \in_s N_d}{2(V_{bi} + V_R)} \right]^{1/2}$

$$= (10^{-5}) \left\{ \frac{(1.6 \times 10^{-19})}{2(V_{bi} + V_R)} \right.$$

$$\left. \times (11.7)(8.85 \times 10^{-14})(2 \times 10^{15}) \right\}^{1/2}$$

$$C = \frac{1.287 \times 10^{-13}}{\sqrt{V_{bi} + V_R}}$$

(i) For $V_R = 1$ V, $C = 9.783 \times 10^{-14}$ F

(ii) For $V_R = 3$ V, $C = 6.663 \times 10^{-14}$ F

(iii) For $V_R = 5$ V, $C = 5.376 \times 10^{-14}$ F

_____

7.31

(a) $V_{bi} = (0.0259) \ln \left[\frac{(8 \times 10^{16}) N_d}{(1.8 \times 10^6)^2} \right] = 1.20$

$$(8 \times 10^{16}) N_d = (1.8 \times 10^6)^2 \exp \left(\frac{1.20}{0.0259} \right)$$

$$\Rightarrow N_d = 5.36 \times 10^{15} \text{ cm}^{-3}$$

(b) $C = AC' = A \left\{ \frac{e \in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$

$$1.10 \times 10^{-12} = A \left\{ \frac{(1.6 \times 10^{-19})}{2(1.20 + 1.0)} \right.$$

$$\left. \times \frac{(13.1)(8.85 \times 10^{-14})(8 \times 10^{16})(5.36 \times 10^{15})}{(8 \times 10^{16} + 5.36 \times 10^{15})} \right\}^{1/2}$$

$$\Rightarrow A = 7.56 \times 10^{-5} \text{ cm}^2$$

(c) $0.80 \times 10^{-12} = (7.56 \times 10^{-5}) \left\{ \frac{(1.6 \times 10^{-19})}{2(V_{bi} + V_R)} \right.$

$$\left. \times \frac{(13.1)(8.85 \times 10^{-14})(8 \times 10^{16})(5.36 \times 10^{15})}{(8 \times 10^{16} + 5.36 \times 10^{15})} \right\}^{1/2}$$

$$1.0582 \times 10^{-8} = \frac{2.1585 \times 10^{-8}}{\sqrt{V_{bi} + V_R}}$$

$$\Rightarrow V_{bi} + V_R = 4.161 = 1.20 + V_R$$

$$V_R = 2.96 \text{ V}$$

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7.32

Plot

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7.33

(a) $V_{bi} = V_t \ln\left(\dfrac{N_{aO} N_{dO}}{n_i^2}\right)$

(c) p-region

$\dfrac{dE}{dx} = \dfrac{\rho(x)}{\epsilon_s} = -\dfrac{eN_{aO}}{\epsilon_s}$

or

$E = -\dfrac{eN_{aO} x}{\epsilon_s} + C_1$

We have

$E = 0$ at $x = -x_p \Rightarrow C_1 = -\dfrac{eN_{aO} x_p}{\epsilon_s}$

Then for $-x_p < x < 0$

$E = -\dfrac{eN_{aO}}{\epsilon_s}\left(x + x_p\right)$

n-region, $0 < x < x_O$

$\dfrac{dE_1}{dx} = \dfrac{\rho(x)}{\epsilon_s} = \dfrac{eN_{dO}}{2\epsilon_s}$

or

$E_1 = \dfrac{eN_{dO} x}{2\epsilon_s} + C_2$

n-region, $x_O < x < x_n$

$\dfrac{dE_2}{dx} = \dfrac{\rho(x)}{\epsilon_s} = \dfrac{eN_{dO}}{\epsilon_s}$

or

$E_2 = \dfrac{eN_{dO} x}{\epsilon_s} + C_3$

We have $E_2 = 0$ at $x = x_n$

$\Rightarrow C_3 = -\dfrac{eN_{dO} x_n}{\epsilon_s}$

so that for $x_O < x < x_n$, we have

$E_2 = -\dfrac{eN_{dO}}{\epsilon_s}\left(x_n - x\right)$

We also have $E_2 = E_1$ at $x = x_O$

Then

$\dfrac{eN_{dO} x_O}{2\epsilon_s} + C_2 = -\dfrac{eN_{dO}}{\epsilon_s}\left(x_n - x_O\right)$

which gives

$C_2 = -\dfrac{eN_{dO}}{\epsilon_s}\left(x_n - \dfrac{x_O}{2}\right)$

Then for $0 < x < x_O$ we have

$E_1 = \dfrac{eN_{dO} x}{2\epsilon_s} - \dfrac{eN_{dO}}{\epsilon_s}\left(x_n - \dfrac{x_O}{2}\right)$

_____

7.34

(a) $\dfrac{d^2\phi(x)}{dx^2} = -\dfrac{\rho(x)}{\epsilon_s} = -\dfrac{dE(x)}{dx}$

For $-2 < x < -1\,\mu$ m, $\rho(x) = +eN_d$

So

$\dfrac{dE}{dx} = \dfrac{eN_d}{\epsilon_s} \Rightarrow E = \dfrac{eN_d x}{\epsilon_s} + C_1$

At $x = -2\,\mu$ m $\equiv -x_O$, $E = 0$

So

$C_1 = \dfrac{eN_d x_O}{\epsilon_s}$

Then

$E = \dfrac{eN_d}{\epsilon_s}\left(x + x_O\right)$

At $x = 0$, $E(0) = E(x = -1)$, so

$E(0) = \dfrac{eN_d}{\epsilon_s}(-1 + 2)\times 10^{-4}$

$= \dfrac{(1.6\times10^{-19})(5\times10^{15})}{(11.7)(8.85\times10^{-14})}(1\times10^{-4})$

or

$E(0) = 7.726\times10^4$ V/cm

(c) Magnitude of potential difference is

$|\phi| = \int E dx = \dfrac{eN_d}{\epsilon_s}\int(x + x_O)dx$

$= \dfrac{eN_d}{\epsilon_s}\left(\dfrac{x^2}{2} + x_O \cdot x\right) + C_2$

Let $\phi = 0$ at $x = -x_O$, then

$0 = \dfrac{eN_d}{\epsilon_s}\left(\dfrac{x_O^2}{2} - x_O^2\right) + C_2 \Rightarrow C_2 = \dfrac{eN_d x_O^2}{2\epsilon_s}$

Then we can write

$|\phi| = \dfrac{eN_d}{2\epsilon_s}\left(x + x_O\right)^2$

At $x = -1\,\mu$ m

$|\phi_1| = \dfrac{(1.6\times10^{-19})(5\times10^{15})}{2(11.7)(8.85\times10^{-14})}\left[(-1 + 2)\times 10^{-4}\right]^2$

or

$|\phi_1| = 3.863$ V

Potential difference across the intrinsic region

$$|\phi_2| = E(0) \cdot d = (7.726 \times 10^4)(2 \times 10^{-4})$$

or

$$|\phi_2| = 15.45 \text{ V}$$

By symmetry, the potential difference across the p-region space-charge region is also 3.863 V. The total reverse-bias voltage is then

$$V_R = 2(3.863) + 15.45 = 23.2 \text{ V}$$

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7.35

(a)　$V_B = \dfrac{\in_s E_{crit}^2}{2eN_B}$

　or

$$N_B = \frac{\in_s E_{crit}^2}{2eV_B} = \frac{(11.7)(8.85 \times 10^{-14})(4 \times 10^5)^2}{2(1.6 \times 10^{-19})(40)}$$

　Then　$N_B = N_a = 1.294 \times 10^{16} \text{ cm}^{-3}$

(b)　$N_B = \dfrac{(11.7)(8.85 \times 10^{-14})(4 \times 10^5)^2}{2(1.6 \times 10^{-19})(20)}$

　Or　$N_B = N_a = 2.59 \times 10^{16} \text{ cm}^{-3}$

_____

7.36

$$N_a = \frac{\in_s E_{crit}^2}{2eV_B} = \frac{(11.7)(8.85 \times 10^{-14})(4 \times 10^5)^2}{2(1.6 \times 10^{-19})(80)}$$

$$= 6.47 \times 10^{15} \text{ cm}^{-3}$$

_____

7.37

(a)　For $N_d = 10^{16} \text{ cm}^{-3}$, from Figure 7.15,

　　$V_B \cong 75 \text{ V}$

(b)　For $N_d = 10^{15} \text{ cm}^{-3}$,

　　$V_B \cong 450 \text{ V}$

_____

7.38

(a)　From Equation (7.36),

$$|E_{max}| = \left\{ \frac{2e(V_{bi} + V_R)}{\in_s} \left(\frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2}$$

　Set $|E_{max}| = E_{crit}$ and $V_R = V_B$

$$V_{bi} = (0.0259) \ln \left[\frac{(2 \times 10^{16})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} \right]$$

$$= 0.7305 \text{ V}$$

Then

$$4 \times 10^5 = \left\{ \frac{2(1.6 \times 10^{-19})(V_{bi} + V_R)}{(11.7)(8.85 \times 10^{-14})} \right.$$

$$\left. \times \left[\frac{(2 \times 10^{16})(2 \times 10^{16})}{2 \times 10^{16} + 2 \times 10^{16}} \right] \right\}^{1/2}$$

$$\Rightarrow V_{bi} + V_B = 51.77 \text{ V}$$

So　$V_B = 51.04 \text{ V}$

(b)

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

$$= 0.6587 \text{ V}$$

Then

$$4 \times 10^5 = \left\{ \frac{2(1.6 \times 10^{-19})(V_{bi} + V_R)}{(11.7)(8.85 \times 10^{-14})} \right.$$

$$\left. \times \left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{5 \times 10^{15} + 5 \times 10^{15}} \right] \right\}^{1/2}$$

$$\Rightarrow V_{bi} + V_R = 207.1$$

So $V_R \cong 206 \text{ V}$

_____

7.39

For a silicon p^+n junction with $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ and $V_B \cong 100 \text{ V}$, then, neglecting V_{bi} we have

$$x_n \cong \left[\frac{2 \in_s V_B}{eN_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(100)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$$

or

$$x_n (\text{min}) = 5.09 \times 10^{-4} \text{ cm} = 5.09 \, \mu\text{m}$$

_____

7.40

We find

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2} \right] = 0.933 \text{ V}$$

Now

$$E_{max} = \frac{eN_d x_n}{\in_s}$$

so

_____

$$10^6 = \frac{(1.6 \times 10^{-19})(10^{18})x_n}{(11.7)(8.85 \times 10^{-14})}$$

which yields

$$x_n = 6.47 \times 10^{-6} \text{ cm}$$

Now

$$x_n = \left[\frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{1/2}$$

Then

$$(6.47 \times 10^{-6})^2 = \left[\frac{2(11.7)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}} \right.$$

$$\left. \times (V_{bi} + V_R) \left(\frac{10^{18}}{10^{18}} \right) \left(\frac{1}{10^{18} + 10^{18}} \right) \right]$$

which yields

$$V_{bi} + V_R = 6.468 \text{ V}$$

or

$$V_R = 5.54 \text{ V}$$

_____

7.41

Assume silicon: For an $n^+ p$ junction

$$x_p = \left[\frac{2 \in_s (V_{bi} + V_R)}{e N_a} \right]^{1/2}$$

Assume $V_{bi} << V_R$

(a) For $x_p = 75 \, \mu m$

$$(75 \times 10^{-4})^2 = \frac{2(11.7)(8.85 \times 10^{-14})V_R}{(1.6 \times 10^{-19})(10^{15})}$$

which yields

$$V_R = 4.35 \times 10^3 \text{ V}$$

(b) For $x_p = 150 \, \mu m$

$$(150 \times 10^{-4})^2 = \frac{2(11.7)(8.85 \times 10^{-14})V_R}{(1.6 \times 10^{-19})(10^{15})}$$

which yields

$$V_R = 1.74 \times 10^4 \text{ V}$$

Note: From Figure 7.15, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

_____

7.42

Impurity gradien

$$a = \frac{2 \times 10^{18}}{2 \times 10^{-4}} = 10^{22} \text{ cm}^{-4}$$

From Figure 7.15, $V_B \cong 15 \text{ V}$

_____

7.43

(a) For the linearly graded junction

$$\rho(x) = eax$$

Then

$$\frac{dE}{dx} = \frac{\rho(x)}{\in_s} = \frac{eax}{\in_s}$$

Now

$$E = \int \frac{eax}{\in_s} dx = \frac{ea}{\in_s} \cdot \frac{x^2}{2} + C_1$$

At $x = +x_O$ and $x = -x_O$, $E = 0$

So

$$0 = \frac{ea}{\in_s} \left(\frac{x_O^2}{2} \right) + C_1 \Rightarrow C_1 = -\frac{ea}{\in_s} \left(\frac{x_O^2}{2} \right)$$

Then

$$E = \frac{ea}{2 \in_s} (x^2 - x_O^2)$$

(b)

$$\phi(x) = -\int E dx = -\frac{ea}{2 \in_s} \left[\frac{x^3}{3} - x_O^2 \cdot x \right] + C_2$$

Set $\phi = 0$ at $x = -x_O$, then

$$0 = -\frac{ea}{2 \in_s} \left[\frac{-x_O^3}{3} + x_O^3 \right] + C_2 \Rightarrow C_2 = \frac{eax_O^3}{3 \in_s}$$

Then

$$\phi(x) = -\frac{ea}{2 \in_s} \left(\frac{x^3}{3} - x_O^2 \cdot x \right) + \frac{eax_O^3}{3 \in_s}$$

_____

7.44

We have that

$$C' = \left[\frac{ea \in_s^2}{12(V_{bi} + V_R)} \right]^{1/3}$$

Then

$$(7.2 \times 10^{-9})^3$$

$$= \left[\frac{a(1.6 \times 10^{-19})[(11.7)(8.85 \times 10^{-14})]^2}{12(0.7 + 3.5)} \right]$$

which yields

$$a = 1.1 \times 10^{20} \text{ cm}^{-4}$$

_____

_____

7.45

(a) $C_j = AC' \cong A \cdot \left[\dfrac{e \in_s N_a}{2(V_{bi} + V_R)} \right]^{1/2}$

Let $N_a = 5 \times 10^{15} \, \text{cm}^{-3} << N_d$

Then $V_{bi} = (0.0259) \ln \left[\dfrac{(3 \times 10^{17})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$

$\qquad = 0.7648 \, \text{V}$

Now

$\quad C_j = 0.45 \times 10^{-12} = A \left\{ \dfrac{(1.6 \times 10^{-19})}{2(0.7648 + 5)} \right.$

$\qquad \left. \times (11.7)(8.85 \times 10^{-14})(5 \times 10^{15}) \right\}^{1/2}$

$\quad 0.45 \times 10^{-12} = A(8.476 \times 10^{-9})$

$\Rightarrow A = 5.31 \times 10^{-5} \, \text{cm}^2$

(b) $C_j = (5.309 \times 10^{-5}) \left\{ \dfrac{(1.6 \times 10^{-19})}{2(V_{bi} + V_R)} \right.$

$\qquad \left. \times (11.7)(8.85 \times 10^{-14})(5 \times 10^{15}) \right\}^{1/2}$

$\quad C_j = \dfrac{1.0805 \times 10^{-12}}{\sqrt{V_{bi} + V_R}}$

(i) For $V_R = 2.5 \, \text{V}$, $C_j = 0.598 \, \text{pF}$

(ii) For $V_R = 0$, $\qquad C_j = 1.24 \, \text{pF}$

_____

Chapter 8

8.1

In forward bias

$$I_f \cong I_S \exp\left(\frac{eV}{kT}\right)$$

Then

$$\frac{I_{f1}}{I_{f2}} = \frac{I_S \exp\left(\frac{eV_1}{kT}\right)}{I_S \exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\left(\frac{e}{kT}\right)(V_1 - V_2)\right]$$

or

$$V_1 - V_2 = \left(\frac{kT}{e}\right)\ln\left(\frac{I_{f1}}{I_{f2}}\right)$$

(a)

For $\dfrac{I_{f1}}{I_{f2}} = 10$, then

$$V_1 - V_2 = (0.0259)\ln(10)$$

or

$$V_1 - V_2 = 59.6\,\text{mV} \cong 60\,\text{mV}$$

(b)

For $\dfrac{I_{f1}}{I_{f2}} = 100$, then

$$V_1 - V_2 = (0.0259)\ln(100)$$

or

$$V_1 - V_2 = 119.3\,\text{mV} \cong 120\,\text{mV}$$

_____

8.2

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{8\times10^{15}} = 2.8125\times10^4\,\text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{2\times10^{15}} = 1.125\times10^5\,\text{cm}^{-3}$$

$$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

$$n_p(-x_p) = n_{po}\exp\left(\frac{V_a}{V_t}\right)$$

(a) $V_a = 0.45\,\text{V}$,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.45}{0.0259}\right)$$

$$= 3.95\times10^{12}\,\text{cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.45}{0.0259}\right)$$

or

$$n_p(-x_p) = 9.88\times10^{11}\,\text{cm}^{-3}$$

(b) $V_a = 0.55\,\text{V}$,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 1.88\times10^{14}\,\text{cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 4.69\times10^{13}\,\text{cm}^{-3}$$

(c) $V_a = -0.55\,\text{V}$

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

_____

8.3

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.8\times10^6)^2}{4\times10^{16}} = 8.1\times10^{-5}\,\text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4}\,\text{cm}^{-3}$$

(a) $V_a = 0.90\,\text{V}$,

$$p_n(x_n) = (3.24\times10^{-4})\exp\left(\frac{0.90}{0.0259}\right)$$

$$= 4.0\times10^{11}\,\text{cm}^{-3}$$

$$n_p(-x_p) = (8.1\times10^{-5})\exp\left(\frac{0.90}{0.0259}\right)$$

$$= 10.0\times10^{10}\,\text{cm}^{-3}$$

(b) $V_a = 1.10\,\text{V}$

$$p_n(x_n) = (3.24\times10^{-4})\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 9.03\times10^{14}\,\text{cm}^{-3}$$

$$n_p(-x_p) = (8.1\times10^{-5})\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 2.26\times10^{14}\,\text{cm}^{-3}$$

(c) $p_n(x_n) \cong 0$
$n_p(-x_p) \cong 0$

_____

_____

8.4

(a) $n_{po} = \dfrac{n_i^2}{N_a} = \dfrac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}}$

$\qquad = 4.5\times10^3 \text{ cm}^{-3}$

$p_{no} = \dfrac{n_i^2}{N_d} = \dfrac{\left(1.5\times10^{10}\right)^2}{5\times10^{15}}$

$\qquad = 4.5\times10^4 \text{ cm}^{-3}$

(i) $p_n\left(x_n\right) = p_{no} \exp\left(\dfrac{V_a}{V_t}\right)$

or $V_a = V_t \ln\left[\dfrac{p_n\left(x_n\right)}{p_{no}}\right]$

$\qquad = (0.0259)\ln\left[\dfrac{(0.1)\left(5\times10^{15}\right)}{4.5\times10^4}\right]$

$\qquad = 0.599 \text{ V}$

(ii) n-region - lower doped side

(b) $n_{po} = \dfrac{n_i^2}{N_a} = \dfrac{\left(1.5\times10^{10}\right)^2}{7\times10^{15}}$

$\qquad = 3.214\times10^4 \text{ cm}^{-3}$

$p_{no} = \dfrac{n_i^2}{N_d} = \dfrac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}}$

$\qquad = 7.5\times10^3 \text{ cm}^{-3}$

(i) $V_a = V_t \ln\left[\dfrac{(0.1)N_a}{n_{po}}\right]$

$\qquad = (0.0259)\ln\left[\dfrac{(0.1)\left(7\times10^{15}\right)}{3.214\times10^4}\right]$

$\qquad = 0.6165 \text{ V}$

(ii) p-region - lower doped side

_____

8.5

(a) $J_n\left(-x_p\right) = \dfrac{eD_n n_{po}}{L_n} \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \dfrac{en_i^2}{N_a}\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \dfrac{\left(1.6\times10^{-19}\right)\left(1.8\times10^6\right)^2}{5\times10^{16}}\sqrt{\dfrac{205}{5\times10^{-8}}}$

$\qquad\qquad \times \exp\left(\dfrac{1.10}{0.0259}\right)$

$\quad = 1.849 \text{ A/cm}^2$

$I_n = AJ_n\left(-x_p\right) = \left(10^{-3}\right)(1.849)\text{ A}$

or $I_n = 1.85 \text{ mA}$

(b) $J_p\left(x_n\right) = \dfrac{eD_p p_{no}}{L_p} \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \dfrac{en_i^2}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}} \cdot \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \dfrac{\left(1.6\times10^{-19}\right)\left(1.8\times10^6\right)^2}{10^{16}}\sqrt{\dfrac{9.80}{10^{-8}}}$

$\qquad\qquad \times \exp\left(\dfrac{1.10}{0.0259}\right)$

$\quad = 4.521 \text{ A/cm}^2$

$I_p = AJ_p\left(x_n\right) = \left(10^{-3}\right)(4.521)\text{ A}$

or $I_p = 4.52 \text{ mA}$

(c) $I = I_n + I_p = 1.85 + 4.52 = 6.37 \text{ mA}$

_____

8.6

For an $n^+ p$ silicon diode

$I_S = Aen_i^2 \cdot \dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{nO}}}$

$\quad = \dfrac{\left(10^{-4}\right)\left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2}{10^{16}}\sqrt{\dfrac{25}{10^{-6}}}$

or

$I_S = 1.8\times10^{-15} \text{ A}$

(a) For $V_a = 0.5 \text{ V}$,

$I_D \cong I_S \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \left(1.8\times10^{-15}\right)\exp\left(\dfrac{0.5}{0.0259}\right)$

or

$I_D = 4.36\times10^{-7} \text{ A}$

(b) For $V_a = -0.5 \text{ V}$,

$I_D = \left(1.8\times10^{-15}\right)\left[\exp\left(\dfrac{-0.5}{0.0259}\right)-1\right]$

or

$I_D \cong -I_S = -1.8\times10^{-15} \text{ A}$

_____

_____

8.7

$$J_s = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{p0}}} \right]$$

$$= \left(1.6\times10^{-19}\right)\left(2.4\times10^{13}\right)^2$$

$$\times \left[\frac{1}{4\times10^{15}} \sqrt{\frac{90}{2\times10^{-6}}} + \frac{1}{2\times10^{17}} \sqrt{\frac{48}{2\times10^{-6}}} \right]$$

$$J_s = 1.568\times10^{-4}\,\text{A/cm}^2$$

(a) $I = AJ_s \exp\left(\dfrac{V_a}{V_t}\right)$

$$= \left(10^{-4}\right)\left(1.568\times10^{-4}\right)\exp\left(\frac{0.25}{0.0259}\right)$$

$$= 2.44\times10^{-4}\,\text{A}$$

or $I = 0.244\,\text{mA}$

(b) $I = -I_s = -AJ_s = -\left(10^{-4}\right)\left(1.568\times10^{-4}\right)$

$$= -1.568\times10^{-8}\,\text{A}$$

_____

8.8

(a) $J_s = en_i^2 \left[\dfrac{1}{N_a} \sqrt{\dfrac{D_n}{\tau_{no}}} + \dfrac{1}{N_d} \sqrt{\dfrac{D_p}{\tau_{p0}}} \right]$

$$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2$$

$$\times \left[\frac{1}{5\times10^{17}} \sqrt{\frac{25}{10^{-7}}} + \frac{1}{8\times10^{15}} \sqrt{\frac{10}{8\times10^{-8}}} \right]$$

$$J_s = 5.145\times10^{-11}\,\text{A/cm}^2$$

$$I_s = AJ_s = \left(2\times10^{-4}\right)\left(5.145\times10^{-11}\right)$$

$$= 1.029\times10^{-14}\,\text{A}$$

(b) $I = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

(i) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.45}{0.0259}\right)$

$$= 3.61\times10^{-7}\,\text{A}$$

(ii) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.55}{0.0259}\right)$

$$= 1.72\times10^{-5}\,\text{A}$$

(iii) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.65}{0.0259}\right)$

$$= 8.16\times10^{-4}\,\text{A}$$

_____

8.9

We have

$$I = I_S \left[\exp\left(\frac{V}{V_t}\right) - 1 \right]$$

or we can write this as

$$\frac{I}{I_S} + 1 = \exp\left(\frac{V}{V_t}\right)$$

so that

$$V = V_t \ln\left(\frac{I}{I_S} + 1\right)$$

In reverse bias, I is negative, so at

$\dfrac{I}{I_S} = -0.90$, we have

$$V = (0.0259)\ln(1 - 0.90)$$

or

$$V = -59.6\,\text{mV}$$

_____

8.10

Case 1: $I = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

$$0.50\times10^{-3} = I_s \exp\left(\frac{0.65}{0.0259}\right)$$

$$\Rightarrow I_s = 6.305\times10^{-15}\,\text{A} = 6.305\times10^{-12}\,\text{mA}$$

$$J_s = \frac{I_s}{A} = \frac{6.305\times10^{-12}}{2\times10^{-4}}$$

$$= 3.153\times10^{-8}\,\text{mA/cm}^2$$

Case 2: $I = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

$$= \left(2\times10^{-12}\right)\exp\left(\frac{0.70}{0.0259}\right)$$

or $I = 1.093\,\text{mA}$

$$J_s = \frac{I_s}{A} = \frac{2\times10^{-12}}{1\times10^{-3}}$$

$$= 2\times10^{-9}\,\text{mA/cm}^2$$

Case 3: $I = AJ_s \exp\left(\dfrac{V_a}{V_t}\right)$

So $V_a = V_t \ln\left[\dfrac{I}{AJ_s}\right]$

$$= (0.0259)\ln\left[\frac{0.80}{\left(10^{-4}\right)\left(10^{-7}\right)}\right]$$

$$V_a = 0.6502\,\text{V}$$

_____

Then
$$I_s = AJ_s = (10^{-4})(10^{-7}) = 10^{-11} \text{ mA}$$

Case 4: $I_s = \dfrac{I}{\exp\left(\dfrac{V_a}{V_t}\right)} = \dfrac{1.20}{\exp\left(\dfrac{0.72}{0.0259}\right)}$

$$I_s = 1.014 \times 10^{-12} \text{ mA}$$

$$A = \frac{I_s}{J_s} = \frac{1.014 \times 10^{-12}}{2 \times 10^{-8}}$$

$$= 5.07 \times 10^{-5} \text{ cm}^2$$

_____

8.11

(a) $\dfrac{J_n}{J_n + J_p} = \dfrac{\dfrac{eD_n n_{po}}{L_n}}{\dfrac{eD_n n_{po}}{L_n} + \dfrac{eD_p p_{no}}{L_p}}$

$$= \frac{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a} + \sqrt{\dfrac{D_p}{\tau_{po}}} \cdot \dfrac{n_i^2}{N_d}}$$

$$0.90 = \frac{1}{1 + \sqrt{\dfrac{D_p \tau_{no}}{D_n \tau_{po}} \cdot \left(\dfrac{N_a}{N_d}\right)}}$$

$$\sqrt{\frac{D_p \tau_{no}}{D_n \tau_{po}} \cdot \left(\frac{N_a}{N_d}\right)} = \frac{1}{0.90} - 1$$

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}\left(\frac{1}{0.90} - 1\right)}$$

$$= \sqrt{\frac{(25)(10^{-7})}{(10)(5 \times 10^{-7})}}(0.1111)$$

$$\frac{N_a}{N_d} = 0.07857 \text{ or } \frac{N_d}{N_a} = 12.73$$

(b) From part (a),

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}\left(\frac{1}{0.20} - 1\right)}$$

$$= \sqrt{\frac{(25)(10^{-7})}{(10)(5 \times 10^{-7})}}(4)$$

$$\frac{N_a}{N_d} = 2.828 \text{ or } \frac{N_d}{N_a} = 0.354$$

_____

8.12

The cross-sectional area is
$$A = \frac{I}{J} = \frac{10 \times 10^{-3}}{20} = 5 \times 10^{-4} \text{ cm}^2$$

We have
$$J \cong J_S \exp\left(\frac{V_D}{V_t}\right) \Rightarrow 20 = J_S \exp\left(\frac{0.65}{0.0259}\right)$$

which yields
$$J_S = 2.522 \times 10^{-10} \text{ A/cm}^2$$

We can write
$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

We want
$$\frac{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{nO}}}}{\dfrac{1}{N_a} \cdot \sqrt{\dfrac{D_n}{\tau_{nO}}} + \dfrac{1}{N_d} \cdot \sqrt{\dfrac{D_p}{\tau_{pO}}}} = 0.10$$

or
$$\frac{\dfrac{1}{N_a} \cdot \sqrt{\dfrac{25}{5 \times 10^{-7}}}}{\dfrac{1}{N_a}\sqrt{\dfrac{25}{5 \times 10^{-7}}} + \dfrac{1}{N_d} \cdot \sqrt{\dfrac{10}{5 \times 10^{-7}}}}$$

$$= \frac{7.071 \times 10^3}{7.071 \times 10^3 + \dfrac{N_a}{N_d}(4.472 \times 10^3)} = 0.10$$

which yields
$$\frac{N_a}{N_d} = 14.23$$

Now
$$J_S = 2.522 \times 10^{-10} = (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$

$$\times \left[\frac{1}{(14.23)N_d} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5 \times 10^{-7}}}\right]$$

We find
$$N_d = 7.09 \times 10^{14} \text{ cm}^{-3}$$

and
$$N_a = 1.01 \times 10^{16} \text{ cm}^{-3}$$

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8.13

Plot

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8.14

(a)

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{eD_n n_{po}}{L_n}}{\dfrac{eD_n n_{po}}{L_n} + \dfrac{eD_p p_{no}}{L_p}}$$

$$= \frac{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a} + \sqrt{\dfrac{D_p}{\tau_{po}}} \cdot \dfrac{n_i^2}{N_d}}$$

$$= \frac{1}{1 + \sqrt{\dfrac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n} = \frac{\mu_p}{\mu_n} = \frac{1}{2.4} \quad \text{and} \quad \frac{\tau_{no}}{\tau_{po}} = \frac{1}{0.1}$$

so

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + \sqrt{\dfrac{1}{2.4} \cdot \dfrac{1}{0.1}} \left(\dfrac{N_a}{N_d}\right)}$$

or

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + (2.04)\left(\dfrac{N_a}{N_d}\right)}$$

(b) Using Einstein's relation, we can write

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a} + \dfrac{e\mu_p}{L_p} \cdot \dfrac{n_i^2}{N_d}}$$

$$= \frac{e\mu_n N_d}{e\mu_n N_d + \dfrac{L_n}{L_p} \cdot e\mu_p N_a}$$

We have

$$\sigma_n = e\mu_n N_d \quad \text{and} \quad \sigma_p = e\mu_p N_a$$

Also

$$\frac{L_n}{L_p} = \sqrt{\frac{D_n \tau_{no}}{D_p \tau_{po}}} = \sqrt{\frac{2.4}{0.1}} = 4.90$$

Then

$$\frac{J_n}{J_n + J_p} = \frac{(\sigma_n / \sigma_p)}{(\sigma_n / \sigma_p) + 4.90}$$

_____

8.15

(a) p-side;

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.329 \, \text{eV}$$

Also on the n-side;

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.407 \, \text{eV}$$

(b) We can find

$$D_n = (1250)(0.0259) = 32.4 \, \text{cm}^2/\text{s}$$

$$D_p = (320)(0.0259) = 8.29 \, \text{cm}^2/\text{s}$$

Now

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$

$$\times \left[\frac{1}{5 \times 10^{15}}\sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}}\right]$$

or

$$J_S = 4.426 \times 10^{-11} \, \text{A/cm}^2$$

Then

$$I_S = AJ_S = (10^{-4})(4.426 \times 10^{-11})$$

or

$$I_S = 4.426 \times 10^{-15} \, \text{A}$$

We find

$$I = I_S \exp\left(\frac{V_D}{V_t}\right)$$

$$= (4.426 \times 10^{-15})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I = 1.07 \times 10^{-6} \, \text{A} = 1.07 \, \mu\text{A}$$

(c) The hole current is

$$I_p = en_i^2 A \cdot \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\exp\left(\frac{V_D}{V_t}\right)$$

_____

$$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2\left(10^{-4}\right)\left(\frac{1}{10^{17}}\right)$$

$$\times\sqrt{\frac{8.29}{10^{-7}}}\exp\left(\frac{V_D}{V_t}\right)$$

or

$$I_p = 3.278\times10^{-16}\exp\left(\frac{V_D}{V_t}\right)\ \ (A)$$

Then

$$\frac{I_p}{I} = \frac{J_p}{J_S} = \frac{3.278\times10^{-16}}{4.426\times10^{-15}} = 0.0741$$

_____

8.16

(a) $I_{sp} = A\left(\frac{eD_p\,p_{no}}{L_p}\right) = eA\sqrt{\frac{D_p}{\tau_{po}}}\cdot\frac{n_i^2}{N_d}$

$$= \left(1.6\times10^{-19}\right)\left(5\times10^{-4}\right)\sqrt{\frac{10}{8\times10^{-8}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}}$$

$$I_{sp} = 1.342\times10^{-14}\ A$$

(b) $I_{sn} = A\left(\frac{eD_n n_{po}}{L_n}\right) = eA\sqrt{\frac{D_n}{\tau_{no}}}\cdot\frac{n_i^2}{N_a}$

$$= \left(1.6\times10^{-19}\right)\left(5\times10^{-4}\right)\sqrt{\frac{25}{2\times10^{-7}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}}$$

$$I_{sn} = 4.025\times10^{-15}\ A$$

(c) $V_{bi} = \left(0.0259\right)\ln\left[\frac{\left(5\times10^{16}\right)\left(1.5\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$

$$= 0.746826\ V$$

$$V_a = \left(0.8\right)V_{bi} = \left(0.8\right)\left(0.746826\right) = 0.59746\ V$$

$$p_n\left(x_n\right) = p_{no}\exp\left(\frac{V_a}{V_t}\right) = \frac{n_i^2}{N_d}\exp\left(\frac{V_a}{V_t}\right)$$

$$= \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}}\exp\left(\frac{0.59746}{0.0259}\right)$$

$$= 1.56\times10^{14}\ cm^{-3}$$

(d) $I_n\left(-x_p\right) = I_n\left(x_n\right) = I_{sn}\exp\left(\frac{V_a}{V_t}\right)$

$$= \left(4.025\times10^{-15}\right)\exp\left(\frac{0.59746}{0.0259}\right)$$

$$= 4.1981\times10^{-5}\ A$$

(e) $I_p\left(x_n\right) = I_{sp}\exp\left(\frac{V_a}{V_t}\right)$

$$= \left(1.342\times10^{-14}\right)\exp\left(\frac{0.59746}{0.0259}\right)$$

$$= 1.3997\times10^{-4}\ A$$

$$I_{Total} = I_n + I_p$$

$$= 4.1981\times10^{-5} + 1.3997\times10^{-4}$$

$$= 1.820\times10^{-4}\ A$$

Now

$$I_p\left(x_n + \frac{1}{2}L_p\right) = I_p\left(x_n\right)\exp\left(\frac{-\left(1/2\right)L_p}{L_p}\right)$$

$$= \left(1.3997\times10^{-4}\right)\exp\left(\frac{-1}{2}\right)$$

$$= 8.4896\times10^{-5}\ A$$

Then

$$I_n\left(x_n + \frac{1}{2}L_p\right) = I_{Total} - I_p\left(x_n + \frac{1}{2}L_p\right)$$

$$= 1.820\times10^{-4} - 8.4896\times10^{-5}$$

$$= 9.710\times10^{-5}\ A$$

_____

8.17

(a) The excess hole concentration is given by

$$\delta p_n = p_n - p_{no}$$

$$= p_{no}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x}{L_p}\right)$$

We find

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4\ cm^{-3}$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{\left(8\right)\left(0.01\times10^{-6}\right)}$$

$$= 2.828\times10^{-4}\ cm = 2.828\ \mu m$$

Then

$$\delta p_n = \left(2.25\times10^4\right)\left[\exp\left(\frac{0.610}{0.0259}\right)-1\right]$$

$$\times\exp\left(\frac{-x}{2.828\times10^{-4}}\right)$$

or

$$\delta p_n = \left(3.81\times10^{14}\right)\exp\left(\frac{-x}{2.828\times10^{-4}}\right)\ cm^{-3}$$

(b) We have

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}$$

$$= \frac{eD_p (3.808 \times 10^{14})}{2.828 \times 10^{-4}} \exp\left(\frac{-x}{2.828 \times 10^{-4}}\right)$$

At $x = 3 \times 10^{-4}$ cm,

$$J_p(3) = \frac{(1.6 \times 10^{-19})(8)(3.808 \times 10^{14})}{2.828 \times 10^{-4}} \exp\left(\frac{-3}{2.828}\right)$$

or

$$J_p(3) = 0.5966 \text{ A/cm}^2$$

(c) We have

$$J_{no} = \frac{eD_n n_{po}}{L_n} \exp\left(\frac{V_a}{V_t}\right)$$

We can determine that

$$n_{po} = 4.5 \times 10^3 \text{ cm}^{-3} \text{ and } L_n = 10.72 \,\mu\text{m}$$

Then

$$J_{no} = \frac{(1.6 \times 10^{-19})(23)(4.5 \times 10^3)}{10.72 \times 10^{-4}}$$

$$\times \exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{no} = 0.2615 \text{ A/cm}^2$$

We can also find

$$J_{po} = 1.724 \text{ A/cm}^2$$

Then at $x = 3 \,\mu\text{m}$,

$$J_n(3) = J_{no} + J_{po} - J_p(3)$$
$$= 0.2615 + 1.724 - 0.5966$$

or

$$J_n(3) = 1.39 \text{ A/cm}^2$$

_____

8.18

(a) Problem 8.7

$$n_p = n_{po} \exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t \ln\left(\frac{n_p}{n_{po}}\right) = V_t \ln\left[\frac{(0.1)N_a}{n_i^2 / N_a}\right]$

$$= V_t \ln\left[\frac{(0.1)N_a^2}{n_i^2}\right]$$

$$= (0.0259) \ln\left[\frac{(0.1)(4 \times 10^{15})^2}{(2.4 \times 10^{13})^2}\right]$$

$$= 0.205 \text{ V}$$

(b) Problem 8.8

$$p_n = p_{no} \exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t \ln\left[\frac{p_n}{p_{no}}\right] = V_t \ln\left[\frac{(0.1)N_d}{n_i^2 / N_d}\right]$

$$= V_t \ln\left[\frac{(0.1)N_d^2}{n_i^2}\right]$$

$$= (0.0259) \ln\left[\frac{(0.1)(8 \times 10^{15})^2}{(1.5 \times 10^{10})^2}\right]$$

$$= 0.623 \text{ V}$$

_____

8.19

The excess electron concentration is given by

$$\delta n_p = n_p - n_{po}$$

$$= n_{po}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] \cdot \exp\left(\frac{-x}{L_n}\right)$$

The total number of excess electrons is

$$N_p = A \int_0^\infty \delta n_p \, dx$$

We may note that

$$\int_0^\infty \exp\left(\frac{-x}{L_n}\right) dx = -L_n \left. \exp\left(\frac{-x}{L_n}\right)\right|_0^\infty = L_n$$

Then

$$N_p = AL_n n_{po}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

We find that

$$D_n = 25 \text{ cm}^2/\text{s} \text{ and } L_n = 50.0 \,\mu\text{m}$$

Also

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{8 \times 10^{15}} = 2.81 \times 10^4 \text{ cm}^{-3}$$

Then

$$N_p = (10^{-3})(50.0 \times 10^{-4})(2.8125 \times 10^4)$$

$$\times \left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

or

$$N_p = (0.1406)\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

Then, we find the total number of excess electrons in the p-region to be:

_____

(a) $V_a = 0.3$ V, $N_p = 1.51 \times 10^4$

(b) $V_a = 0.4$ V, $N_p = 7.17 \times 10^5$

(c) $V_a = 0.5$ V, $N_p = 3.40 \times 10^7$

Similarly, the total number of excess holes in the n-region is found to be

$$P_n = AL_p p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

We find that

$D_p = 10.0\,\text{cm}^2/\text{s}$ and $L_p = 10.0\,\mu\text{m}$

Also

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{16}} = 2.25 \times 10^4\,\text{cm}^{-3}$$

Then

$$P_n = \left(2.25 \times 10^{-2}\right)\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

So

(a) $V_a = 0.3$ V, $P_n = 2.41 \times 10^3$

(b) $V_a = 0.4$ V, $P_n = 1.15 \times 10^5$

(c) $V_a = 0.5$ V, $P_n = 5.45 \times 10^6$

_____

8.20

$$I \propto n_i^2 \exp\left(\frac{V_a}{V_t}\right) \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_a}{kT}\right)$$

Then

$$I \propto \exp\left(\frac{eV_a - E_g}{kT}\right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp\left(\dfrac{eV_{a1} - E_{g1}}{kT}\right)}{\exp\left(\dfrac{eV_{a2} - E_{g2}}{kT}\right)}$$

or

$$\frac{I_1}{I_2} = \exp\left(\frac{eV_{a1} - eV_{a2} - E_{g1} + E_{g2}}{kT}\right)$$

We then have

$$\frac{10 \times 10^{-3}}{10 \times 10^{-6}} = \exp\left(\frac{0.255 - 0.32 - 0.525 + E_{g2}}{0.0259}\right)$$

or

$$10^3 = \exp\left(\frac{E_{g2} - 0.59}{0.0259}\right)$$

Then

$$E_{g2} = 0.59 + \left(0.0259\right)\ln\left(10^3\right)$$

or

$$E_{g2} = 0.769\,\text{eV}$$

_____

8.21

(a) We have

$$I_S = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

which can be written in the form

$$I_S = C'n_i^2$$

$$= C'N_{cO}N_{vO}\left(\frac{T}{300}\right)^3\exp\left(\frac{-E_g}{kT}\right)$$

or

$$I_S = CT^3\exp\left(\frac{-E_g}{kT}\right)$$

(b) Taking the ratio

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3\cdot\frac{\exp\left(\dfrac{-E_g}{kT_2}\right)}{\exp\left(\dfrac{-E_g}{kT_1}\right)}$$

$$= \left(\frac{T_2}{T_1}\right)^3\cdot\exp\left[+E_g\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

For $T_1 = 300$ K, $kT_1 = 0.0259$, $\dfrac{1}{kT_1} = 38.61$

For $T_2 = 400$ K, $kT_2 = 0.03453$, $\dfrac{1}{kT_2} = 28.96$

(i) Germanium: $E_g = 0.66\,\text{eV}$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3\exp\left[(0.66)(38.61 - 28.96)\right]$$

or $\dfrac{I_{S2}}{I_{S1}} = 1383$

(ii) Silicon: $E_g = 1.12\,\text{eV}$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3\exp\left[(1.12)(38.61 - 28.96)\right]$$

or $\dfrac{I_{S2}}{I_{S1}} = 1.17 \times 10^5$

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8.22

Plot

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8.23

First case:

$$\left| \frac{I_f}{I_s} \right| = \exp\left(\frac{V_a}{V_t} \right)$$

or $V_t = \dfrac{V_a}{\ln\left| \dfrac{I_f}{I_s} \right|} = \dfrac{0.50}{\ln\left(2 \times 10^4 \right)} = 0.05049 \text{ V}$

Now $0.05049 = (0.0259)\left(\dfrac{T}{300} \right)$

$\Rightarrow T = 584.8 \text{ K}$

Second case:

$$I_s = Aen_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \right]$$

$1.2 \times 10^{-6} = \left(5 \times 10^{-4} \right)\left(1.6 \times 10^{-19} \right)\left(n_i^2 \right)$

$\times \left[\dfrac{1}{4 \times 10^{15}} \sqrt{\dfrac{25}{5 \times 10^{-7}}} + \dfrac{1}{2 \times 10^{17}} \sqrt{\dfrac{10}{10^{-7}}} \right]$

or $n_i^2 = 8.2519 \times 10^{27}$

Now

$$n_i^2 = N_c N_\nu \exp\left(\frac{-E_g}{kT} \right)$$

$\left(8.2519 \times 10^{27} \right) = \left(2.8 \times 10^{19} \right)\left(1.04 \times 10^{19} \right)\left(\dfrac{T}{300} \right)^3$

$\times \exp\left[\dfrac{-1.12}{(0.0259)(T/300)} \right]$

$2.8337 \times 10^{-11} = \left(\dfrac{T}{300} \right)^3 \exp\left[\dfrac{-(1.12)(300)}{(0.0259)(T)} \right]$

By trial and error,

$T \cong 502 \text{ K}$

The reverse-bias current is limiting factor.

_____

8.24

$$L_p = \sqrt{D_p \tau_{po}} = \sqrt{(10)\left(10^{-7} \right)} = 10^{-3} \text{ cm}$$

or $L_p = 10 \, \mu\text{m}; \quad \Rightarrow W_n \ll L_p$

(a) $J_p(x_n) = \dfrac{eD_p p_{no}}{W_n} \exp\left(\dfrac{V_a}{V_t} \right)$

(i) $p_n(x_n) = (0.1)N_d = p_{no} \exp\left(\dfrac{V_a}{V_t} \right)$

$$= \left(\frac{n_i^2}{N_d} \right) \exp\left(\frac{V_a}{V_t} \right)$$

or $V_a = V_t \ln\left[\dfrac{(0.1)N_d^2}{n_i^2} \right]$

$= (0.0259)\ln\left[\dfrac{(0.1)\left(2 \times 10^{15} \right)^2}{\left(1.5 \times 10^{10} \right)^2} \right]$

$V_a = 0.5516 \text{ V}$

(ii) $I_p = \dfrac{AeD_p}{W_n} \left(\dfrac{n_i^2}{N_d} \right) \exp\left(\dfrac{V_a}{V_t} \right)$

$= \dfrac{\left(10^{-3} \right)\left(1.6 \times 10^{-19} \right)(10)\left(1.5 \times 10^{10} \right)^2}{\left(0.7 \times 10^{-4} \right)\left(2 \times 10^{15} \right)}$

$\times \exp\left(\dfrac{0.5516}{0.0259} \right)$

$I_p = 4.565 \times 10^{-3} \text{ A}$

$I_n = \dfrac{AeD_n n_{po}}{L_n} \exp\left(\dfrac{V_a}{V_t} \right)$

$= Ae \sqrt{\dfrac{D_n}{\tau_{no}}} \left(\dfrac{n_i^2}{N_a} \right) \exp\left(\dfrac{V_a}{V_t} \right)$

$= \left(10^{-3} \right)\left(1.6 \times 10^{-19} \right)\sqrt{\dfrac{25}{5 \times 10^{-7}}} \cdot \dfrac{\left(1.5 \times 10^{10} \right)^2}{2 \times 10^{17}}$

$\times \exp\left(\dfrac{0.5516}{0.0259} \right)$

$I_n = 2.26 \times 10^{-6} \text{ A}$

$I = I_n + I_p$

$= 2.26 \times 10^{-6} + 4.565 \times 10^{-3}$

$= 4.567 \times 10^{-3} \text{ A}$

or $I = 4.567 \text{ mA}$

(b) (i) $n_p(-x_p) = (0.1)N_a = n_{po} \exp\left(\dfrac{V_a}{V_t} \right)$

$$= \left(\frac{n_i^2}{N_a} \right) \exp\left(\frac{V_a}{V_t} \right)$$

or $V_a = V_t \ln\left[\dfrac{(0.1)N_a^2}{n_i^2} \right]$

$= (0.0259)\ln\left[\dfrac{(0.1)\left(2 \times 10^{15} \right)^2}{\left(1.5 \times 10^{10} \right)^2} \right]$

$V_a = 0.5516 \text{ V}$

_____

(ii) $I_p = \dfrac{AeD_p}{W_n}\left(\dfrac{n_i^2}{N_d}\right)\exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \dfrac{\left(10^{-3}\right)\left(1.6\times10^{-19}\right)\left(10\right)\left(1.5\times10^{10}\right)^2}{\left(0.7\times10^{-4}\right)\left(2\times10^{17}\right)}$

$\qquad\qquad\qquad\qquad \exp\left(\dfrac{0.5516}{0.0259}\right)$

$\quad I_p = 4.565\times10^{-5}\,\mathrm{A}$

$\quad I_n = Ae\sqrt{\dfrac{D_n}{\tau_{no}}}\left(\dfrac{n_i^2}{N_a}\right)\exp\left(\dfrac{V_a}{V_t}\right)$

$\quad = \left(10^{-3}\right)\left(1.6\times10^{-19}\right)\sqrt{\dfrac{25}{5\times10^{-7}}}\cdot\dfrac{\left(1.5\times10^{10}\right)^2}{2\times10^{15}}$

$\qquad\qquad\qquad\qquad \times\exp\left(\dfrac{0.5516}{0.0259}\right)$

$\quad I_n = 2.2597\times10^{-4}\,\mathrm{A}$

$\quad I = I_n + I_p$

$\qquad = 2.2597\times10^{-4} + 4.565\times10^{-5}$

$\qquad = 2.716\times10^{-4}\,\mathrm{A}$

or $\quad I = 0.2716\,\mathrm{mA}$

_____

8.25

(a) We can write for the n-region

$\dfrac{d^2(\delta p_n)}{dx^2} - \dfrac{\delta p_n}{L_p^2} = 0$

The general solution is of the form

$\delta p_n = A\exp\left(\dfrac{+x}{L_p}\right) + B\exp\left(\dfrac{-x}{L_p}\right)$

The boundary condition at $x = x_n$ gives

$\delta p_n(x_n) = p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]$

$\qquad = A\exp\left(\dfrac{+x_n}{L_p}\right) + B\exp\left(\dfrac{-x_n}{L_p}\right)$

and the boundary condition at $x = x_n + W_n$
gives

$\delta p_n(x_n + W_n) = 0$

$\qquad = A\exp\left(\dfrac{x_n + W_n}{L_p}\right) + B\exp\left(\dfrac{-(x_n + W_n)}{L_p}\right)$

From this equation, we have

$A = -B\exp\left[\dfrac{-2(x_n + W_n)}{L_p}\right]$

Then, from the first boundary condition, we obtain

$p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]$

$\quad = -B\exp\left[\dfrac{-(x_n + 2W_n)}{L_p}\right] + B\exp\left(\dfrac{-x_n}{L_p}\right)$

$\quad = B\exp\left(\dfrac{-x_n}{L_p}\right)\left[1 - \exp\left(\dfrac{-2W_n}{L_p}\right)\right]$

We then obtain

$B = \dfrac{p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]}{\exp\left(\dfrac{-x_n}{L_p}\right)\left[1 - \exp\left(\dfrac{-2W_n}{L_p}\right)\right]}$

which can be written as

$B = \dfrac{p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]\cdot\exp\left[\dfrac{x_n + W_n}{L_p}\right]}{\exp\left(\dfrac{W_n}{L_p}\right) - \exp\left(\dfrac{-W_n}{L_p}\right)}$

We can also find

$A = \dfrac{-p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]\cdot\exp\left[\dfrac{-(x_n + W_n)}{L_p}\right]}{\exp\left(\dfrac{W_n}{L_p}\right) - \exp\left(\dfrac{-W_n}{L_p}\right)}$

The solution can now be written as

$\delta p_n = \dfrac{p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]}{2\sinh\left(\dfrac{W_n}{L_p}\right)}$

$\times\left\{\exp\left[\dfrac{x_n + W_n - x}{L_p}\right] - \exp\left[\dfrac{-(x_n + W_n - x)}{L_p}\right]\right\}$

or finally

$\delta p_n = p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]\cdot\dfrac{\sinh\left(\dfrac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\dfrac{W_n}{L_p}\right)}$

_____

(b)

$$J_p = -eD_p \left. \frac{d(\delta p_n)}{dx} \right|_{x=x_n}$$

$$= \frac{-eD_p p_{no}\left[\exp\left(\dfrac{V_a}{V_t}\right)-1\right]}{\sinh\left(\dfrac{W_n}{L_p}\right)}$$

$$\times \left.\left(\frac{-1}{L_p}\right)\cosh\left(\frac{x_n+W_n-x}{L_p}\right)\right|_{x=x_n}$$

Then

$$J_p = \frac{eD_p p_{no}}{L_p}\coth\left(\frac{W_n}{L_p}\right)\cdot\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$$

_____

8.26

$$I_D \propto n_i^2 \exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \le T \le 320\,\text{K}$, neglect the change in N_c and N_υ.
Then

$$I_D \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_D}{kT}\right)$$

$$\propto \exp\left[\frac{-(E_g-eV_D)}{kT}\right]$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\dfrac{-(E_g-eV_{D1})}{kT_1}\right]}{\exp\left[\dfrac{-(E_g-eV_{D2})}{kT_2}\right]}$$

We then have

$$\frac{E_g-eV_{D1}}{kT_1} = \frac{E_g-eV_{D2}}{kT_2}$$

We have
$T = 300\,\text{K}$, $V_{D1} = 0.60\,\text{V}$ and

$$kT_1 = 0.0259\,\text{eV}, \quad \frac{kT_1}{e} = 0.0259\,\text{V}$$

$T = 310\,\text{K}$,

$$kT_2 = 0.02676\,\text{eV}, \quad \frac{kT_2}{e} = 0.02676\,\text{V}$$

$T = 320\,\text{K}$,

$$kT_3 = 0.02763\,\text{eV}, \quad \frac{kT_3}{e} = 0.02763\,\text{V}$$

For $T = 310\,\text{K}$,

$$\frac{1.12-0.60}{0.0259} = \frac{1.12-V_{D2}}{0.02676}$$

which yields
$$V_{D2} = 0.5827\,\text{V}$$
For $T = 320\,\text{K}$,

$$\frac{1.12-0.60}{0.0259} = \frac{1.12-V_{D3}}{0.02763}$$

which yields
$$V_{D3} = 0.5653\,\text{V}$$

_____

8.27

(a) We can write

$$I_D = C\cdot n_i^2 \cdot\exp\left(\frac{eV_a}{kT}\right)$$

where C is a constant, independent of temperature.
As a first approximation, neglect the variation of N_c and N_υ with temperature over the range of interest. We can then write

$$I_D = C_1 \cdot\exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{V_a}{V_t}\right)$$

$$= C_1 \cdot\exp\left[\frac{-(E_g-eV_a)}{kT}\right]$$

where C_1 is another constant, independent of temperature. We find

$$\frac{E_g-eV_a}{kT} = \ln\left(\frac{C_1}{I_D}\right)$$

or

$$V_a = E_g - \left(\frac{kT}{e}\right)\cdot\ln\left(\frac{C_1}{I_D}\right)$$

_____

8.28

(a) $\displaystyle I_s = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$

$$= (10^{-4})(1.6\times10^{-19})(1.5\times10^{10})^2$$

$$\times\left[\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}}\right]$$

$$I_s = 2.323\times10^{-15}\,\text{A}$$

(b) $\displaystyle I_{gen} = \frac{Aen_iW}{2\tau_0}$

We find

_____

$$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.7665 \text{ V}$$

and

$$W = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7665+5)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left[\frac{4\times10^{16}+4\times10^{16}}{(4\times10^{16})(4\times10^{16})}\right]\right\}^{1/2}$$

$$W = 6.109\times10^{-5} \text{ cm}$$

Then

$$I_{gen} = \frac{(10^{-4})(1.6\times10^{-19})(1.5\times10^{10})(6.109\times10^{-5})}{2(10^{-7})}$$

$$= 7.331\times10^{-11} \text{ A}$$

(c) $\dfrac{I_{gen}}{I_s} = \dfrac{7.331\times10^{-11}}{2.323\times10^{-15}} = 3.16\times10^4$

_____

8.29

(a)   Set $I_S = I_{gen}$,

$$Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] = \frac{Aen_iW}{2\tau_0}$$

$$n_i\left[\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}}+\frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}}\right]$$

$$= \frac{W}{2\tau_0} = \frac{6.109\times10^{-5}}{2(10^{-7})}$$

so $n_i = \dfrac{3.0545\times10^2}{3.9528\times10^{-13}+2.50\times10^{-13}}$

$$= 4.734\times10^{14} \text{ cm}^{-3}$$

Then

$$n_i^2 = 2.2407\times10^{29}$$

$$= (2.8\times10^{19})(1.04\times10^{19})\left(\frac{T}{300}\right)^3$$

$$7.6947\times10^{-10} = \left(\frac{T}{300}\right)^3\exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error,

$$T \cong 567 \text{ K}$$

We have

$$I_s = I_{gen} = \frac{Aen_iW}{2\tau_0}$$

$$= \frac{(10^{-4})(1.6\times10^{-19})(4.734\times10^{14})(6.109\times10^{-5})}{2(10^{-7})}$$

Then

$$I_s + I_{gen} = 2.314\times10^{-6} \text{ A}$$

or $I_s = I_{gen} = 2.314\,\mu\text{A}$

(b)   From Problem 8.28

$$I_s = 2.323\times10^{-15} \text{ A}$$

$$I_{gen} = 7.331\times10^{-11} \text{ A}$$

So $I = I_s\exp\left(\dfrac{V_a}{V_t}\right) = I_{gen}\exp\left(\dfrac{V_a}{2V_t}\right)$

$$(2.323\times10^{-15})\exp\left(\frac{V_a}{V_t}\right)$$

$$= (7.331\times10^{-11})\exp\left(\frac{V_a}{2V_t}\right)$$

$$\frac{\exp\left(\dfrac{V_a}{V_t}\right)}{\exp\left(\dfrac{V_a}{2V_t}\right)} = \frac{7.331\times10^{-11}}{2.323\times10^{-15}}$$

$$\exp\left(\frac{V_a}{2V_t}\right) = 3.1558\times10^4$$

$$V_a = 2V_t\ln(3.1558\times10^4)$$

$$= 0.5366 \text{ V}$$

_____

8.30

$$D_n = \left(\frac{kT}{e}\right)\cdot\mu_n = (0.0259)(5500)$$

$$= 142.5 \text{ cm}^2/\text{s}$$

$$D_p = (0.0259)(220) = 5.70 \text{ cm}^2/\text{s}$$

(a)

(i) $I_s = Aen_i^2\left[\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}+\dfrac{1}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}\right]$

$$= (2\times10^{-4})(1.6\times10^{-19})(1.8\times10^6)^2$$

$$\times\left[\frac{1}{7\times10^{16}}\sqrt{\frac{142.5}{2\times10^{-8}}}+\frac{1}{7\times10^{16}}\sqrt{\frac{5.70}{2\times10^{-8}}}\right]$$

$$I_s = 1.50\times10^{-22} \text{ A}$$

_____

(ii) $I_D = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

$\qquad = \left(1.50 \times 10^{-22}\right) \exp\left(\dfrac{0.6}{0.0259}\right)$

$\qquad = 1.726 \times 10^{-12}$ A

(iii) $I_D = \left(1.50 \times 10^{-22}\right) \exp\left(\dfrac{0.8}{0.0259}\right)$

$\qquad = 3.896 \times 10^{-9}$ A

(iv) $I_D = \left(1.50 \times 10^{-22}\right) \exp\left(\dfrac{1.0}{0.0259}\right)$

$\qquad = 8.795 \times 10^{-6}$ A

(b) $I_{gen} = \dfrac{A e n_i W}{2\tau_0}$

$\qquad V_{bi} = (0.0259) \ln\left[\dfrac{\left(7 \times 10^{16}\right)\left(7 \times 10^{16}\right)}{\left(1.8 \times 10^6\right)^2}\right]$

$\qquad = 1.263$ V

$\qquad W = \left\{\dfrac{2(13.1)\left(8.85 \times 10^{-14}\right)(1.263 + 3)}{1.6 \times 10^{-19}}\right.$

$\qquad\qquad \left. \times \left[\dfrac{7 \times 10^{16} + 7 \times 10^{16}}{\left(7 \times 10^{16}\right)\left(7 \times 10^{16}\right)}\right]\right\}^{1/2}$

$\qquad = 4.201 \times 10^{-5}$ cm

(i) Then

$I_{gen} = \dfrac{\left(2 \times 10^{-4}\right)\left(1.6 \times 10^{-19}\right)\left(1.8 \times 10^6\right)\left(4.201 \times 10^{-5}\right)}{2\left(2 \times 10^{-8}\right)}$

$\qquad = 6.049 \times 10^{-14}$ A

(ii) $I_{rec} = I_{ro} \exp\left(\dfrac{V_a}{2V_t}\right)$

$\qquad = \left(6 \times 10^{-14}\right) \exp\left(\dfrac{0.6}{2(0.0259)}\right)$

$\qquad = 6.436 \times 10^{-9}$ A

(iii) $I_{rec} = \left(6 \times 10^{-14}\right) \exp\left(\dfrac{0.8}{2(0.0259)}\right)$

$\qquad = 3.058 \times 10^{-7}$ A

(iv) $I_{rec} = \left(6 \times 10^{-14}\right) \exp\left(\dfrac{1.0}{2(0.0259)}\right)$

$\qquad = 1.453 \times 10^{-5}$ A

_____

8.31

Using results from Problem 8.30, we find

$V_a = 0.4$ V, $\quad I_d = 7.64 \times 10^{-16}$ A,

$\qquad I_{rec} = 1.35 \times 10^{-10}$ A, $\quad I_T \cong 1.35 \times 10^{-10}$ A

$V_a = 0.6$ V, $\quad I_d = 1.73 \times 10^{-12}$ A

$\qquad I_{rec} = 6.44 \times 10^{-9}$ A, $\quad I_T \cong 6.44 \times 10^{-9}$ A

$V_a = 0.8$ V, $\quad I_d = 3.90 \times 10^{-9}$ A

$\qquad I_{rec} = 3.06 \times 10^{-7}$ A, $\quad I_T = 3.10 \times 10^{-7}$ A

$V_a = 1.0$ V, $\quad I_d = 8.80 \times 10^{-6}$ A

$\qquad I_{rec} = 1.45 \times 10^{-5}$ A, $\quad I_T = 2.33 \times 10^{-5}$ A

$V_a = 1.2$ V. $\quad I_d = 1.99 \times 10^{-2}$ A

$\qquad I_{rec} = 6.90 \times 10^{-4}$ A, $\quad I_T = 2.06 \times 10^{-2}$ A

_____

8.32

Plot

_____

8.33

Plot

_____

8.34

We have that

$\qquad R = \dfrac{np - n_i^2}{\tau_{pO}(n + n') + \tau_{nO}(p + p')}$

Let $\tau_{pO} = \tau_{nO} \equiv \tau_O$ and $n' = p' = n_i$

We can write

$\qquad n = n_i \exp\left(\dfrac{E_{Fn} - E_{Fi}}{kT}\right)$

and

$\qquad p = n_i \exp\left(\dfrac{E_{Fi} - E_{Fp}}{kT}\right)$

We also have

$\qquad \left(E_{Fn} - E_{Fi}\right) + \left(E_{Fi} - E_{Fp}\right) = eV_a$

so that

$\qquad \left(E_{Fi} - E_{Fp}\right) = eV_a - \left(E_{Fn} - E_{Fi}\right)$

Then

$\qquad p = n_i \exp\left[\dfrac{eV_a - \left(E_{Fn} - E_{Fi}\right)}{kT}\right]$

$\qquad = n_i \exp\left(\dfrac{eV_a}{kT}\right) \cdot \exp\left[\dfrac{-\left(E_{Fn} - E_{Fi}\right)}{kT}\right]$

_____

Define

$$\eta_a = \frac{eV_a}{kT} \quad \text{and} \quad \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT} \right)$$

Then the recombination rate can be written as

$$R = \frac{\left(n_i e^\eta \right) \left(n_i e^{\eta_a} \cdot e^{-\eta} \right) - n_i^2}{\tau_O \left[n_i e^\eta + n_i + n_i e^{\eta_a} \cdot e^{-\eta} + n_i \right]}$$

or

$$R = \frac{n_i \left(e^{\eta_a} - 1 \right)}{\tau_O \left[2 + e^\eta + e^{\eta_a} \cdot e^{-\eta} \right]}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i \left(e^{\eta_a} - 1 \right)}{\tau_O} \cdot \frac{d}{d\eta} \left[2 + e^\eta + e^{\eta_a} \cdot e^{-\eta} \right]^{-1}$$

or

$$0 = \frac{n_i \left(e^{\eta_a} - 1 \right)}{\tau_O} \cdot (-1) \left[2 + e^\eta + e^{\eta_a} \cdot e^{-\eta} \right]^{-2}$$

$$\times \left[e^\eta - e^{\eta_a} \cdot e^{-\eta} \right]$$

which simplifies to

$$0 = \frac{-n_i \left(e^{\eta_a} - 1 \right)}{\tau_O} \cdot \frac{\left[e^\eta - e^{\eta_a} \cdot e^{-\eta} \right]}{\left[2 + e^\eta + e^{\eta_a} \cdot e^{-\eta} \right]^2}$$

The denominator is not zero, so we have

$$0 = e^\eta - e^{\eta_a} \cdot e^{-\eta}$$

or

$$e^{2\eta} = e^{\eta_a} \quad \Rightarrow \eta = \frac{\eta_a}{2}$$

Then the maximum recombination rate
becomes

$$R_{max} = \frac{n_i \left(e^{\eta_a} - 1 \right)}{\tau_O \left[2 + e^{\eta_a/2} + e^{\eta_a} \cdot e^{-\eta_a/2} \right]}$$

$$= \frac{n_i \left(e^{\eta_a} - 1 \right)}{\tau_O \left[2 + e^{\eta_a/2} + e^{\eta_a/2} \right]}$$

or

$$R_{max} = \frac{n_i \left(e^{\eta_a} - 1 \right)}{2\tau_O \left(e^{\eta_a/2} + 1 \right)}$$

which can be written as

$$R_{max} = \frac{n_i \left[\exp\left(\dfrac{eV_a}{kT} \right) - 1 \right]}{2\tau_O \left[\exp\left(\dfrac{eV_a}{2kT} \right) + 1 \right]}$$

If $V_a \gg (kT/e)$, then we can neglect the (-1)
term in the numerator and the (+1) term in the
denominator, so we finally have

$$R_{max} = \frac{n_i}{2\tau_O} \exp\left(\frac{eV_a}{2kT} \right)$$

Q.E.D.

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8.35

We have

$$J_{gen} = \int_0^W eG \, dx$$

In this case, $G = g' = 4 \times 10^{19} \, \text{cm}^{-3} \, \text{s}^{-1}$ and is
a constant through the space charge region.
Then

$$J_{gen} = eg'W$$

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2} \right)$$

$$= (0.0259) \ln\left[\frac{\left(5 \times 10^{15} \right) \left(5 \times 10^{15} \right)}{\left(1.5 \times 10^{10} \right)^2} \right]$$

or

$$V_{bi} = 0.659 \, \text{V}$$

Also

$$W = \left[\frac{2 \in_s \left(V_{bi} + V_R \right)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85 \times 10^{-14} \right)(0.659 + 10)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times \left(\frac{5 \times 10^{15} + 5 \times 10^{15}}{\left(5 \times 10^{15} \right)\left(5 \times 10^{15} \right)} \right) \right]^{1/2}$$

or

$$W = 2.35 \times 10^{-4} \, \text{cm}$$

Then

$$J_{gen} = \left(1.6 \times 10^{-19} \right)\left(4 \times 10^{19} \right)\left(2.35 \times 10^{-4} \right)$$

or

$$J_{gen} = 1.5 \times 10^{-3} \, \text{A/cm}^2$$

_____

8.36

$$J_S = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^2 \left[\frac{1}{3 \times 10^{16}} \sqrt{\frac{18}{10^{-7}}} \right. $$
$$ \left. + \frac{1}{10^{18}} \sqrt{\frac{6}{10^{-7}}} \right]$$

or

$$J_S = 1.638 \times 10^{-11} \text{ A/cm}^2$$

Now

$$J_D = J_S \exp\left(\frac{V_D}{V_t}\right)$$

We want

$$J = 0 = J_G - J_D$$

or

$$0 = 25 \times 10^{-3} - 1.638 \times 10^{-11} \exp\left(\frac{V_D}{V_t}\right)$$

which can be written as

$$\exp\left(\frac{V_D}{V_t}\right) = \frac{25 \times 10^{-3}}{1.638 \times 10^{-11}} = 1.526 \times 10^9$$

We find

$$V_D = V_t \ln(1.526 \times 10^9)$$

or

$$V_D = 0.548 \text{ V}$$

8.37

(a) $r_d = \dfrac{V_t}{I_{DQ}} = \dfrac{0.0259}{1.2 \times 10^{-3}} = 21.6 \, \Omega$

$$C_d = \frac{I_{DQ} \tau_0}{2V_t} = \frac{(1.2 \times 10^{-3})(0.5 \times 10^{-6})}{2(0.0259)}$$
$$= 1.16 \times 10^{-8} \text{ F}$$

or $C_d = 11.6 \text{ nF}$

(b) $r_d = \dfrac{0.0259}{0.12 \times 10^{-3}} = 216 \, \Omega$

$$C_d = \frac{(0.12 \times 10^{-3})(0.5 \times 10^{-6})}{2(0.0259)}$$
$$= 1.16 \times 10^{-9} \text{ F}$$

or $C_d = 1.16 \text{ nF}$

8.38

(a) $C_d = \dfrac{\Delta Q}{\Delta V}$, For $I_D = 1.2$ mA

$$\Delta Q = C_d \cdot \Delta V = (1.158 \times 10^{-8})(50 \times 10^{-3})$$
$$= 5.79 \times 10^{-10} \text{ C}$$

(b) For $I_D = 0.12$ mA

$$\Delta Q = C_d \cdot \Delta V = (1.158 \times 10^{-9})(50 \times 10^{-3})$$
$$= 5.79 \times 10^{-11} \text{ C}$$

8.39

For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t}, \quad C_d = \frac{I_{DQ} \tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86 \times 10^{-2} \text{ S}$$

and

$$C_d = \frac{(10^{-3})(10^{-7})}{2(0.0259)} = 1.93 \times 10^{-9} \text{ F}$$

We have

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

where $\omega = 2\pi f$

We obtain

$f = 10 \text{ kHz}, \quad Z = 25.9 - j0.0814$

$f = 100 \text{ kHz}, \quad Z = 25.9 - j0.814$

$f = 1 \text{ MHz}, \quad Z = 23.6 - j7.41$

$f = 10 \text{ MHz}, \quad Z = 2.38 - j7.49$

8.40

Reverse bias

$$C_j = AC' = A \cdot \left\{ \frac{e \in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$$

$$V_{bi} = (0.0259) \ln \left[\frac{(5 \times 10^{17})(8 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$$

$$= 0.790 \text{ V}$$

$$C_j = (2 \times 10^{-4}) \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(V_{bi} + V_R)} \right.$$

$$\left. \times \left[\frac{(5 \times 10^{17})(8 \times 10^{15})}{5 \times 10^{17} + 8 \times 10^{15}} \right] \right\}^{1/2}$$

_____

$$C_j = \frac{5.1078 \times 10^{-12}}{\sqrt{V_{bi} + V_r}} \text{ F}$$

V_R (V)	C_j (pF)
10	1.555
5	2.123
3	2.624
1	3.818
0	5.747
−0.20	6.650
−0.40	8.179

Forward bias
For $N_a \gg N_d \Rightarrow I_{po} \gg I_{no}$
Then

$$C_d = \frac{I_{po}\tau_{p0}}{2V_t} = \frac{I_{po}(8 \times 10^{-8})}{2(0.0259)} = (1.544 \times 10^{-6})I_{po}$$

$$I_{po} = Ae\sqrt{\frac{D_p}{\tau_{p0}}} \cdot \left(\frac{n_i^2}{N_d}\right) \cdot \exp\left(\frac{V_a}{V_t}\right)$$

$$= (2 \times 10^{-4})(1.6 \times 10^{-19})\sqrt{\frac{10}{8 \times 10^{-8}}} \cdot \frac{(1.5 \times 10^{10})^2}{8 \times 10^{15}}$$

$$\times \exp\left(\frac{V_a}{V_t}\right)$$

$$I_{po} = (1.006 \times 10^{-14})\exp\left(\frac{V_a}{V_t}\right) \text{ A}$$

V_a (V)	C_d (F)	+	C_j (F)	= C_{Total} (F)
0.20	3.51×10^{-17} +		6.650×10^{-12}	
				$\cong 6.650 \times 10^{-12}$
0.40	7.92×10^{-14} +		8.179×10^{-12}	
				$= 8.258 \times 10^{-12}$
0.60	1.79×10^{-10} +	...		
				$\cong 1.79 \times 10^{-10}$

_____

8.41

For a p^+n diode, $I_{pO} \gg I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t}\right)(I_{pO}\tau_{pO})$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5 \times 10^{-6} \text{ F/A}$$

Then

$$\tau_{pO} = 2(0.0259)(2.5 \times 10^{-6})$$

or

$$\tau_{pO} = 1.3 \times 10^{-7} \text{ s}$$

At 1 mA,

$$C_d = (2.5 \times 10^{-6})(10^{-3})$$

or

$$C_d = 2.5 \times 10^{-9} \text{ F}$$

_____

8.42

(a) $N_a \gg N_d \Rightarrow I_{po} \gg I_{no}$

(i) $C_d = \dfrac{I_{po}\tau_{p0}}{2V_t}$

or $I_{po} = \dfrac{2V_t(C_d)}{\tau_{p0}} = \dfrac{2(0.0259)(10^{-9})}{10^{-7}}$

$$= 5.18 \times 10^{-4} \text{ A}$$

or $I_{po} = 0.518 \text{ mA}$

(ii) $I_{po} = Ae\sqrt{\dfrac{D_p}{\tau_{p0}}} \cdot \dfrac{n_i^2}{N_d} \exp\left(\dfrac{V_a}{V_t}\right)$

$$0.518 \times 10^{-3} = (5 \times 10^{-4})(1.6 \times 10^{-19})\sqrt{\frac{10}{10^{-7}}}$$

$$\times \frac{(1.5 \times 10^{10})^2}{8 \times 10^{15}} \exp\left(\frac{V_a}{V_t}\right)$$

$$V_a = (0.0259)\ln\left(\frac{0.518 \times 10^{-3}}{2.25 \times 10^{-14}}\right)$$

$$= 0.618 \text{ V}$$

(iii) $r_d = \dfrac{V_t}{I_D} = \dfrac{0.0259}{0.518 \times 10^{-3}} = 50 \,\Omega$

(b)

(i) $I_{po} = \dfrac{2V_t(C_d)}{\tau_{p0}} = \dfrac{2(0.0259)(0.25 \times 10^{-9})}{10^{-7}}$

$$= 1.295 \times 10^{-4} \text{ A}$$

or $I_{po} = 0.1295 \text{ mA}$

(ii) $V_a = (0.0259)\ln\left(\dfrac{0.1295 \times 10^{-3}}{2.25 \times 10^{-14}}\right)$

$$= 0.5821 \text{ V}$$

(iii) $r_d = \dfrac{0.0259}{0.1295 \times 10^{-3}} = 200 \,\Omega$

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8.43

(a) p-region:

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{(e\mu_p N_a)A}$$

so

$$R_p = \frac{0.2}{(1.6\times10^{-19})(480)(10^{16})(10^{-2})}$$

or

$$R_p = 26\,\Omega$$

n-region:

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{(e\mu_n N_d)A}$$

so

$$R_n = \frac{0.10}{(1.6\times10^{-19})(1350)(10^{15})(10^{-2})}$$

or

$$R_n = 46.3\,\Omega$$

The total resistance is

$$R = R_p + R_n = 26 + 46.3$$

or

$$R = 72.3\,\Omega$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

which yields

$$I = 1.38\,\text{mA}$$

_____

8.44

$$R = \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)}$$

$$= \frac{(0.2)(10^{-2})}{2\times10^{-5}} + \frac{(0.1)(10^{-2})}{2\times10^{-5}}$$

or

$$R = 150\,\Omega$$

We can write

$$V = I_D R + V_t \ln\left(\frac{I_D}{I_S}\right)$$

(a) (i) For $I_D = 1\,\text{mA}$,

$$V = (10^{-3})(150) + (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or $V = 0.567\,\text{V}$

(ii) For $I_D = 10\,\text{mA}$,

$$V = (10^{-2})(150) + (0.0259)\ln\left(\frac{10^{-2}}{10^{-10}}\right)$$

or $V = 1.98\,\text{V}$

(b) Set $R = 0$

(i) For $I_D = 1\,\text{mA}$,

$$V = (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or $V = 0.417\,\text{V}$

(ii) For $I_D = 10\,\text{mA}$,

$$V = (0.0259)\ln\left(\frac{10^{-2}}{10^{-10}}\right)$$

or $V = 0.477\,\text{V}$

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8.45

(a) $\quad r_d = \dfrac{V_t}{I_D} \Rightarrow I_D = \dfrac{V_t}{r_d} = \dfrac{0.0259}{32}$

or $I_D = 8.09375\times10^{-4}\,\text{A}$

$$V_a = V_t \ln\left(\frac{I_D}{I_s}\right)$$

$$= (0.0259)\ln\left(\frac{8.09375\times10^{-4}}{5\times10^{-12}}\right)$$

$$V_a = 0.4896\,\text{V}$$

(b) $\quad I_D = \dfrac{V_t}{r_d} = \dfrac{0.0259}{60} = 4.3167\times10^{-4}\,\text{A}$

$$V_a = (0.0259)\ln\left(\frac{4.3167\times10^{-4}}{5\times10^{-12}}\right)$$

$$= 0.4733\,\text{V}$$

_____

8.46

(a) $\quad \dfrac{1}{r_d} = \dfrac{dI_D}{dV_a} = I_S\left(\dfrac{1}{V_t}\right)\exp\left(\dfrac{V_a}{V_t}\right)$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$r_d = 1.2\times10^{11}\,\Omega$$

(b)

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{-0.02}{0.0259}\right)$$

which yields

$$r_d = 5.6\times10^{11}\,\Omega$$

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8.47

(a) If $\dfrac{I_R}{I_F} = 0.2$

Then we have

$$erf \sqrt{\dfrac{t_s}{\tau_{pO}}} = \dfrac{I_F}{I_F + I_R} = \dfrac{1}{1 + \dfrac{I_R}{I_F}} = \dfrac{1}{1 + 0.2}$$

or

$$erf \sqrt{\dfrac{t_s}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\dfrac{t_s}{\tau_{pO}}} = 0.978 \Rightarrow \dfrac{t_s}{\tau_{pO}} = 0.956$$

(b) If $\dfrac{I_R}{I_F} = 1.0$, then

$$erf \sqrt{\dfrac{t_s}{\tau_{pO}}} = \dfrac{1}{1 + 1} = 0.50$$

which yields

$$\dfrac{t_s}{\tau_{pO}} = 0.228$$

_____

8.48

(a) $$erf \sqrt{\dfrac{t_s}{\tau_p}} = \dfrac{I_F}{I_F + I_R}$$

$$erf \sqrt{0.3} = erf(0.5477)$$

$$\cong erf(0.55) = 0.56332$$

Then $$0.56332 = \dfrac{1}{1 + \dfrac{I_R}{I_F}}$$

$$\dfrac{I_R}{I_F} = \dfrac{1}{0.56332} - 1 = 0.775$$

(b) $$erf \sqrt{\dfrac{t_2}{\tau_{p0}}} + \dfrac{\exp\left(\dfrac{-t_2}{\tau_{p0}}\right)}{\sqrt{\pi\left(\dfrac{t_2}{\tau_{p0}}\right)}} = 1 + (0.1)\left(\dfrac{I_R}{I_F}\right)$$

$$= 1 + (0.1)(0.775)$$

$$= 1.0775$$

By trial and error, $$\dfrac{t_2}{\tau_{p0}} \cong 0.80$$

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8.49

$$C_j = 18 \text{ pF at } V_R = 0$$

$$C_j = 4.2 \text{ pF at } V_R = 10 \text{ V}$$

We have

$$\tau_{nO} = \tau_{pO} = 10^{-7} \text{ s} , \ I_F = 2 \text{ mA}$$

and

$$I_R \cong \dfrac{V_R}{R} = \dfrac{10}{10} = 1 \text{ mA}$$

So

$$t_s \cong \tau_{pO} \ln\left(1 + \dfrac{I_F}{I_R}\right) = \left(10^{-7}\right) \ln\left(1 + \dfrac{2}{1}\right)$$

or

$$t_s = 1.1 \times 10^{-7} \text{ s}$$

Also

$$C_{avg} = \dfrac{18 + 4.2}{2} = 11.1 \text{ pF}$$

The time constant is

$$\tau_S = RC_{avg} = \left(10^4\right)\left(11.1 \times 10^{-12}\right)$$

$$= 1.11 \times 10^{-7} \text{ s}$$

Now, the turn-off time is

$$t_{off} = t_s + \tau_S = (1.1 + 1.11) \times 10^{-7}$$

or

$$t_{off} = 2.21 \times 10^{-7} \text{ s}$$

_____

8.50

$$V_{bi} = (0.0259) \ln\left[\dfrac{\left(5 \times 10^{19}\right)^2}{\left(1.5 \times 10^{10}\right)^2}\right] = 1.136 \text{ V}$$

We find

$$W = \left[\dfrac{2 \in_s (V_{bi} - V_a)}{e}\left(\dfrac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\dfrac{2(11.7)\left(8.85 \times 10^{-14}\right)(1.136 - 0.40)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\dfrac{5 \times 10^{19} + 5 \times 10^{19}}{\left(5 \times 10^{19}\right)^2}\right)\right]^{1/2}$$

which yields

$$W = 6.17 \times 10^{-7} \text{ cm} = 61.7 \ \overset{o}{A}$$

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8.51

Sketch

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8.53

From Figure 7.15, $N_d \cong 9 \times 10^{15}$ cm^{-3}

Let $N_a = 5 \times 10^{17}$ cm^{-3}

$$p_n(x_n) = (0.1)N_d = 9 \times 10^{14} = p_{no} \exp\left(\frac{V_a}{V_t}\right)$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{9 \times 10^{15}} = 2.5 \times 10^4 \text{ cm}^{-3}$$

Then $V_a = (0.0259) \ln\left(\dfrac{9 \times 10^{14}}{2.5 \times 10^4}\right) = 0.6295$ V

$$I_s = \frac{I}{\exp\left(\dfrac{V_a}{V_t}\right)} = \frac{50 \times 10^{-3}}{\exp\left(\dfrac{0.6295}{0.0259}\right)}$$

$$= 1.389 \times 10^{-12} \text{ A}$$

$$I_s \cong \frac{AeD_p p_{no}}{L_p} = \frac{Aen_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}$$

1.389×10^{-12}

$$= \frac{A(1.6 \times 10^{-19})(1.5 \times 10^{10})^2}{9 \times 10^{15}}\sqrt{\frac{10}{2 \times 10^{-7}}}$$

$1.389 \times 10^{-12} = A(2.828 \times 10^{-11})$

or $A = 4.91 \times 10^{-2}$ cm^2

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Chapter 9

9.1

(a) We have

$$e\phi_n = eV_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right) = 0.206 \text{ eV}$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\phi_{BO} = 0.27 \text{ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064 \text{ V}$$

Also

$$x_d = \left[\frac{2\in_s V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.064)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 9.1\times10^{-6} \text{ cm}$$

Then

$$|E_{max}| = \frac{eN_d x_d}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(10^{16})(9.1\times10^{-6})}{(11.7)(8.85\times10^{-14})}$$

or

$$|E_{max}| = 1.41\times10^4 \text{ V/cm}$$

(d)

Using the figure, $\phi_{Bn} = 0.55 \text{ V}$

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344 \text{ V}$$

We then find

$$x_n = 2.11\times10^{-5} \text{ cm}$$

and

$$|E_{max}| = 3.26\times10^4 \text{ V/cm}$$

_____

9.2

(a) $V_{bi} = \phi_{B0} - \phi_n$

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{5\times10^{15}}\right)$$

$$= 0.2235 \text{ V}$$

$$V_{bi} = 0.65 - 0.2235 = 0.4265 \text{ V}$$

(b) $\phi_n = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right)$

$$= 0.2056 \text{ V}$$

$$V_{bi} = 0.65 - 0.2056 = 0.4444 \text{ V}$$

V_{bi} increases, ϕ_{B0} remains constant

(c) $\phi_n = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$

$$= 0.2652 \text{ V}$$

$$V_{bi} = 0.65 - 0.2652 = 0.3848 \text{ V}$$

V_{bi} decreases, ϕ_{B0} remains constant

_____

9.3

(a) $\phi_{B0} = \phi_m - \chi = 5.1 - 4.01 = 1.09 \text{ V}$

(b) $V_{bi} = \phi_{B0} - \phi_n$

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right) = 0.2056 \text{ V}$$

$$V_{bi} = 1.09 - 0.2056 = 0.8844 \text{ V}$$

(c) $x_n = \left[\frac{2\in_s (V_{bi}+V_R)}{eN_d}\right]^{1/2}$

(i) $x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8844+1)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$

$$= 4.939\times10^{-5} \text{ cm}$$

or $x_n = 0.4939 \, \mu\text{m}$

$$|E_{max}| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(10^{16})(4.939\times10^{-5})}{(11.7)(8.85\times10^{-14})}$$

$$= 7.63\times10^4 \text{ V/cm}$$

(ii) $x_n = \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(0.8844 + 5)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$

$\qquad = 8.727 \times 10^{-5}$ cm

$\quad$ or $\; x_n = 0.8728 \, \mu$m

$\qquad \left| E_{max} \right| = \dfrac{(1.6 \times 10^{-19})(10^{16})(8.727 \times 10^{-5})}{(11.7)(8.85 \times 10^{-14})}$

$\qquad\qquad = 1.35 \times 10^5$ V/cm

(ii) $x_n = \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(0.7623 + 5)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$

$\qquad = 1.292 \times 10^{-4}$ cm

$\quad$ or $\; x_n = 1.292 \, \mu$m

$\qquad \left| E_{max} \right| = \dfrac{(1.6 \times 10^{-19})(5 \times 10^{15})(1.292 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})}$

$\qquad\qquad = 8.92 \times 10^4$ V/cm

9.4

(a) $\; \phi_{B0} = \phi_m - \chi = 5.1 - 4.07 = 1.03$ V

(b) $\; \phi_n = (0.0259) \ln\left(\dfrac{4.7 \times 10^{17}}{5 \times 10^{15}} \right) = 0.1177$ V

(c) $\; V_{bi} = 1.03 - 0.1177 = 0.9123$ V

(d)

$\quad$ (i) $x_n = \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(0.9123 + 1)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$

$\qquad = 7.445 \times 10^{-5}$ cm

$\quad$ or $\; x_n = 0.7445 \, \mu$m

$\qquad \left| E_{max} \right| = \dfrac{(1.6 \times 10^{-19})(5 \times 10^{15})(7.445 \times 10^{-5})}{(13.1)(8.85 \times 10^{-14})}$

$\qquad\qquad = 5.14 \times 10^4$ V/cm

$\quad$ (ii) $x_n = \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(0.9123 + 5)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$

$\qquad = 1.309 \times 10^{-4}$ cm

$\quad$ or $\; x_n = 1.309 \, \mu$m

$\qquad \left| E_{max} \right| = \dfrac{(1.6 \times 10^{-19})(5 \times 10^{15})(1.309 \times 10^{-4})}{(13.1)(8.85 \times 10^{-14})}$

$\qquad\qquad = 9.03 \times 10^4$ V/cm

9.5

(b) $\phi_n = 0.1177$ V

(c) $V_{bi} = 0.88 - 0.1177 = 0.7623$ V

(d)

$\quad$ (i) $\; x_n = \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(0.7623 + 1)}{(1.6 \times 10^{-19})(5 \times 10^{15})} \right]^{1/2}$

$\qquad = 7.147 \times 10^{-5}$ cm

$\quad$ or $\; x_n = 0.7147 \, \mu$m

$\qquad \left| E_{max} \right| = \dfrac{(1.6 \times 10^{-19})(5 \times 10^{15})(7.147 \times 10^{-5})}{(13.1)(8.85 \times 10^{-14})}$

$\qquad\qquad = 4.93 \times 10^4$ V/cm

9.6

(a) $\; C' = \left[\dfrac{e \in_s N_d}{2(V_{bi} + V_R)} \right]^{1/2}$

We have $\phi_{B0} = 0.88$ V

$\qquad \phi_n = (0.0259) \ln\left(\dfrac{2.8 \times 10^{19}}{10^{15}} \right) = 0.265$ V

$\qquad V_{bi} = 0.88 - 0.265 = 0.615$ V

(i)

$C = (10^{-4}) \left[\dfrac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}{2(0.615 + 1)} \right]^{1/2}$

$\qquad = 7.16 \times 10^{-13}$ F

$\quad$ or $\; C = 0.716$ pF

(ii)

$C = (10^{-4}) \left[\dfrac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}{2(0.615 + 5)} \right]^{1/2}$

$\qquad = 3.84 \times 10^{-13}$ F

$\quad$ or $\; C = 0.384$ pF

(b) $\; \phi_n = (0.0259) \ln\left(\dfrac{2.8 \times 10^{19}}{10^{16}} \right) = 0.206$ V

$\qquad V_{bi} = 0.88 - 0.206 = 0.674$ V

(i)

$C = (10^{-4}) \left[\dfrac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})}{2(0.674 + 1)} \right]^{1/2}$

$\qquad = 2.22 \times 10^{-12}$ F

$\quad$ or $\; C = 2.22$ pF

(ii)

$C = (10^{-4}) \left[\dfrac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})}{2(0.6745 + 5)} \right]^{1/2}$

$\qquad = 1.21 \times 10^{-12}$ F

$\quad$ or $\; C = 1.21$ pF

_____

9.7

(a) From the figure, $V_{bi} = 0.90$ V

(b) We find

$$\frac{\Delta\left(\dfrac{1}{C'}\right)^2}{\Delta V_R} = \frac{3\times10^{15}-0}{2-(-0.90)} = 1.034\times10^{15}$$

and

$$1.034\times10^{15} = \frac{2}{e \in_s N_d}$$

We can then write

$$N_d = \frac{2}{\left(1.6\times10^{-19}\right)\left(13.1\right)\left(8.85\times10^{-14}\right)\left(1.034\times10^{15}\right)}$$

or

$$N_d = 1.04\times10^{16}\ \text{cm}^{-3}$$

(c)

$$\phi_n = V_t\,\ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7\times10^{17}}{1.04\times10^{16}}\right)$$

or

$$\phi_n = 0.0986\ \text{V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0986$$

or

$$\phi_{Bn} = 0.9986\ \text{V}$$

_____

9.8

From Figure 9.5, $\phi_{BO} \cong 0.63$ V

(a) $\quad \phi_n = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{5\times10^{15}}\right) = 0.224$ V

$$V_{bi} = \phi_{B0} - \phi_n = 0.63 - 0.224 = 0.406\ \text{V}$$

(i) $x_n = \left[\dfrac{2(11.7)\left(8.85\times10^{-14}\right)(0.406+1)}{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)}\right]^{1/2}$

$$= 6.033\times10^{-5}\ \text{cm}$$

or $\ x_n = 0.6033\ \mu\text{m}$

$$\left|\text{E}_{max}\right| = \frac{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)\left(6.033\times10^{-5}\right)}{(11.7)\left(8.85\times10^{-14}\right)}$$

$$= 4.66\times10^4\ \text{V/cm}$$

(ii) $x_n = \left[\dfrac{2(11.7)\left(8.85\times10^{-14}\right)(0.406+5)}{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)}\right]^{1/2}$

$$= 1.183\times10^{-4}\ \text{cm}$$

or $\ x_n = 1.183\ \mu\text{m}$

$$\left|\text{E}_{max}\right| = \frac{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)\left(1.183\times10^{-4}\right)}{(11.7)\left(8.85\times10^{-14}\right)}$$

$$= 9.14\times10^4\ \text{V/cm}$$

(b)

(i) $\Delta\phi = \sqrt{\dfrac{e\text{E}}{4\pi\in_s}}$

$$= \left[\frac{\left(1.6\times10^{-19}\right)\left(4.66\times10^4\right)}{4\pi(11.7)\left(8.85\times10^{-14}\right)}\right]^{1/2}$$

$$= 0.0239\ \text{V}$$

$$x_m = \sqrt{\frac{e}{16\pi\in_s\text{E}}}$$

$$= \left[\frac{\left(1.6\times10^{-19}\right)}{16\pi(11.7)\left(8.85\times10^{-14}\right)\left(4.66\times10^4\right)}\right]^{1/2}$$

or

$$x_m = 2.57\times10^{-7}\ \text{cm}$$

(ii) $\Delta\phi = \left[\dfrac{\left(1.6\times10^{-19}\right)\left(9.14\times10^4\right)}{4\pi(11.7)\left(8.85\times10^{-14}\right)}\right]^{1/2}$

$$= 0.0335\ \text{V}$$

$$x_m = \left[\frac{1.6\times10^{-19}}{16\pi(11.7)\left(8.85\times10^{-14}\right)\left(9.14\times10^4\right)}\right]^{1/2}$$

$$= 1.83\times10^{-7}\ \text{cm}$$

_____

9.9

We have

$$-\phi(x) = \frac{-e}{16\pi\in_s x} - \text{E}x$$

or

$$e\phi(x) = \frac{e^2}{16\pi\in_s x} + \text{E}ex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi\in_s x^2} + \text{E}e$$

Solving for x, we find

$$x = x_m = \sqrt{\frac{e}{16\pi\in_s\text{E}}}$$

Substituting this value of $x = x_m$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi\in_s\sqrt{\dfrac{e}{16\pi\in_s\text{E}}}} + \text{E}\sqrt{\frac{e}{16\pi\in_s\text{E}}}$$

which yields

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in_s}}$$

9.10

From Figure 9.5, $\phi_{BO} \cong 0.88$ V

(a) $\phi_n = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{16}}\right) = 0.0997$ V

$V_{bi} = \phi_{B0} - \phi_n = 0.88 - 0.0997 \cong 0.780$ V

$x_n = \left[\frac{2(13.1)(8.85\times10^{-14})(0.780)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$

$= 3.362\times10^{-5}$ cm

or $x_n = 0.3362\,\mu$m

$|\mathrm{E}_{max}| = \frac{(1.6\times10^{-19})(10^{16})(3.362\times10^{-5})}{(13.1)(8.85\times10^{-14})}$

$= 4.64\times10^{4}$ V/cm

(b) $\Delta\phi = (0.05)(0.88) = 0.044$ V

$= \sqrt{\frac{e\mathrm{E}}{4\pi \in_s}}$

$(0.044)^2 = \frac{(1.6\times10^{-19})\mathrm{E}}{4\pi(13.1)(8.85\times10^{-14})}$

$\mathrm{E} = \frac{(0.044)^2(4\pi)(13.1)(8.85\times10^{-14})}{1.6\times10^{-19}}$

$= 1.763\times10^{5}$ V/cm

Now

$\mathrm{E} = 1.763\times10^{5} = \frac{(1.6\times10^{-19})(10^{16})x_n}{(13.1)(8.85\times10^{-14})}$

$\Rightarrow x_n = 1.277\times10^{-4}$ cm

And

$x_n^2 = (1.277\times10^{-4})^2$

$= \frac{2(13.1)(8.85\times10^{-14})(0.780+V_R)}{(1.6\times10^{-19})(10^{16})}$

$\Rightarrow V_R = 10.5$ V

9.11

Plot

9.12

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$\phi_{BO} = 1.13$ V

(b) We have

$(E_g - e\phi_O - e\phi_{Bn})$

$= \frac{1}{eD_{it}}\sqrt{2e\in_s N_d(\phi_{Bn} - \phi_n)}$

$- \frac{\in_i}{eD_{it}\delta}[\phi_m - (\chi + \phi_{Bn})]$

which becomes

$e(1.43 - 0.60 - \phi_{Bn})$

$= \frac{1}{e\left(\frac{10^{13}}{e}\right)}\left[2(1.6\times10^{-19})(13.1)(8.85\times10^{-14})\right.$

$\left. \times(10^{16})(\phi_{Bn} - 0.10)\right]^{1/2}$

$- \frac{(8.85\times10^{-14})}{e\left(\frac{10^{13}}{e}\right)(25\times10^{-8})}[5.2 - (4.07 + \phi_{Bn})]$

or

$0.83 - \phi_{Bn}$

$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221(1.13 - \phi_{Bn})$

We find

$\phi_{Bn} = 0.858$ V

(c)

If $\phi_m = 4.5$ V, then

$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$

or

$\phi_{BO} = 0.43$ V

From part (b), we have

$0.83 - \phi_{Bn}$

$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221[4.5 - (4.07 + \phi_{Bn})]$

We then find

$\phi_{Bn} = 0.733$ V

With interface states, the barrier height is less sensitive to the metal work function.

9.13

We have that

$(E_g - e\phi_O - e\phi_{Bn})$

$= \frac{1}{eD_{it}}\sqrt{2e\in_s N_d(\phi_{Bn} - \phi_n)}$

$- \frac{\in_i}{eD_{it}\delta}[\phi_m - (\chi + \phi_{Bn})]$

Let $eD_{it} = D'_{it}\ (\mathrm{cm}^{-2}\,\mathrm{eV}^{-1})$

Then we can write

$$e(1.12 - 0.230 - 0.60)$$

$$= \frac{1}{D'_{it}} \left[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14}) \right.$$

$$\times (5 \times 10^{16})(0.60 - 0.164) \big]^{1/2}$$

$$\left. - \frac{(8.85 \times 10^{-14})}{D'_{it}(20 \times 10^{-8})} [4.75 - (4.01 + 0.60)] \right]$$

We then find

$$D'_{it} = 4.97 \times 10^{11} \text{ cm}^{-2} \text{eV}^{-1}$$

_____

9.14

(a) $\phi_n = (0.0259) \ln \left(\dfrac{2.8 \times 10^{19}}{5 \times 10^{15}} \right) = 0.224 \text{ V}$

(b) $V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.224 = 0.666 \text{ V}$

(c) $J_{sT} = A^* T^2 \exp \left(\dfrac{-e\phi_{Bn}}{kT} \right)$

$$= (120)(300)^2 \exp \left(\frac{-0.89}{0.0259} \right)$$

$$J_{sT} = 1.29 \times 10^{-8} \text{ A/cm}^2$$

(d)

$$V_a = V_t \ln \left(\frac{J}{J_{sT}} \right) = (0.0259) \ln \left(\frac{5}{1.29 \times 10^{-8}} \right)$$

$$V_a = 0.512 \text{ V}$$

_____

9.15

(a) $\phi_{B0} \cong 0.63 \text{ V}$

$$J_{sT} = (120)(300)^2 \exp \left(\frac{-0.63}{0.0259} \right)$$

$$= 2.948 \times 10^{-4} \text{ A/cm}^2$$

$$I_{sT} = (10^{-4})(2.948 \times 10^{-4}) = 2.948 \times 10^{-8} \text{ A}$$

(i) $V_a = V_t \ln \left(\dfrac{I}{I_{sT}} \right)$

$$= (0.0259) \ln \left(\frac{10 \times 10^{-6}}{2.948 \times 10^{-8}} \right)$$

$$= 0.151 \text{ V}$$

(ii) $V_a = (0.0259) \ln \left(\dfrac{100 \times 10^{-6}}{2.948 \times 10^{-8}} \right)$

$$= 0.211 \text{ V}$$

(iii) $V_a = (0.0259) \ln \left(\dfrac{10^{-3}}{2.948 \times 10^{-8}} \right)$

$$= 0.270 \text{ V}$$

(b) $kT = (0.0259) \left(\dfrac{350}{300} \right) = 0.030217 \text{ eV}$

$$I_{sT} = (10^{-4})(120)(350)^2 \exp \left(\frac{-0.63}{0.030217} \right)$$

$$= 1.296 \times 10^{-6} \text{ A}$$

(i) $I = I_{sT} \left[\exp \left(\dfrac{V_a}{V_t} \right) - 1 \right]$

$$V_a = (0.030217) \ln \left[\frac{10 \times 10^{-6}}{1.296 \times 10^{-6}} + 1 \right]$$

$$= 0.0654 \text{ V}$$

(ii) $V_a = (0.030217) \ln \left[\dfrac{100 \times 10^{-6}}{1.296 \times 10^{-6}} + 1 \right]$

$$= 0.1317 \text{ V}$$

(iii) $V_a \cong (0.030217) \ln \left(\dfrac{10^{-3}}{1.296 \times 10^{-6}} \right)$

$$= 0.201 \text{ V}$$

_____

9.16

(a) $\phi_{Bn} \cong 0.88 \text{ V}$

(b) $J_{sT} = (1.12)(300)^2 \exp \left(\dfrac{-0.88}{0.0259} \right)$

$$= 1.768 \times 10^{-10} \text{ A/cm}^2$$

(c) $V_a = (0.0259) \ln \left(\dfrac{10}{1.768 \times 10^{-10}} \right)$

$$= 0.641 \text{ V}$$

(d) $\Delta V_a = V_t \ln(2) = (0.0259) \ln(2)$
$$= 0.0180 \text{ V}$$

_____

9.17
Plot

_____

9.18
From the figure, $\phi_{Bn} = 0.68 \text{ V}$

$$J_{ST} = A^* T^2 \exp \left(\frac{-\phi_{Bn}}{V_t} \right) \cdot \exp \left(\frac{\Delta\phi}{V_t} \right)$$

$$= (120)(300)^2 \exp \left(\frac{-0.68}{0.0259} \right) \cdot \exp \left(\frac{\Delta\phi}{V_t} \right)$$

or

$$J_{ST} = 4.277 \times 10^{-5} \exp \left(\frac{\Delta\phi}{V_t} \right)$$

We have

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \in_s}}$$

Now

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right) = 0.2056 \text{ V}$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.2056 = 0.4744 \text{ V}$$

(a) We find for $V_R = 2$ V,

$$x_d = \left[\frac{2\in_s (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(2.4744)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.566\times10^{-4} \text{ cm} = 0.566 \,\mu\text{m}$$

Then

$$|E_{max}| = \frac{eN_d x_d}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(10^{16})(0.566\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$$|E_{max}| = 8.745\times10^4 \text{ V/cm}$$

Now

$$\Delta\phi = \left[\frac{(1.6\times10^{-19})(8.745\times10^4)}{4\pi(11.7)(8.85\times10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0328 \text{ V}$$

Then

$$J_{ST1} = (4.277\times10^{-5})\exp\left(\frac{0.0328}{0.0259}\right)$$

or

$$J_{ST1} = 1.52\times10^{-4} \text{ A/cm}^2$$

For $A = 10^{-4}$ cm^2, we find

$$I_{R1} = 1.52\times10^{-8} \text{ A}$$

(b) For $V_R = 4$ V, then

$$x_d = \left[\frac{2(11.7)(8.85\times10^{-14})(4.4744)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.761\times10^{-4} \text{ cm} = 0.761 \,\mu\text{m}$$

Also

$$|E_{max}| = \frac{(1.6\times10^{-19})(10^{16})(0.761\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$$|E_{max}| = 1.176\times10^5 \text{ V/cm}$$

and

$$\Delta\phi = \left[\frac{(1.6\times10^{-19})(1.176\times10^5)}{4\pi(11.7)(8.85\times10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.03803 \text{ V}$$

Then

$$J_{ST2} = (4.277\times10^{-5})\exp\left(\frac{0.03803}{0.0259}\right)$$

or

$$J_{ST2} = 1.86\times10^{-4} \text{ A/cm}^2$$

Finally,

$$I_{R2} = 1.86\times10^{-8} \text{ A}$$

_____

9.19

We have that

$$J_{s\to m}^- = \int_{E_c}^{\infty} \upsilon_x \, dn$$

The incremental electron concentration is

$$dn = g_c(E) f_F(E) dE$$

where

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

and assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E - E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

$$\times \exp\left[\frac{-(E - E_F)}{kT}\right]dE$$

If the energy above E_c is kinetic energy, then

$$\frac{1}{2}m_n^* \upsilon^2 = E - E_c$$

We can then write

$$\sqrt{E - E_c} = \upsilon\sqrt{\frac{m_n^*}{2}}$$

and

_____

$$dE = \frac{1}{2}m_n^* \cdot 2\upsilon d\upsilon = m_n^* \upsilon d\upsilon$$

We can also write

$$E - E_F = (E - E_c) + (E_c - E_F)$$

$$= \frac{1}{2}m_n^* \upsilon^2 + e\phi_n$$

so that

$$dn = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)$$

$$\times \exp\left(\frac{-m_n^* \upsilon^2}{2kT}\right) \cdot 4\pi\upsilon^2 d\upsilon$$

We can write

$$\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2$$

The differential volume element is

$$4\pi\upsilon^2 d\upsilon = d\upsilon_x d\upsilon_y d\upsilon_z$$

The current is due to all x-directed velocities that are greater than υ_{Ox} and for all y- and z- directed velocities. Then

$$J_{s\to m}^- = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)$$

$$\times \int_{\upsilon_{Ox}}^{\infty} \upsilon_x \exp\left(\frac{-m_n^* \upsilon_x^2}{2kT}\right) d\upsilon_x$$

$$\times \int_{-\infty}^{\infty} \exp\left(\frac{-m_n^* \upsilon_y^2}{2kT}\right) d\upsilon_y$$

$$\times \int_{-\infty}^{\infty} \exp\left(\frac{-m_n^* \upsilon_z^2}{2kT}\right) d\upsilon_z$$

We can write

$$\frac{1}{2}m_n^* \upsilon_{Ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^* \upsilon_x^2}{2kT} = \alpha^2 + \frac{2(V_{bi} - V_a)}{kT}$$

or

$$\upsilon_x^2 = \frac{2kT}{m_n^*}\left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT}\right]$$

Taking the differential, we find

$$\upsilon_x d\upsilon_x = \left(\frac{2kT}{m_n^*}\right)\alpha d\alpha$$

We may note that when $\upsilon_x = \upsilon_{Ox}$, $\alpha = 0$.
We may define other change of variables,

$$\frac{m_n^* \upsilon_y^2}{2kT} = \beta^2 \Rightarrow \upsilon_y = \left(\frac{2kT}{m_n^*}\right)^{1/2} \cdot \beta$$

$$\frac{m_n^* \upsilon_z^2}{2kT} = \gamma^2 \Rightarrow \upsilon_z = \left(\frac{2kT}{m_n^*}\right)^{1/2} \cdot \gamma$$

Substituting the new variables, we have

$$J_{s\to m}^- = 2\left(\frac{m_n^*}{h}\right)^3 \cdot \left(\frac{2kT}{m_n^*}\right)^2 \cdot \exp\left(\frac{-e\phi_n}{kT}\right)$$

$$\times \exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right] \cdot \int_0^{\infty} \alpha \exp(-\alpha^2) d\alpha$$

$$\times \int_{-\infty}^{\infty} \exp(-\beta^2) d\beta \cdot \int_{-\infty}^{\infty} \exp(-\gamma^2) d\gamma$$

_____

9.20

For the Schottky diode,

$$0.80 \times 10^{-3} = (10^{-4})(6 \times 10^{-8})\exp\left(\frac{V_a}{V_t}\right)$$

(a) $V_a(SB) = (0.0259)\ln\left[\dfrac{0.80 \times 10^{-3}}{(10^{-4})(6 \times 10^{-8})}\right]$

$$= 0.4845 \text{ V}$$

Then

$$V_a(pn) = 0.4845 + 0.285 = 0.7695 \text{ V}$$

(b) $0.80 \times 10^{-3} = A_{pn}(10^{-11})\exp\left(\dfrac{0.7695}{0.0259}\right)$

$$\Rightarrow A_{pn} = 0.998 \times 10^{-5} \cong 10^{-5} \text{ cm}^2$$

_____

9.21

For the pn junction,

$$I_s = (8 \times 10^{-4})(8 \times 10^{-13}) = 6.4 \times 10^{-16} \text{ A}$$

(a) $V_a = (0.0259)\ln\left(\dfrac{150 \times 10^{-6}}{6.4 \times 10^{-16}}\right)$

$$= 0.678 \text{ V}$$

(b) $V_a = (0.0259)\ln\left(\dfrac{700 \times 10^{-6}}{6.4 \times 10^{-16}}\right)$

$$= 0.718 \text{ V}$$

(c) $V_a = (0.0259)\ln\left(\dfrac{1.2 \times 10^{-3}}{6.4 \times 10^{-16}}\right)$

$$= 0.732 \text{ V}$$

For the Schottky junction,
$$I_{sT} = (8 \times 10^{-4})(6 \times 10^{-9}) = 4.8 \times 10^{-12} \text{ A}$$

(a) $V_a = (0.0259) \ln \left(\dfrac{150 \times 10^{-6}}{4.8 \times 10^{-12}} \right)$

$\qquad = 0.447 \text{ V}$

(b) $V_a = (0.0259) \ln \left(\dfrac{700 \times 10^{-6}}{4.8 \times 10^{-12}} \right)$

$\qquad = 0.487 \text{ V}$

(c) $V_a = (0.0259) \ln \left(\dfrac{1.2 \times 10^{-3}}{4.8 \times 10^{-12}} \right)$

$\qquad = 0.501 \text{ V}$

_____

9.22

(a) (i) $I = 0.80 \text{ mA}$ in each diode

(ii)

$$V_a(SB) = (0.0259) \ln \left[\frac{0.8 \times 10^{-3}}{(8 \times 10^{-4})(6 \times 10^{-9})} \right]$$

$\qquad = 0.490 \text{ V}$

$$V_a(pn) = (0.0259) \ln \left[\frac{0.8 \times 10^{-3}}{(8 \times 10^{-4})(8 \times 10^{-13})} \right]$$

$\qquad = 0.721 \text{ V}$

(b) Same voltage across each diode

$I = 0.8 \times 10^{-3} = I_{SB} + I_{pn}$

$\qquad = (8 \times 10^{-4})(6 \times 10^{-9}) \exp \left(\dfrac{V_a}{V_t} \right)$

$\qquad + (8 \times 10^{-4})(8 \times 10^{-13}) \exp \left(\dfrac{V_a}{V_t} \right)$

$\qquad = (4.8 \times 10^{-12} + 6.4 \times 10^{-16}) \exp \left(\dfrac{V_a}{V_t} \right)$

Then

$$V_a = (0.0259) \ln \left[\frac{0.8 \times 10^{-3}}{4.8 \times 10^{-12} + 6.4 \times 10^{-16}} \right]$$

$V_a = 0.49032 \text{ V}$

$I_{SB} = (4.8 \times 10^{-12}) \exp \left(\dfrac{0.49032}{0.0259} \right)$

$\Rightarrow I_{SB} = 0.7998 \text{ mA}$

$I_{pn} = (6.4 \times 10^{-16}) \exp \left(\dfrac{0.49032}{0.0259} \right)$

$\Rightarrow I_{pn} \cong 0.107 \, \mu \text{A}$

_____

9.23

(a) For $I = 0.8 \text{ mA}$, we find

$$J = \frac{0.8 \times 10^{-3}}{7 \times 10^{-4}} = 1.143 \text{ A/cm}^2$$

We have

$$V_a = V_t \ln \left(\frac{J}{J_S} \right)$$

For the pn junction diode,

$$V_a = (0.0259) \ln \left(\frac{1.143}{3 \times 10^{-12}} \right) = 0.6907 \text{ V}$$

For the Schottky diode,

$$V_a = (0.0259) \ln \left(\frac{1.143}{4 \times 10^{-8}} \right) = 0.4447 \text{ V}$$

(b) For the pn junction diode,

$$J_S \propto n_i^2 \propto \left(\frac{T}{300} \right)^3 \exp \left(\frac{-E_g}{kT} \right)$$

Then

$$\frac{J_S(400)}{J_S(300)} = \left(\frac{400}{300} \right)^3$$

$$\times \exp \left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259} \right]$$

$$= 2.37 \exp \left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453} \right]$$

or

$$\frac{J_S(400)}{J_S(300)} = 1.17 \times 10^5$$

Now

$$I = (7 \times 10^{-4})(1.17 \times 10^5)(3 \times 10^{-12})$$

$$\times \exp \left(\frac{0.6907}{0.03453} \right)$$

or

$I = 120 \text{ mA}$

For the Schottky diode,

$$J_{ST} \propto T^2 \exp \left(\frac{-e\phi_{BO}}{kT} \right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)} = \left(\frac{400}{300} \right)^2$$

$$\times \exp \left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259} \right]$$

$$= 1.778 \exp \left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453} \right]$$

or

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.856 \times 10^3$$

Then

$$I = \left(7 \times 10^{-4}\right)\left(4.856 \times 10^3\right)\left(4 \times 10^{-8}\right)$$
$$\times \exp\left(\frac{0.4447}{0.03453}\right)$$

or

$$I = 53.3 \, \text{mA}$$

_____

9.24

Plot

_____

9.25

(a) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-3}} = 0.1\,\Omega$

(b) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-4}} = 1\,\Omega$

(c) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-5}} = 10\,\Omega$

_____

9.26

(a) $R = \dfrac{R_c}{A} = \dfrac{5 \times 10^{-5}}{10^{-5}} = 5\,\Omega$

(i) $V = IR = (1)(5) = 5\,\text{mV}$

(ii) $V = IR = (0.1)(5) = 0.5\,\text{mV}$

(b) $R = \dfrac{5 \times 10^{-5}}{10^{-6}} = 50\,\Omega$

(i) $V = IR = (1)(50) = 50\,\text{mV}$

(ii) $V = IR = (0.1)(50) = 5\,\text{mV}$

_____

9.27

$$R_c = \frac{V_t \exp\left(\dfrac{\phi_{Bn}}{V_t}\right)}{A^* T^2}$$

or $\phi_{Bn} = V_t \ln\left[\dfrac{R_c A^* T^2}{V_t}\right]$

(a) $\phi_{Bn} = (0.0259)\ln\left[\dfrac{\left(5 \times 10^{-5}\right)(120)(300)^2}{0.0259}\right]$

$$= 0.258 \, \text{V}$$

(b) $\phi_{Bn} = (0.0259)\ln\left[\dfrac{\left(5 \times 10^{-6}\right)(120)(300)^2}{0.0259}\right]$

$$= 0.198 \, \text{V}$$

_____

9.28

(b) We need $\phi_n = \phi_m - \chi = 4.2 - 4.0 = 0.20 \, \text{V}$

And

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

or

$$0.20 = (0.0259)\ln\left(\frac{2.8 \times 10^{19}}{N_d}\right)$$

which yields

$$N_d = 1.24 \times 10^{16} \, \text{cm}^{-3}$$

(c)

Barrier height = 0.20 V

_____

9.29

We have that

$$\text{E} = \frac{-eN_d}{\in_s}\left(x_n - x\right)$$

Then

$$\phi = -\int \text{E}dx = \frac{eN_d}{\in_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$, so

$$\phi = \frac{eN_d}{\in_s}\left(x_n \cdot x - \frac{x^2}{2}\right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\in_s} \cdot \frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2 \in_s V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

Now for

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35 \, \text{V}$$

we have

$$0.35 = \frac{(1.6\times10^{-19})N_d}{(11.7)(8.85\times10^{-14})}\left[x_n(50\times10^{-8})\right.$$
$$\left.-\frac{(50\times10^{-8})^2}{2}\right]$$

or

$$0.35 = 7.73\times10^{-14}\,N_d\left(x_n - 25\times10^{-8}\right)$$

We have

$$x_n = \left[\frac{2(11.7)(8.85\times10^{-14})V_{bi}}{(1.6\times10^{-19})N_d}\right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error, we find

$$N_d = 3.5\times10^{18}\,\text{cm}^{-3}$$

_____

9.30

(b) $\phi_{BO} = \phi_p = V_t\,\ln\left(\dfrac{N_v}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{16}}\right)$$

or

$$\phi_{BO} = 0.138\ \text{V}$$

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9.31

Sketches

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9.32

Sketches

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9.33

Electron affinity rule

$$\Delta E_c = e(\chi_n - \chi_p)$$

For GaAs, $\chi = 4.07$ and for AlAs, $\chi = 3.5$.
If we assume a linear extrapolation between
GaAs and AlAs, then for

$$\text{Al}_{0.3}\,\text{Ga}_{0.7}\,\text{As} \Rightarrow \chi = 3.90$$

Then

$$\left|\Delta E_c\right| = 4.07 - 3.90 = 0.17\ \text{eV}$$

_____

9.34

Consider an n-P heterojunction in thermal
equilibrium. Poisson's equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\in} = -\frac{d\text{E}}{dx}$$

In the n-region,

$$\frac{d\text{E}_n}{dx} = \frac{\rho(x)}{\in_n} = \frac{eN_{dn}}{\in_n}$$

For uniform doping, we have

$$\text{E}_n = \frac{eN_{dn}x}{\in_n} + C_1$$

The boundary condition is
$\text{E}_n = 0$ at $x = -x_n$, so we obatin

$$C_1 = \frac{eN_{dn}x_n}{\in_n}$$

Then

$$\text{E}_n = \frac{eN_{dn}}{\in_n}(x + x_n)$$

In the P-region,

$$\frac{d\text{E}_p}{dx} = -\frac{eN_{aP}}{\in_P}$$

which gives

$$\text{E}_P = -\frac{eN_{aP}x}{\in_P} + C_2$$

We have the boundary condition that
$\text{E}_P = 0$ at $x = x_P$, so that

$$C_2 = \frac{eN_{aP}x_P}{\in_P}$$

Then

$$\text{E}_P = \frac{eN_{aP}}{\in_P}(x_P - x)$$

Assuming zero surface charge density at
$x = 0$, the electric flux density D is
continuous, so $\in_n \text{E}_n(0) = \in_P \text{E}_P(0)$, which
yields

$$N_{dn}x_n = N_{aP}x_P$$

We can determine the electric potential as

$$\phi_n(x) = -\int \text{E}_n\,dx$$

$$= -\left[\frac{eN_{dn}x^2}{2\in_n} + \frac{eN_{dn}x_n x}{\in_n}\right] + C_3$$

Now

$$V_{bin} = \left| \phi_n(0) - \phi_n(-x_n) \right|$$

$$= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2 \in_n} + \frac{eN_{dn}x_n^2}{\in_n} \right]$$

or

$$V_{bin} = \frac{eN_{dn}x_n^2}{2 \in_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_P^2}{2 \in_P}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2 \in_n} + \frac{eN_{aP}x_P^2}{2 \in_P}$$

We can write

$$x_P = x_n \left(\frac{N_{dn}}{N_{aP}} \right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e \in_P N_{dn}N_{aP} + e \in_n N_{dn}^2}{2 \in_n \in_P N_{aP}} \right] \cdot x_n^2$$

Solving for x_n, we have

$$x_n = \left[\frac{2 \in_n \in_P N_{aP}V_{bi}}{eN_{dn}(\in_P N_{aP} + \in_n N_{dn})} \right]^{1/2}$$

Similarly on the P-side, we have

$$x_P = \left[\frac{2 \in_n \in_P N_{dn}V_{bi}}{eN_{aP}(\in_P N_{aP} + \in_n N_{dn})} \right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_P$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2 \in_n \in_P V_{bi}(N_{aP} + N_{dn})}{eN_{dn}N_{aP}(\in_P N_{aP} + \in_n N_{dn})} \right]^{1/2}$$

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Chapter 10

10.1
(a) p-type; inversion
(b) p-type; depletion
(c) p-type; accumulation
(d) n-type; inversion

_____

10.2

(a) (i) $\phi_{fp} = V_t \ln\left(\dfrac{N_a}{n_i}\right)$

$= (0.0259)\ln\left(\dfrac{7\times10^{15}}{1.5\times10^{10}}\right)$

$= 0.3381\,\text{V}$

$x_{dT} = \left[\dfrac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$

$= \left[\dfrac{4(11.7)(8.85\times10^{-14})(0.3381)}{(1.6\times10^{-19})(7\times10^{15})}\right]^{1/2}$

$= 3.54\times10^{-5}\,\text{cm}$

or $x_{dT} = 0.354\,\mu\text{m}$

(ii) $\phi_{fp} = (0.0259)\ln\left(\dfrac{3\times10^{16}}{1.5\times10^{10}}\right)$

$= 0.3758\,\text{V}$

$x_{dT} = \left[\dfrac{4(11.7)(8.85\times10^{-14})(0.3758)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$= 1.80\times10^{-5}\,\text{cm}$

or $x_{dT} = 0.180\,\mu\text{m}$

(b) $kT = (0.0259)\left(\dfrac{350}{300}\right) = 0.03022\,\text{V}$

$n_i^2 = N_c N_\upsilon \exp\left(\dfrac{-E_g}{kT}\right)$

$= (2.8\times10^{19})(1.04\times10^{19})\left(\dfrac{350}{300}\right)^3$

$\times \exp\left(\dfrac{-1.12}{0.03022}\right)$

$= 3.71\times10^{22}$

so $n_i = 1.93\times10^{11}\,\text{cm}^{-3}$

(i) $\phi_{fp} = (0.03022)\ln\left(\dfrac{7\times10^{15}}{1.93\times10^{11}}\right)$

$= 0.3173\,\text{V}$

$x_{dT} = \left[\dfrac{4(11.7)(8.85\times10^{-14})(0.3173)}{(1.6\times10^{-19})(7\times10^{15})}\right]^{1/2}$

$= 3.43\times10^{-5}\,\text{cm}$

or $x_{dT} = 0.343\,\mu\text{m}$

(ii) $\phi_{fp} = (0.03022)\ln\left(\dfrac{3\times10^{16}}{1.93\times10^{11}}\right)$

$= 0.3613\,\text{V}$

$x_{dT} = \left[\dfrac{4(11.7)(8.85\times10^{-14})(0.3613)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$= 1.77\times10^{-5}\,\text{cm}$

or $x_{dT} = 0.177\,\mu\text{m}$

_____

10.3

(a) $|Q'_{SD}(\max)| = eN_d x_{dT} = eN_d\left[\dfrac{4\in_s \phi_{fn}}{eN_d}\right]^{1/2}$

$= [(eN_d)(4\in_s \phi_{fn})]^{1/2}$

1st approximation: Let $\phi_{fn} = 0.30\,\text{V}$

Then

$(1.25\times10^{-8})^2$
$= [(1.6\times10^{-19})(N_d)(4)(11.7)(8.85\times10^{-14})(0.30)]$

$\Rightarrow N_d = 7.86\times10^{14}\,\text{cm}^{-3}$

2nd approximation:

$\phi_{fn} = (0.0259)\ln\left(\dfrac{7.86\times10^{14}}{1.5\times10^{10}}\right) = 0.2814\,\text{V}$

Then

$(1.25\times10^{-8})^2$
$= [(1.6\times10^{-19})(N_d)(4)(11.7)(8.85\times10^{-14})(0.2814)]$

$\Rightarrow N_d = 8.38\times10^{14}\,\text{cm}^{-3}$

(b) $\phi_{fn} = (0.0259)\ln\left(\dfrac{8.38\times10^{14}}{1.5\times10^{10}}\right) = 0.2831\,\text{V}$

$\phi_s = 2\phi_{fn} = 2(0.2831) = 0.566\,\text{V}$

_____

10.4

p-type silicon

(a) Aluminum gate

$$\phi_{ms} = \left[\phi_m' - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right) \right]$$

We have

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i} \right)$$

$$= (0.0259)\ln\left(\frac{6 \times 10^{15}}{1.5 \times 10^{10}} \right) = 0.334 \text{ V}$$

Then

$$\phi_{ms} = [3.20 - (3.25 + 0.56 + 0.334)]$$

or

$$\phi_{ms} = -0.944 \text{ V}$$

(b) n^+ polysilicon gate

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp} \right) = -(0.56 + 0.334)$$

or

$$\phi_{ms} = -0.894 \text{ V}$$

(c) p^+ polysilicon gate

$$\phi_{ms} = \left(\frac{E_g}{2e} - \phi_{fp} \right) = (0.56 - 0.334)$$

or

$$\phi_{ms} = +0.226 \text{ V}$$

_____

10.5

$$\phi_{fp} = (0.0259)\ln\left(\frac{4 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3832 \text{ V}$$

$$\phi_{ms} = \phi_m' - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right)$$

$$= 3.20 - (3.25 + 0.56 + 0.3832)$$

$$\phi_{ms} = -0.9932 \text{ V}$$

_____

10.6

(a) $N_d \cong 2 \times 10^{17} \text{ cm}^{-3}$

(b) Not possible - ϕ_{ms} is always positive.

(c) $N_d \cong 2 \times 10^{15} \text{ cm}^{-3}$

_____

10.7

From Problem 10.5, $\phi_{ms} = -0.9932$ V

$$V_{FB} = \phi_{ms} - \frac{Q_{ss}'}{C_{ox}}$$

(a) $C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$

$$= 1.726 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = -0.9932 - \frac{(5 \times 10^{10})(1.6 \times 10^{-19})}{1.726 \times 10^{-7}}$$

$$= -1.040 \text{ V}$$

(b) $C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{80 \times 10^{-8}}$

$$= 4.314 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = -0.9932 - \frac{(5 \times 10^{10})(1.6 \times 10^{-19})}{4.314 \times 10^{-7}}$$

$$= -1.012 \text{ V}$$

_____

10.8

(a) $\phi_{ms} \cong -0.42$ V

$$V_{FB} = \phi_{ms} = -0.42 \text{ V}$$

(b)

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}} = 1.726 \times 10^{-7} \text{ F/cm}^2$$

(i) $\Delta V_{FB} = -\frac{Q_{ss}'}{C_{ox}} = -\frac{(4 \times 10^{10})(1.6 \times 10^{-19})}{1.726 \times 10^{-7}}$

$$= -0.0371 \text{ V}$$

(ii) $\Delta V_{FB} = -\frac{(10^{11})(1.6 \times 10^{-19})}{1.726 \times 10^{-7}}$

$$= -0.0927 \text{ V}$$

(c) $V_{FB} = \phi_{ms} = -0.42$ V

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{120 \times 10^{-8}} = 2.876 \times 10^{-7} \text{ F/cm}^2$$

(i) $\Delta V_{FB} = -\frac{(4 \times 10^{10})(1.6 \times 10^{-19})}{2.876 \times 10^{-7}}$

$$= -0.0223 \text{ V}$$

(ii) $\Delta V_{FB} = -\frac{(10^{11})(1.6 \times 10^{-19})}{2.876 \times 10^{-7}}$

$$= -0.0556 \text{ V}$$

_____

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10.9

$$\phi_{ms} = \phi_m' - \left(\chi' + \frac{E_g}{2e} + \phi_{fp} \right)$$

where

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}} \right) = 0.365 \text{ V}$$

Then

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.365)$$

or

$$\phi_{ms} = -0.975 \text{ V}$$

Now

$$V_{FB} = \phi_{ms} - \frac{Q_{ss}'}{C_{ox}}$$

or

$$Q_{ss}' = (\phi_{ms} - V_{FB})C_{ox}$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{450\times10^{-8}}$$

or

$$C_{ox} = 7.67\times10^{-8} \text{ F/cm}^2$$

So now

$$Q_{ss}' = [-0.975 - (-1)]\cdot(7.67\times10^{-8})$$
$$= 1.92\times10^{-9} \text{ C/cm}^2$$

or

$$\frac{Q_{ss}'}{e} = 1.2\times10^{10} \text{ cm}^{-2}$$

_____

10.10

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}} \right) = 0.3653 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3653)}{(1.6\times10^{-19})(2\times10^{16})} \right]^{1/2}$$
$$= 2.174\times10^{-5} \text{ cm}$$

$$|Q_{SD}'(\max)| = eN_a x_{dT}$$
$$= (1.6\times10^{-19})(2\times10^{16})(2.174\times10^{-5})$$
$$= 6.958\times10^{-8} \text{ C/cm}^2$$

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}} = 2.301\times10^{-7} \text{ F/cm}^2$$

_____

$$V_{TN} = \frac{|Q_{SD}'(\max)| - Q_{ss}'}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{6.958\times10^{-8} - (7\times10^{10})(1.6\times10^{-19})}{2.301\times10^{-7}}$$
$$\qquad + \phi_{ms} + 2(0.3653)$$

$$= 0.9843 + \phi_{ms}$$

(a) n^+ poly gate on p-type: $\phi_{ms} \cong -1.12$ V

$$V_{TN} = 0.9843 - 1.12 = -0.136 \text{ V}$$

(b) p^+ poly gate on p-type: $\phi_{ms} \cong +0.28$ V

$$V_{TN} = 0.9843 + 0.28 = +1.26 \text{ V}$$

(c) Al gate on p-type: $\phi_{ms} \cong -0.95$ V

$$V_{TN} = 0.9843 - 0.95 = +0.0343 \text{ V}$$

_____

10.11

$$\phi_{fn} = (0.0259)\ln\left(\frac{3\times10^{15}}{1.5\times10^{10}} \right) = 0.3161 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3161)}{(1.6\times10^{-19})(3\times10^{15})} \right]^{1/2}$$
$$= 5.223\times10^{-5} \text{ cm}$$

$$|Q_{SD}'(\max)| = eN_d x_{dT}$$
$$= (1.6\times10^{-19})(3\times10^{15})(5.223\times10^{-5})$$
$$= 2.507\times10^{-8} \text{ C/cm}^2$$

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}} = 2.301\times10^{-7} \text{ F/cm}^2$$

$$V_{TP} = -\left[\frac{|Q_{SD}'(\max)| + Q_{ss}'}{C_{ox}} \right] + \phi_{ms} - 2\phi_{fn}$$

$$= -\left[\frac{2.507\times10^{-8} + (1.6\times10^{-19})(7\times10^{10})}{2.301\times10^{-7}} \right]$$
$$\qquad + \phi_{ms} - 2(0.3161)$$

$$V_{TP} = -0.7898 + \phi_{ms}$$

(a) n^+ poly gate on n-type: $\phi_{ms} \cong -0.41$ V

$$V_{TP} = -0.7898 - 0.41 = -1.20 \text{ V}$$

(b) p^+ poly gate on n-type: $\phi_{ms} \cong +1.0$ V

$$V_{TP} = -0.7898 + 1.0 = +0.210 \text{ V}$$

(c) Al gate on n-type: $\phi_{ms} \cong -0.29$ V

$$V_{TP} = -0.7898 - 0.29 = -1.08 \text{ V}$$

_____

10.12

$$\phi_{fp} = (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right) = 0.3294 \text{ V}$$

The surface potential is
$$\phi_s = 2\phi_{fp} = 2(0.3294) = 0.659 \text{ V}$$

We have
$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} = -0.90 \text{ V}$$

Now
$$V_T = \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + \phi_s + V_{FB}$$

We obtain
$$x_{dT} = \left[\frac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$$
$$= \left[\frac{4(11.7)(8.85\times10^{-14})(0.3294)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$$

or
$$x_{dT} = 0.413\times10^{-4} \text{ cm}$$

Then
$$|Q'_{SD}(\text{max})| = (1.6\times10^{-19})(5\times10^{15})(0.413\times10^{-4})$$

or
$$|Q'_{SD}(\text{max})| = 3.304\times10^{-8} \text{ C/cm}^2$$

We also find
$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{400\times10^{-8}}$$

or
$$C_{ox} = 8.629\times10^{-8} \text{ F/cm}^2$$

Then
$$V_T = \frac{3.304\times10^{-8}}{8.629\times10^{-8}} + 0.659 - 0.90$$

or
$$V_T = +0.142 \text{ V}$$

_____

10.13

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{220\times10^{-8}}$$
$$= 1.569\times10^{-7} \text{ F/cm}^2$$
$$Q'_{ss} = (1.6\times10^{-19})(4\times10^{10})$$
$$= 6.4\times10^{-9} \text{ C/cm}^2$$

By trial and error, let $N_a = 4\times10^{16} \text{ cm}^{-3}$.

Now
$$\phi_{fp} = (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right)$$
$$= 0.3832 \text{ V}$$
$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3832)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$
$$= 1.575\times10^{-5} \text{ cm}$$
$$|Q'_{SD}(\text{max})|$$
$$= (1.6\times10^{-19})(4\times10^{16})(1.575\times10^{-5})$$
$$= 1.008\times10^{-7} \text{ C/cm}^2$$
$$\phi_{ms} \cong -0.94 \text{ V}$$

Then
$$V_{TN} = \frac{|Q'_{SD}(\text{max})| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$
$$= \frac{1.008\times10^{-7} - 6.4\times10^{-9}}{1.569\times10^{-7}}$$
$$- 0.94 + 2(0.3832)$$

Then $V_{TN} = 0.428 \text{ V} \cong 0.45 \text{ V}$

_____

10.14

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$
$$= 1.9175\times10^{-7} \text{ F/cm}^{-3}$$
$$Q'_{ss} = (1.6\times10^{-19})(4\times10^{10})$$
$$= 6.4\times10^{-9} \text{ C/cm}^2$$

By trial and error, let $N_d = 5\times10^{16} \text{ cm}^{-3}$

Now
$$\phi_{fn} = (0.0259)\ln\left(\frac{5\times10^{16}}{1.5\times10^{10}}\right)$$
$$= 0.3890 \text{ V}$$
$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3890)}{(1.6\times10^{-19})(5\times10^{16})}\right]^{1/2}$$
$$= 1.419\times10^{-5} \text{ cm}$$
$$|Q'_{SD}(\text{max})|$$
$$= (1.6\times10^{-19})(5\times10^{16})(1.419\times10^{-5})$$
$$= 1.135\times10^{-7} \text{ C/cm}^{-3}$$
$$\phi_{ms} \cong +1.10 \text{ V}$$

Then

_____

$$V_{TP} = -\frac{\left(\left|Q'_{SD}(\max)\right| + Q'_{ss}\right)}{C_{ox}} + \phi_{ms} - 2\phi_{fn}$$

$$= -\frac{\left(1.135\times10^{-7} + 6.4\times10^{-9}\right)}{1.9175\times10^{-7}}$$
$$+ 1.10 - 2(0.3890)$$

Then $V_{TP} = -0.303$ V, which is within the specified value.

_____

10.15

We have $C_{ox} = 1.569\times10^{-7}$ F/cm^2

$$Q'_{ss} = 6.4\times10^{-9} \text{ C/cm}^2$$

By trial and error, let $N_d = 5\times10^{14}$ cm^{-3}
Now

$$\phi_{fn} = (0.0259)\ln\left(\frac{5\times10^{14}}{1.5\times10^{10}}\right)$$
$$= 0.2697 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.2697)}{(1.6\times10^{-19})(5\times10^{14})}\right]^{1/2}$$
$$= 1.182\times10^{-4} \text{ cm}$$

$$\left|Q'_{SD}(\max)\right|$$
$$= (1.6\times10^{-19})(5\times10^{14})(1.182\times10^{-4})$$
$$= 9.456\times10^{-9} \text{ C/cm}^2$$

$$\phi_{ms} \cong -0.33 \text{ V}$$

Then

$$V_{TP} = -\frac{\left(\left|Q'_{SD}(\max)\right| + Q'_{ss}\right)}{C_{ox}} + \phi_{ms} - 2\phi_{fn}$$

$$= -\left(\frac{9.456\times10^{-9} + 6.4\times10^{-9}}{1.569\times10^{-7}}\right)$$
$$- 0.33 - 2(0.2697)$$

$$= 0.970 \text{ V}$$

Then $V_{TP} = -0.970 \text{ V} \cong -0.975$ V which meets the specification.

_____

10.16

(a) $\phi_{ms} \cong -1.03$ V

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$
$$= 1.9175\times10^{-7} \text{ F/cm}^2$$

Now

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

$$= -1.03 - \frac{(1.6\times10^{-19})(6\times10^{10})}{1.9175\times10^{-7}}$$

$$V_{FB} = -1.08 \text{ V}$$

(b) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$

$$= 0.2877 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.2877)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2}$$

$$= 8.630\times10^{-5} \text{ cm}$$

$$\left|Q'_{SD}(\max)\right|$$
$$= (1.6\times10^{-19})(10^{15})(8.630\times10^{-5})$$
$$= 1.381\times10^{-8} \text{ C/cm}^2$$

Now

$$V_{TN} = \frac{\left|Q'_{SD}(\max)\right|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

$$= \frac{1.381\times10^{-8}}{1.9175\times10^{-7}} - 1.08 + 2(0.2877)$$

or $V_{TN} = -0.433$ V

_____

10.17

(a) We have n-type material under the gate, so

$$x_{dT} = t_C = \left[\frac{4\in_s \phi_{fn}}{eN_d}\right]^{1/2}$$

where

$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = 0.288 \text{ V}$$

Then

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.288)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2}$$

or

$$x_{dT} = t_C = 0.863\times10^{-4} \text{ cm} = 0.863\,\mu\text{m}$$

(b)

$$V_T = -\left(\left|Q'_{SD}(\max)\right| + Q'_{ss}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

For an n^+ polysilicon gate,

$$\phi_{ms} = -\left(\frac{E_g}{2e} - \phi_{fn}\right) = -(0.56 - 0.288)$$

or

$$\phi_{ms} = -0.272 \text{ V}$$

Now

$$\left| Q'_{SD}(\max) \right| = \left(1.6 \times 10^{-19}\right)\left(10^{15}\right)\left(0.863 \times 10^{-4}\right)$$

or

$$\left| Q'_{SD}(\max) \right| = 1.38 \times 10^{-8} \text{ C/cm}^2$$

We have

$$Q'_{ss} = \left(1.6 \times 10^{-19}\right)\left(10^{10}\right) = 1.6 \times 10^{-9} \text{ C/cm}^2$$

We now find

$$V_T = \frac{-\left(1.38 \times 10^{-8} + 1.6 \times 10^{-9}\right)}{(3.9)\left(8.85 \times 10^{-14}\right)}\left(500 \times 10^{-8}\right)$$
$$-0.272 - 2(0.288)$$

or

$$V_T = -1.07 \text{ V}$$

_____

10.18

(b) $\phi_{ms} = \phi'_m - \left(\chi' + \dfrac{E_g}{2e} + \phi_{fp}\right)$

where

$$\phi'_m - \chi' = -0.20 \text{ V}$$

and

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right) = 0.3473 \text{ V}$$

Then

$$\phi_{ms} = -0.20 - (0.56 + 0.3473)$$

or

$$\phi_{ms} = -1.107 \text{ V}$$

(c) For $Q'_{ss} = 0$

$$V_{TN} = \left| Q'_{SD}(\max) \right|\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

We find

$$x_{dT} = \left[\frac{4(11.7)\left(8.85 \times 10^{-14}\right)(0.3473)}{\left(1.6 \times 10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.30 \times 10^{-4} \text{ cm} = 0.30 \,\mu\text{m}$$

Now

$$\left| Q'_{SD}(\max) \right| = \left(1.6 \times 10^{-19}\right)\left(10^{16}\right)\left(0.30 \times 10^{-4}\right)$$

or

$$\left| Q'_{SD}(\max) \right| = 4.797 \times 10^{-8} \text{ C/cm}^2$$

Then

$$V_T = \frac{\left(4.797 \times 10^{-8}\right)\left(300 \times 10^{-8}\right)}{(3.9)\left(8.85 \times 10^{-14}\right)}$$
$$-1.107 + 2(0.3473)$$

or

$$V_T = +0.00455 \text{ V} \cong 0 \text{ V}$$

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10.19

   Plot

_____

10.20

   Plot

_____

10.21

   Plot

_____

10.22

   Plot

_____

10.23

(a)   For $f = 1\,\text{Hz}$ (low freq),

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85 \times 10^{-14}\right)}{120 \times 10^{-8}}$$
$$= 2.876 \times 10^{-7} \text{ F/cm}^2$$

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right)\sqrt{\dfrac{V_t \in_s}{eN_a}}}$$

$$= \frac{(3.9)\left(8.85 \times 10^{-14}\right)}{120 \times 10^{-8} + \left(\dfrac{3.9}{11.7}\right)\sqrt{\dfrac{(0.0259)(11.7)\left(8.85 \times 10^{-14}\right)}{\left(1.6 \times 10^{-19}\right)\left(10^{16}\right)}}}$$

$$C'_{FB} = 1.346 \times 10^{-7} \text{ F/cm}^2$$

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right) \cdot x_{dT}}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right) = 0.3473 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)\left(8.85 \times 10^{-14}\right)(0.3473)}{\left(1.6 \times 10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

$$= 3.00 \times 10^{-5} \text{ cm}$$

_____

Then

$$C'_{min} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)(3.00\times10^{-5})}$$

$$= 3.083\times10^{-8}\,\text{F/cm}^2$$

$$C'(\text{inv}) = C_{ox} = 2.876\times10^{-7}\,\text{F/cm}^2$$

(b) $f = 1\,\text{MHz}$ (high freq),

$C_{ox} = 2.876\times10^{-7}\,\text{F/cm}^2$ (unchanged)

$C'_{FB} = 1.346\times10^{-7}\,\text{F/cm}^2$ (unchanged)

$C'_{min} = 3.083\times10^{-8}\,\text{F/cm}^2$ (unchanged)

$C'(\text{inv}) = C'_{min} = 3.083\times10^{-8}\,\text{F/cm}^2$

(c) $V_{FB} = \phi_{ms} \cong = -1.10\,\text{V}$

$$V_{TN} = \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

Now

$$|Q'_{SD}(\text{max})| = eN_a x_{dT}$$

$$= (1.6\times10^{-19})(10^{16})(3.00\times10^{-5})$$

$$= 4.80\times10^{-8}\,\text{C/cm}^2$$

$$V_{TN} = \frac{4.80\times10^{-8}}{2.876\times10^{-7}} - 1.10 + 2(0.3473)$$

$$V_{TN} = -0.2385\,\text{V}$$

_____

10.24

(a) $f = 1\,\text{Hz}$ (low freq),

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}}$$

$$= 2.876\times10^{-7}\,\text{F/cm}^2$$

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)\sqrt{\frac{V_t \epsilon_s}{eN_a}}}$$

$$= \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259)(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(5\times10^{14})}}}$$

$$C'_{FB} = 4.726\times10^{-8}\,\text{F/cm}^2$$

$$C'_{min} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)\cdot x_{dT}}$$

Now

$$\phi_{fn} = (0.0259)\ln\left(\frac{5\times10^{14}}{1.5\times10^{10}}\right) = 0.2697\,\text{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.2697)}{(1.6\times10^{-19})(5\times10^{14})}\right]^{1/2}$$

$$= 1.182\times10^{-4}\,\text{cm}$$

Then

$$C'_{min} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)(1.182\times10^{-4})}$$

$$= 8.504\times10^{-9}\,\text{F/cm}^2$$

$$C'(\text{inv}) = C_{ox} = 2.876\times10^{-7}\,\text{F/cm}^2$$

(b) $f = 1\,\text{MHz}$ (high freq),

$C_{ox} = 2.876\times10^{-7}\,\text{F/cm}^2$ (unchanged)

$C'_{FB} = 4.726\times10^{-8}\,\text{F/cm}^2$ (unchanged)

$C'_{min} = 8.504\times10^{-9}\,\text{F/cm}^2$ (unchanged)

$C'(\text{inv}) = C'_{min} = 8.504\times10^{-9}\,\text{F/cm}^2$

(c) $V_{FB} = \phi_{ms} \cong 0.95\,\text{V}$

$$V_{TP} = -\frac{|Q'_{SD}(\text{max})|}{C_{ox}} + V_{FB} - 2\phi_{fn}$$

Now

$$|Q'_{SD}(\text{max})| = eN_d x_{dT}$$

$$= (1.6\times10^{-19})(5\times10^{14})(1.182\times10^{-4})$$

$$= 9.456\times10^{-9}\,\text{C/cm}^2$$

Then

$$V_{TP} = -\frac{9.456\times10^{-9}}{2.876\times10^{-7}} + 0.95 - 2(0.2697)$$

$$V_{TP} = +0.378\,\text{V}$$

_____

10.25

The amount of fixed oxide charge at x is

$\rho(x)\Delta x$ C/cm^2

By lever action, the effect of this oxide charge on the flatband voltage is

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\left(\frac{x}{t_{ox}}\right)\rho(x)\Delta x$$

If we add the effect at each point, we must integrate so that

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}}\frac{x\rho(x)}{t_{ox}}dx$$

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10.26

(a) We have $\rho(x) = \dfrac{Q'_{SS}}{\Delta t}$

Then

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{\rho(x)}{t_{ox}} x \, dx$$

$$\approx -\frac{1}{C_{ox}} \int_{t_{ox}-\Delta t}^{t_{ox}} \left(\frac{x}{t_{ox}} \right) \left(\frac{Q'_{SS}}{\Delta t} \right) dx$$

$$= -\frac{1}{C_{ox}} \left[\frac{Q'_{SS}}{\Delta t} \right] \left[t_{ox} - \left(t_{ox} - \Delta t \right) \right] = -\frac{Q'_{SS}}{C_{ox}}$$

or

$$\Delta V_{FB} = -Q'_{SS} \left(\frac{t_{ox}}{\in_{ox}} \right)$$

$$= \frac{-\left(1.6 \times 10^{-19} \right)\left(10^{10} \right)\left(200 \times 10^{-8} \right)}{(3.9)\left(8.85 \times 10^{-14} \right)}$$

or

$$\Delta V_{FB} = -0.0742 \text{ V}$$

(b)
We have

$$\rho(x) = \frac{Q'_{SS}}{t_{ox}} = \frac{\left(1.6 \times 10^{-19} \right)\left(10^{10} \right)}{200 \times 10^{-8}}$$

$$= 6.4 \times 10^{-3} = \rho_o$$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{\rho(x)}{t_{ox}} x \, dx = -\frac{\rho_o}{C_{ox} t_{ox}} \int_0^{t_{ox}} x \, dx$$

or

$$\Delta V_{FB} = -\frac{\rho_o t_{ox}^2}{2 \in_{ox}}$$

$$= \frac{-\left(6.4 \times 10^{-3} \right)\left(200 \times 10^{-8} \right)}{2(3.9)\left(8.85 \times 10^{-14} \right)}$$

or

$$\Delta V_{FB} = -0.0371 \text{ V}$$

(c)

$$\rho(x) = \rho_o \left(\frac{x}{t_{ox}} \right)$$

We find

$$\frac{1}{2} t_{ox} \rho_o = Q'_{SS} \Rightarrow \rho_o = \frac{2\left(1.6 \times 10^{-19} \right)\left(10^{10} \right)}{200 \times 10^{-8}}$$

or $\rho_o = 1.28 \times 10^{-2}$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}} \int_0^{t_{ox}} \frac{1}{t_{ox}} \cdot x \cdot \rho_o \left(\frac{x}{t_{ox}} \right) dx$$

$$= -\frac{1}{C_{ox}} \cdot \frac{\rho_o}{t_{ox}^2} \int_0^{t_{ox}} x^2 \, dx$$

which becomes

$$\Delta V_{FB} = -\frac{1}{\left(\frac{\in_{ox}}{t_{ox}} \right)} \cdot \frac{\rho_o}{t_{ox}^2} \cdot \frac{x^3}{3} \Big|_0^{t_{ox}} = -\frac{\rho_o t_{ox}^2}{3 \in_{ox}}$$

Then

$$\Delta V_{FB} = \frac{-\left(1.28 \times 10^{-2} \right)\left(200 \times 10^{-8} \right)^2}{3(3.9)\left(8.85 \times 10^{-14} \right)}$$

or $\Delta V_{FB} = -0.0494 \text{ V}$

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10.27
Sketch

_____

10.28
Sketch

_____

10.29
(b)

$$V_{FB} = -V_{bi} = -V_t \ln\left(\frac{N_a N_d}{n_i^2} \right)$$

$$= -(0.0259)\ln\left[\frac{\left(10^{16} \right)\left(10^{16} \right)}{\left(1.5 \times 10^{10} \right)^2} \right]$$

or

$$V_{FB} = -0.695 \text{ V}$$

(c) Apply $V_G = -3 \text{ V}$, $|V_{ox}| \cong 3 \text{ V}$

For $V_G = +3 \text{ V}$,

$$\frac{d\text{E}}{dx} = -\frac{\rho}{\in_s}$$

n-side: $\rho = eN_d$

$$\frac{d\text{E}}{dx} = -\frac{eN_d}{\in_s} \Rightarrow \text{E} = -\frac{eN_d x}{\in_s} + C_1$$

_____

$E = 0$ at $x = -x_n$, then $C_1 = -\dfrac{eN_d x_n}{\epsilon_s}$

so

$E = -\dfrac{eN_d}{\epsilon_s}(x + x_n)$ for $-x_n \le x \le 0$

In the oxide, $\rho = 0$, so

$\dfrac{dE}{dx} = 0 \Rightarrow E = $ constant. From the

boundary conditions, in the oxide

$E = -\dfrac{eN_d x_n}{\epsilon_s}$

In the p-region,

$\dfrac{dE}{dx} = -\dfrac{\rho}{\epsilon_s} = +\dfrac{eN_a}{\epsilon_s} \Rightarrow E = \dfrac{eN_a x}{\epsilon_s} + C_2$

$E = 0$ at $x = (t_{ox} + x_p)$, then

$E = -\dfrac{eN_a}{\epsilon_s}\left[(t_{ox} + x_p) - x\right]$

At $x = t_{ox}$, $E = -\dfrac{eN_a x_p}{\epsilon_s} = -\dfrac{eN_d x_n}{\epsilon_s}$

So that $N_a x_p = N_d x_n$

Since $N_a = N_d$, then $x_n = x_p$

The potential is

$\phi = -\displaystyle\int E\,dx$

For zero bias, we can write

$V_n + V_{ox} + V_p = V_{bi}$

where V_n, V_{ox}, V_p are the voltage drops
across
the n-region, the oxide, and the p-region,
respectively. For the oxide:

$V_{ox} = E \cdot t_{ox} = \dfrac{eN_d x_n t_{ox}}{\epsilon_s}$

For the n-region:

$V_n(x) = \dfrac{eN_d}{\epsilon_s}\left(\dfrac{x^2}{2} + x_n \cdot x\right) + C'$

Arbitrarily, set $V_n = 0$ at $x = -x_n$, then

$C' = \dfrac{eN_d x_n^2}{2\epsilon_s}$ so that

$V_n(x) = \dfrac{eN_d}{2\epsilon_s}(x + x_n)^2$

At $x = 0$, $V_n = \dfrac{eN_d x_n^2}{2\epsilon_s}$ which is the voltage

drop across the n-region. Because of
symmetry, $V_n = V_p$. Then for zero bias, we

have

$2V_n + V_{ox} = V_{bi}$

which can be written as

$\dfrac{eN_d x_n^2}{\epsilon_s} + \dfrac{eN_d x_n t_{ox}}{\epsilon_s} = V_{bi}$

or

$x_n^2 + x_n t_{ox} - \dfrac{V_{bi}\,\epsilon_s}{eN_d} = 0$

Solving for x_n, we obtain

$x_n = -\dfrac{t_{ox}}{2} + \sqrt{\left(\dfrac{t_{ox}}{2}\right)^2 + \dfrac{\epsilon_s V_{bi}}{eN_d}}$

If we apply a voltage V_G, then replace V_{bi} by
$V_{bi} + V_G$, so

$x_n = x_p = -\dfrac{t_{ox}}{2} + \sqrt{\left(\dfrac{t_{ox}}{2}\right)^2 + \dfrac{\epsilon_s (V_{bi} + V_G)}{eN_d}}$

We find

$x_n = x_p = -\dfrac{500 \times 10^{-8}}{2}$

$+ \sqrt{\left(\dfrac{500 \times 10^{-8}}{2}\right)^2 + \dfrac{(11.7)(8.85 \times 10^{-14})(3.695)}{(1.6 \times 10^{-19})(10^{16})}}$

which yields

$x_n = x_p = 4.646 \times 10^{-5}$ cm

Now

$V_{ox} = \dfrac{eN_d x_n t_{ox}}{\epsilon_s}$

$= \dfrac{(1.6 \times 10^{-19})(10^{16})(4.646 \times 10^{-5})(500 \times 10^{-8})}{(11.7)(8.85 \times 10^{-14})}$

or

$V_{ox} = 0.359$ V

We also find

$V_n = V_p = \dfrac{eN_d x_n^2}{2\epsilon_s}$

$= \dfrac{(1.6 \times 10^{-19})(10^{16})(4.646 \times 10^{-5})^2}{2(11.7)(8.85 \times 10^{-14})}$

or

$V_n = V_p = 1.67$ V

_____

10.30

(a) n-type

(b) We have

$C_{ox} = \dfrac{200 \times 10^{-12}}{2 \times 10^{-3}} = 1 \times 10^{-7}$ F/cm^2

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{1 \times 10^{-7}}$$

or

$$t_{ox} = 3.45 \times 10^{-6} \text{ cm} = 34.5 \text{ nm} = 345 \overset{o}{A}$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

or

$$-0.80 = -0.50 - \frac{Q'_{ss}}{10^{-7}}$$

which yields

$$Q'_{ss} = 3 \times 10^{-8} \text{ C/cm}^2 = 1.875 \times 10^{11} \text{ cm}^{-2}$$

(d)

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\sqrt{\left(\frac{kT}{e}\right)\left(\frac{\in_s}{eN_d}\right)}}$$

$$= \left[(3.9)(8.85 \times 10^{-14}) \right] \div \left[3.45 \times 10^{-6} \right.$$

$$\left. + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259)(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(2 \times 10^{16})}} \right]$$

which yields

$$C'_{FB} = 7.82 \times 10^{-8} \text{ F/cm}^2$$

or

$$C_{FB} = 156 \text{ pF}$$

_____

10.31

(a) Point 1: Inversion
　　　 2: Threshold
　　　 3: Depletion
　　　 4: Flat-band
　　　 5: Accumulation

_____

10.32

We have

$$Q'_n = -C_{ox}\left[(V_{GS} - V_x) - (\phi_{ms} + 2\phi_{fp}) \right]$$
$$- (Q'_{ss} + Q'_{SD}(\max))$$

Now let $V_x = V_{DS}$, so

$$Q'_n = -C_{ox}\left\{ (V_{GS} - V_{DS}) \right.$$

$$\left. + \left[\frac{|Q'_{SD}(\max)| + Q'_{ss}}{C_{ox}} - (\phi_{ms} + 2\phi_{fp}) \right] \right\}$$

For a p-type substrate, $Q'_{SD}(\max)$ is a
negative value, so we can write

$$Q'_n = -C_{ox}\left\{ (V_{GS} - V_{DS}) \right.$$

$$\left. - \left[\frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp} \right] \right\}$$

Using the definition of threshold voltage V_T,
we have

$$Q'_n = -C_{ox}\left[(V_{GS} - V_{DS}) - V_T \right]$$

At saturation

$$V_{DS} = V_{DS}(sat) = V_{GS} - V_T$$

which then makes Q'_n equal to zero at the
drain terminal.

_____

10.33

(a) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L} \left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$

$$= \left(\frac{0.18}{2}\right)(8)\left[2(0.8 - 0.4)(0.2) - (0.2)^2 \right]$$

$$= 0.0864 \text{ mA}$$

(b) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L}(V_{GS} - V_T)^2$

$$= \left(\frac{0.18}{2}\right)(8)(0.8 - 0.4)^2$$

$$= 0.1152 \text{ mA}$$

(c) Same as (b), $I_D = 0.1152$ mA

(d) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L}(V_{GS} - V_T)^2$

$$= \left(\frac{0.18}{2}\right)(8)(1.2 - 0.4)^2$$

$$= 0.4608 \text{ mA}$$

_____

10.34

(a) $I_D = \frac{k'_p}{2} \cdot \frac{W}{L} \left[2(V_{SG} + V_T)V_{SD} - V_{SD}^2 \right]$

$$= \left(\frac{0.10}{2}\right)(15)\left[2(0.8 - 0.4)(0.25) - (0.25)^2 \right]$$

$$I_D = 0.103 \text{ mA}$$

(b) $I_D = \frac{k'_p}{2} \cdot \frac{W}{L}(V_{SG} + V_T)^2$

$$= \left(\frac{0.10}{2}\right)(15)(0.8 - 0.4)^2$$

$$= 0.12 \text{ mA}$$

(c) $I_D = \dfrac{k'_p}{2} \cdot \dfrac{W}{L} (V_{SG} + V_T)^2$

$\quad = \left(\dfrac{0.10}{2}\right)(15)(1.2 - 0.4)^2$

$\quad = 0.48 \text{ mA}$

(d) Same as (c), $I_D = 0.48 \text{ mA}$

_____

10.35

(a) $I_D = \dfrac{k'_n}{2} \cdot \dfrac{W}{L} (V_{GS} - V_T)^2$

$\quad 1.0 = \left(\dfrac{0.6}{2}\right)\left(\dfrac{W}{L}\right)(1.4 - 0.8)^2$

$\quad \Rightarrow \dfrac{W}{L} = 9.26$

(b) $I_D = \left(\dfrac{0.6}{2}\right)(9.26)(1.85 - 0.8)^2$

$\quad = 3.06 \text{ mA}$

(c) $I_D = \dfrac{k'_n}{2} \cdot \dfrac{W}{L} \left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$

$\quad = \left(\dfrac{0.6}{2}\right)(9.26)\left[2(1.2 - 0.8)(0.15) - (0.15)^2\right]$

$\quad = 0.271 \text{ mA}$

_____

10.36

(a) Assume biased in saturation region

$\quad I_D = \dfrac{k'_p}{2} \cdot \dfrac{W}{L} (V_{SG} + V_T)^2$

$\quad 0.10 = \left(\dfrac{0.12}{2}\right)(20)(0 + V_T)^2$

$\quad \Rightarrow V_T = +0.289 \text{ V}$

Note: $V_{SD} = 1.0 \text{ V} > V_{SG} + V_T = 0 + 0.289 \text{ V}$

So the transistor is biased in the saturation region.

(b) $I_D = \left(\dfrac{0.12}{2}\right)(20)(0.4 + 0.289)^2$

$\quad = 0.570 \text{ mA}$

(c) $I_D = \left(\dfrac{0.12}{2}\right)(20)\left[2(0.6 + 0.289)(0.15)\right.$

$\left. \qquad\qquad\qquad\qquad - (0.15)^2\right]$

or

$\quad I_D = 0.293 \text{ mA}$

_____

10.37

$C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{110 \times 10^{-8}} = 3.138 \times 10^{-7} \text{ F/cm}^2$

$K_n = \dfrac{\mu_n C_{ox} W}{2L} = \dfrac{(425)(3.138 \times 10^{-7})(20)}{2(1.2)}$

$\quad = 1.111 \times 10^{-3} \text{ A/V}^2 = 1.111 \text{ mA/V}^2$

(a) $V_{GS} = 0$, $I_D = 0$

$\quad V_{GS} = 0.6 \text{ V}, \ V_{DS}(sat) = 0.15 \text{ V},$

$\qquad I_D(sat) = (1.111)(0.6 - 0.45)^2$

$\qquad\qquad = 0.025 \text{ mA}$

$\quad V_{GS} = 1.2 \text{ V}, \ V_{DS}(sat) = 0.75 \text{ V},$

$\qquad I_D(sat) = (1.111)(1.2 - 0.45)^2$

$\qquad\qquad = 0.625 \text{ mA}$

$\quad V_{GS} = 1.8 \text{ V}, \ V_{DS}(sat) = 1.35 \text{ V},$

$\qquad I_D(sat) = (1.111)(1.8 - 0.45)^2$

$\qquad\qquad = 2.025 \text{ mA}$

$\quad V_{GS} = 2.4 \text{ V}, \ V_{DS}(sat) = 1.95 \text{ V},$

$\qquad I_D(sat) = (1.111)(2.4 - 0.45)^2$

$\qquad\qquad = 4.225 \text{ mA}$

(c) $I_D = 0$ for $V_{GS} \le 0.45 \text{ V}$

$\quad V_{GS} = 0.6 \text{ V},$

$\qquad I_D = (1.111)\left[2(0.6 - 0.45)(0.1) - (0.1)^2\right]$

$\qquad\quad = 0.0222 \text{ mA}$

$\quad V_{GS} = 1.2 \text{ V},$

$\qquad I_D = (1.111)\left[2(1.2 - 0.45)(0.1) - (0.1)^2\right]$

$\qquad\quad = 0.156 \text{ mA}$

$\quad V_{GS} = 1.8 \text{ V},$

$\qquad I_D = (1.111)\left[2(1.8 - 0.45)(0.1) - (0.1)^2\right]$

$\qquad\quad = 0.289 \text{ mA}$

$\quad V_{GS} = 2.4 \text{ V},$

$\qquad I_D = (1.111)\left[2(2.4 - 0.45)(0.1) - (0.1)^2\right]$

$\qquad\quad = 0.422 \text{ mA}$

_____

10.38

$C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{(3.9)(8.85 \times 10^{-14})}{110 \times 10^{-8}}$

$\quad = 3.138 \times 10^{-7} \text{ F/cm}^2$

$K_p = \dfrac{\mu_p C_{ox} W}{2L}$

$\quad = \dfrac{(210)(3.138 \times 10^{-7})(35)}{2(1.2)}$

$\quad = 9.61 \times 10^{-4} \text{ A/V}^2 = 0.961 \text{ mA/V}^2$

_____

(a) $V_{SG} = 0$, $I_D = 0$

$V_{SG} = 0.6$ V, $V_{SD}(sat) = 0.25$ V

$$I_D(sat) = (0.961)(0.6 - 0.35)^2$$
$$= 0.060 \, \text{mA}$$

$V_{SG} = 1.2$ V, $V_{SD}(sat) = 0.85$ V

$$I_D(sat) = (0.961)(1.2 - 0.35)^2$$
$$= 0.694 \, \text{mA}$$

$V_{SG} = 1.8$ V, $V_{SD}(sat) = 1.45$ V

$$I_D(sat) = (0.961)(1.8 - 0.35)^2$$
$$= 2.02 \, \text{mA}$$

$V_{SG} = 2.4$ V, $V_{SD}(sat) = 2.05$ V

$$I_D(sat) = (0.961)(2.4 - 0.35)^2$$
$$= 4.04 \, \text{mA}$$

(c) $I_D = 0$ for $V_{SG} \leq 0.35$ V

$V_{SG} = 0.6$ V

$$I_D = (0.961)\left[2(0.6 - 0.35)(0.1) - (0.1)^2\right]$$
$$= 0.0384 \, \text{mA}$$

$V_{SG} = 1.2$ V

$$I_D = (0.961)\left[2(1.2 - 0.35)(0.1) - (0.1)^2\right]$$
$$= 0.154 \, \text{mA}$$

$V_{SG} = 1.8$ V

$$I_D = (0.961)\left[2(1.8 - 0.35)(0.1) - (0.1)^2\right]$$
$$= 0.269 \, \text{mA}$$

$V_{SG} = 2.4$ V

$$I_D = (0.961)\left[2(2.4 - 0.35)(0.1) - (0.1)^2\right]$$
$$= 0.384 \, \text{mA}$$

_____

10.39

(a) From Problem 10.37, $K_n = 1.111 \, \text{mA/V}^2$

For $V_{GS} = -0.8$ V, $I_D = 0$

$V_{GS} = 0$, $V_{DS}(sat) = 0.8$ V

$$I_D(sat) = (1.111)(0 + 0.8)^2$$
$$= 0.711 \text{mA}$$

$V_{GS} = +0.8$ V, $V_{DS}(sat) = 1.6$ V

$$I_D(sat) = (1.111)(0.8 + 0.8)^2$$
$$= 2.84 \, \text{mA}$$

$V_{GS} = 1.6$ V, $V_{DS}(sat) = 2.4$ V

$$I_D(sat) = (1.111)(1.6 + 0.8)^2$$
$$= 6.40 \, \text{mA}$$

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10.40

Sketch

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10.41

Sketch

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10.42

We have
$$V_{DS}(sat) = V_{GS} - V_T = V_{DS} - V_T$$
so that
$$V_{DS} = V_{DS}(sat) + V_T$$
Since $V_{DS} > V_{DS}(sat)$, the transistor is always biased in the saturation region. Then
$$I_D = K_n(V_{GS} - V_T)^2$$
where, from Problem 10.37,
$$K_n = 1.111 \, \text{mA/V}^2 \text{ and } V_T = 0.45 \text{ V}$$
Then

$V_{DS} = V_{GS}$	I_D (mA)
0	0
1	0.336
2	2.67
3	7.22
4	14.0
5	23.0

_____

10.43

From Problem 10.38, $K_p = 0.961 \, \text{mA/V}^2$

$$I_D = K_p\left[2(V_{SG} + V_T)(V_{SD}) - V_{SD}^2\right]$$

$$g_d = \left.\frac{\partial I_D}{\partial V_{SD}}\right|_{V_{SD} \to 0} = 2K_p(V_{SG} + V_T)$$

For $V_{SG} \leq 0.35$ V, $g_d = 0$

For $V_{SG} > 0.35$ V,
$$g_d = 2(0.961)(V_{SG} - 0.35)$$
For $V_{SG} = 2.4$ V,
$$g_d = 2(0.961)(2.4 - 0.35)$$
$$= 3.94 \, \text{mA/V}$$

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10.44

(a) $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

$= \dfrac{\partial}{\partial V_{GS}} \left\{ K_n \left[2(V_{GS} - V_T)(V_{DS}) - V_{DS}^2 \right] \right\}$

$= K_n (2V_{DS})$

$1.25 = K_n (2)(0.05)$

$\Rightarrow K_n = 12.5 \text{ mA/V}^2$

(b) $I_D = (12.5)\left[2(0.8 - 0.3)(0.05) - (0.05)^2 \right]$

$= 0.594 \text{ mA}$

(c) $I_D = (12.5)(0.8 - 0.3)^2$

$= 3.125 \text{ mA}$

_____

10.45

We find that $V_T \cong 0.2$ V

Now

$$\sqrt{I_D(sat)} = \sqrt{\dfrac{W\mu_n C_{ox}}{2L}} \cdot (V_{GS} - V_T)$$

where

$$C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{(3.9)(8.85 \times 10^{-14})}{425 \times 10^{-8}}$$

or

$C_{ox} = 8.12 \times 10^{-8} \text{ F/cm}^2$

We are given $W/L = 10$. From the graph, for $V_{GS} = 3$ V, we have

$\sqrt{I_D(sat)} \cong 0.033$,

then

$$0.033 = \sqrt{\dfrac{W\mu_n C_{ox}}{2L}} \cdot (3 - 0.2)$$

or

$$\dfrac{W\mu_n C_{ox}}{2L} = 0.139 \times 10^{-3}$$

or

$$\dfrac{1}{2}(10)\mu_n (8.12 \times 10^{-8}) = 0.139 \times 10^{-3}$$

which yields

$\mu_n = 342 \text{ cm}^2/\text{V-s}$

_____

10.46

(a)

$V_{DS}(sat) = V_{GS} - V_T$

or

$4 = V_{GS} - 0.8 \Rightarrow V_{GS} = 4.8 \text{ V}$

(b)

$I_D(sat) = K_n (V_{GS} - V_T)^2 = K_n V_{DS}^2(sat)$

so

$2 \times 10^{-4} = K_n (4)^2$

which yields

$K_n = 12.5 \, \mu\text{A/V}^2$

(c)

$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2 \text{ V}$

so $V_{DS} > V_{DS}(sat)$

$I_D(sat) = (1.25 \times 10^{-5})(2 - 0.8)^2$

or

$I_D(sat) = 18 \, \mu\text{A}$

(d)

$V_{DS} < V_{DS}(sat)$

$I_D = K_n \left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$

$= (1.25 \times 10^{-5})\left[2(3 - 0.8)(1) - (1)^2 \right]$

or

$I_D = 42.5 \, \mu\text{A}$

_____

10.47

(a) $C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8}}$

$= 1.9175 \times 10^{-7} \text{ F/cm}^2$

(i) $k_n' = \mu_n C_{ox} = (450)(1.9175 \times 10^{-7})$

$= 8.629 \times 10^{-5} \text{ A/V}^2$

or $k_n' = 86.29 \, \mu\text{A/V}^2$

(ii) $I_D(sat) = \left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)(V_{GS} - V_T)^2$

$0.8 = \left(\dfrac{0.08629}{2}\right)\left(\dfrac{W}{L}\right)(2 - 0.4)^2$

$\Rightarrow \dfrac{W}{L} = 7.24$

(b) (i) $k_p' = \mu_p C_{ox} = (210)(1.9175 \times 10^{-7})$

$= 4.027 \times 10^{-5} \text{ A/V}^2$

or $k_p' = 40.27 \, \mu\text{A/V}^2$

(ii) $I_D(sat) = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)(V_{SG} + V_T)^2$

$0.8 = \left(\dfrac{0.04027}{2}\right)\left(\dfrac{W}{L}\right)(2 - 0.4)^2$

$\Rightarrow \dfrac{W}{L} = 15.5$

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10.48

From Problem 10.37, $K_n = 1.111 \, \text{mA/V}^2$

(a) $g_{mL} = \dfrac{\partial}{\partial V_{GS}} \left\{ K_n \left[2(V_{GS} - V_T)(V_{DS}) - V_{DS}^2 \right] \right\}$

$\qquad = K_n(2V_{DS}) = (1.111)(2)(0.1)$

so $g_{mL} = 0.222 \, \text{mA/V}$

(b) $g_{ms} = \dfrac{\partial}{\partial V_{GS}} \left\{ K_n (V_{GS} - V_T)^2 \right\}$

$\qquad = 2K_n(V_{GS} - V_T) = 2(1.111)(1.5 - 0.45)$

so $g_{ms} = 2.33 \, \text{mA/V}$

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10.49

From Problem 10.38, $K_p = 0.961 \, \text{mA/V}^2$

(a) $g_{mL} = \dfrac{\partial}{\partial V_{SG}} \left\{ K_p \left[2(V_{SG} + V_T)(V_{SD}) - V_{SD}^2 \right] \right\}$

$\qquad = K_p(2V_{SD}) = (0.961)(2)(0.1)$

or $g_{mL} = 0.192 \, \text{mA/V}$

(b) $g_{ms} = \dfrac{\partial}{\partial V_{SG}} \left[K_p (V_{SG} + V_T)^2 \right]$

$\qquad = 2K_p(V_{SG} + V_T) = 2(0.961)(1.5 - 0.35)$

or $g_{ms} = 2.21 \, \text{mA/V}$

_____

10.50

(a) $\gamma = \dfrac{\sqrt{2e \in_s N_a}}{C_{ox}}$

Now $C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{150 \times 10^{-8}}$

$\qquad = 2.301 \times 10^{-7} \, \text{F/cm}^2$

Then

$\gamma = \dfrac{\sqrt{2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{16})}}{2.301 \times 10^{-7}}$

$\gamma = 0.5594 \, \text{V}^{1/2}$

(b) $\phi_{fp} = (0.0259)\ln\left(\dfrac{5 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3890 \, \text{V}$

(i) $x_{dT} = \left[\dfrac{4(11.7)(8.85 \times 10^{-14})(0.3890)}{(1.6 \times 10^{-19})(5 \times 10^{16})} \right]^{1/2}$

$\qquad = 1.419 \times 10^{-5} \, \text{cm}$

$\left| Q'_{SD}(\text{max}) \right|$

$\qquad = (1.6 \times 10^{-19})(5 \times 10^{16})(1.419 \times 10^{-5})$

$= 1.135 \times 10^{-7} \, \text{C/cm}^2$

$V_{TO} = \dfrac{\left| Q'_{SD}(\text{max}) \right|}{C_{ox}} + V_{FB} + 2\phi_{fp}$

$\qquad = \dfrac{1.135 \times 10^{-7}}{2.301 \times 10^{-7}} - 0.5 + 2(0.3890)$

$\qquad = 0.7713 \, \text{V}$

$K_n = \dfrac{\mu_n C_{ox} W}{2L}$

$\qquad = \dfrac{(450)(2.301 \times 10^{-7})(8)}{2(1.2)}$

$\qquad = 3.452 \times 10^{-4} \, \text{A/V}^2$

or $K_n = 0.3452 \, \text{mA/V}^2$

For $I_D = 0$, $V_{GS} = V_{TO} = 0.7713 \, \text{V}$

For $I_D = 0.5 = (0.3452)(V_{GS} - 0.7713)^2$

$\Rightarrow V_{GS} = 1.975 \, \text{V}$

(c) (i) For $V_{SB} = 0$, $V_T = V_{TO} = 0.7713 \, \text{V}$

(ii) $V_{SB} = 1 \, \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389) + 1} \right.$

$\qquad\qquad\qquad \left. - \sqrt{2(0.389)} \right]$

$\qquad = 0.2525 \, \text{V}$

$V_T = 0.7713 + 0.2525 = 1.024 \, \text{V}$

(iii) $V_{SB} = 2 \, \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389) + 2} \right.$

$\qquad\qquad\qquad \left. - \sqrt{2(0.389)} \right]$

$\qquad = 0.4390 \, \text{V}$

$V_T = 0.7713 + 0.4390 = 1.210 \, \text{V}$

(iv) $V_{SB} = 4 \, \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389) + 4} \right.$

$\qquad\qquad\qquad \left. - \sqrt{2(0.389)} \right]$

$\qquad = 0.7294 \, \text{V}$

$V_T = 0.7713 + 0.7294 = 1.501 \, \text{V}$

_____

10.51

$\phi_{fp} = (0.0259)\ln\left(\dfrac{10^{16}}{1.5 \times 10^{10}}\right) = 0.3473 \, \text{V}$

$\Delta V_T = \gamma \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$

$\qquad = (0.12)\left[\sqrt{2(0.3473) + 2.5} \right.$

$\qquad\qquad\qquad \left. - \sqrt{2(0.3473)} \right]$

or

$\Delta V_T = 0.114 \, \text{V}$

_____

Now $V_T = V_{TO} + \Delta V_T$

$0.5 = V_{TO} + 0.114$

$\Rightarrow V_{TO} = 0.386 \, \text{V}$

_____

10.52

(a) $C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$

$= 1.726 \times 10^{-7} \, \text{F/cm}^2$

$\gamma = \dfrac{\sqrt{2e \in_s N_d}}{C_{ox}}$

$= \dfrac{\sqrt{2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(5 \times 10^{15})}}{1.726 \times 10^{-7}}$

$\gamma = 0.2358 \, \text{V}^{1/2}$

(b) $\phi_{fn} = (0.0259) \ln\left(\dfrac{5 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.3294 \, \text{V}$

$\Delta V_T = -\gamma\left[\sqrt{2\phi_{fn} + V_{BS}} - \sqrt{2\phi_{fn}}\right]$

$-0.22 = -(0.2358)\left[\sqrt{2(0.3294) + V_{BS}} - \sqrt{2(0.3294)}\right]$

$\Rightarrow V_{BS} = 2.39 \, \text{V}$

_____

10.53

(a) n^+ poly-to-p-type $\Rightarrow \phi_{ms} = -1.0 \, \text{V}$

$\phi_{fp} = (0.0259) \ln\left(\dfrac{10^{15}}{1.5 \times 10^{10}}\right) = 0.288 \, \text{V}$

also

$x_{dT} = \left[\dfrac{4 \in_s \phi_{fp}}{eN_a}\right]^{1/2}$

$= \left[\dfrac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})}\right]^{1/2}$

or

$x_{dT} = 0.863 \times 10^{-4} \, \text{cm}$

Now

$|Q'_{SD}(\text{max})| = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$

or

$|Q'_{SD}(\text{max})| = 1.38 \times 10^{-8} \, \text{C/cm}^2$

Also

$C_{ox} = \dfrac{\in_{ox}}{t_{ox}} = \dfrac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$

or

$C_{ox} = 8.63 \times 10^{-8} \, \text{F/cm}^2$

We find

$Q'_{ss} = (1.6 \times 10^{-19})(5 \times 10^{10}) = 8 \times 10^{-9} \, \text{C/cm}^2$

Then

$V_T = \dfrac{|Q'_{SD}(\text{max})| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$

$= \left(\dfrac{1.38 \times 10^{-8} - 8 \times 10^{-9}}{8.63 \times 10^{-8}}\right) - 1.0 + 2(0.288)$

or

$V_T = -0.357 \, \text{V}$

(b) For NMOS, apply V_{SB} and V_T shifts in a positive direction, so for $V_T = 0$, we want

$\Delta V_T = +0.357 \, \text{V}$.

So

$\Delta V_T = \dfrac{\sqrt{2e \in_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$

or

$+0.357 = \dfrac{\sqrt{2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}}{8.63 \times 10^{-8}}$

$\times \left[\sqrt{2(0.288) + V_{SB}} - \sqrt{2(0.288)}\right]$

or

$0.357 = 0.211\left[\sqrt{0.576 + V_{SB}} - \sqrt{0.576}\right]$

which yields

$V_{SB} = 5.43 \, \text{V}$

_____

10.54

Plot

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10.55

(a)

$g_m = \dfrac{W\mu_n C_{ox}}{L}(V_{GS} - V_T)$

$= \dfrac{W\mu_n \in_{ox}}{L t_{ox}}(V_{GS} - V_T)$

$= \dfrac{(10)(400)(3.9)(8.85 \times 10^{-14})}{475 \times 10^{-8}}(5 - 0.65)$

or

$g_m = 1.26 \, \text{mS}$

Now

$g'_m = \dfrac{g_m}{1 + g_m r_s} \Rightarrow \dfrac{g'_m}{g_m} = 0.8 = \dfrac{1}{1 + g_m r_s}$

which yields

_____

$$r_s = \frac{1}{g_m}\left(\frac{1}{0.8}-1\right) = \frac{1}{1.26}\left(\frac{1}{0.8}-1\right)$$

or

$$r_s = 0.198\,\mathrm{k}\,\Omega$$

(b) For $V_{GS} = 3$ V, $g_m = 0.683$ mS

Then

$$g_m' = \frac{0.683}{1+(0.683)(0.198)} = 0.602\ \mathrm{mS}$$

or

$$\frac{g_m'}{g_m} = \frac{0.602}{0.683} = 0.88$$

which is a 12% reduction.

_____

10.56

(a) The ideal cutoff frequency for no overlap capacitance is,

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\mu_n(V_{GS}-V_T)}{2\pi L^2}$$

$$= \frac{(400)(4-0.75)}{2\pi(2\times10^{-4})^2}$$

or

$$f_T = 5.17\ \mathrm{GHz}$$

(b) Now

$$f_T = \frac{g_m}{2\pi(C_{gsT}+C_M)}$$

where

$$C_M = C_{gdT}(1+g_m R_L)$$

We find

$$C_{gdT} = C_{ox}(0.75\times10^{-4})(20\times10^{-4})$$

$$= \frac{(3.9)(8.85\times10^{-14})}{500\times10^{-8}}$$

$$\times(0.75\times10^{-4})(20\times10^{-4})$$

or

$$C_{gdT} = 1.035\times10^{-14}\ \mathrm{F}$$

Also

$$g_m = \frac{W\mu_n C_{ox}}{L}(V_{GS}-V_T)$$

$$= \frac{(20\times10^{-4})(400)(3.9)(8.85\times10^{-14})}{(2\times10^{-4})(500\times10^{-8})}$$

$$\times(4-0.75)$$

or

$$g_m = 0.8974\times10^{-3}\ \mathrm{S}$$

Then

$$C_M = (1.035\times10^{-14})$$

$$\times\left[1+(0.8974\times10^{-3})(10\times10^{3})\right]$$

or

$$C_M = 1.032\times10^{-13}\ \mathrm{F}$$

Now

$$C_{gsT} = C_{ox}(L+0.75\times10^{-4})(W)$$

$$= \frac{(3.9)(8.85\times10^{-14})}{500\times10^{-8}}$$

$$\times(2\times10^{-4}+0.75\times10^{-4})(20\times10^{-4})$$

or

$$C_{gsT} = 3.797\times10^{-14}\ \mathrm{F}$$

We now find

$$f_T = \frac{g_m}{2\pi(C_{gsT}+C_M)}$$

$$= \frac{0.8974\times10^{-3}}{2\pi(3.797\times10^{-14}+1.032\times10^{-13})}$$

or

$$f_T = 1.01\ \mathrm{GHz}$$

_____

10.57

(a) For the ideal case

$$f_T = \frac{\upsilon_{ds}}{2\pi L} = \frac{4\times10^6}{2\pi(2\times10^{-4})}$$

or

$$f_T = 3.18\ \mathrm{GHz}$$

(b) With overlap capacitance (using the values from Problem 10.56),

$$f_T = \frac{g_m}{2\pi(C_{gdT}+C_M)}$$

We find

$$g_m = WC_{ox}\upsilon_{ds}$$

$$= \frac{(20\times10^{-4})(3.9)(8.85\times10^{-14})(4\times10^6)}{500\times10^{-8}}$$

or

$$g_m = 0.5522\times10^{-3}\ \mathrm{S}$$

We have

$$C_M = C_{gdT}(1+g_m R_L)$$

$$= (1.035\times10^{-14})$$

$$\times\left[1+(0.5522\times10^{-3})(10\times10^{3})\right]$$

or

$$C_M = 6.750\times10^{-14}\ \mathrm{F}$$

_____

Then

$$f_T = \frac{0.5522 \times 10^{-3}}{2\pi \left(3.797 \times 10^{-14} + 6.75 \times 10^{-14}\right)}$$

or

$$f_T = 0.833 \text{ GHz}$$

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Chapter 11

11.1

(a)

$$I_D = 10^{-15} \exp\left(\frac{V_{GS}}{(2.1)V_t}\right)$$

For $V_{GS} = 0.5\,\text{V}$,

$$I_D = 10^{-15} \exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$

$$I_D = 9.83 \times 10^{-12}\,\text{A}$$

For $V_{GS} = 0.7\,\text{V}$,

$$I_D = 3.88 \times 10^{-10}\,\text{A}$$

For $V_{GS} = 0.9\,\text{V}$,

$$I_D = 1.54 \times 10^{-8}\,\text{A}$$

Then the total current is:

$$I_T = I_D\left(10^6\right)$$

For $V_{GS} = 0.5\,\text{V}$, $I_T = 9.83\,\mu\text{A}$

For $V_{GS} = 0.7\,\text{V}$, $I_T = 0.388\,\text{mA}$

For $V_{GS} = 0.9\,\text{V}$, $I_T = 15.4\,\text{mA}$

(b)

Power: $P = I_T \cdot V_{DD}$

Then

For $V_{GS} = 0.5\,\text{V}$, $P = 49.2\,\mu\text{W}$

For $V_{GS} = 0.7\,\text{V}$, $P = 1.94\,\text{mW}$

For $V_{GS} = 0.9\,\text{V}$, $P = 77\,\text{mW}$

_____

11.2

$$\frac{I_{D2}}{I_{D1}} = \frac{\exp\left(\dfrac{V_{GS2}}{nV_t}\right)}{\exp\left(\dfrac{V_{GS1}}{nV_t}\right)} = \exp\left[\frac{(V_{GS2} - V_{GS1})}{nV_t}\right]$$

$$V_{GS2} - V_{GS1} = nV_t \ln\left(\frac{I_{D2}}{I_{D1}}\right)$$

(a) $V_{GS2} - V_{GS1} = (0.0259)\ln(10)$
$$= 0.0596\,\text{V}$$

(b) $V_{GS2} - V_{GS1} = (1.5)(0.0259)\ln(10)$
$$= 0.0895\,\text{V}$$

(c) $V_{GS2} - V_{GS1} = (2.1)(0.0259)\ln(10)$
$$= 0.125\,\text{V}$$

_____

11.3

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\,\text{V}$$

We find that

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(2\times10^{16})}}$$

$$= 2.544\times10^{-5}\,\text{cm/V}^{1/2}$$

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

(a) $V_{DS}(sat) = V_{GS} - V_T = 1.0 - 0.4 = 0.6\,\text{V}$
$$\Delta L = (2.544\times10^{-5})$$
$$\times\left[\sqrt{0.3653 + 2} - \sqrt{0.3653 + 0.6}\right]$$
$$\Delta L = 1.413\times10^{-5}\,\text{cm} = 0.1413\,\mu\text{m}$$

(b) $V_{DS}(sat) = V_{GS} - V_T = 1.0 - 0.4 = 0.6\,\text{V}$
$$\Delta L = (2.544\times10^{-5})$$
$$\times\left[\sqrt{0.3653 + 4} - \sqrt{0.3653 + 0.6}\right]$$
$$\Delta L = 2.816\times10^{-5}\,\text{cm} = 0.2816\,\mu\text{m}$$

(c) $V_{DS}(sat) = V_{GS} - V_T = 2.0 - 0.4 = 1.6\,\text{V}$
$$\Delta L = (2.544\times10^{-5})$$
$$\times\left[\sqrt{0.3653 + 2} - \sqrt{0.3653 + 1.6}\right]$$
$$\Delta L = 3.461\times10^{-6}\,\text{cm} = 0.0346\,\mu\text{m}$$

(d) $V_{DS}(sat) = V_{GS} - V_T = 2.0 - 0.4 = 1.6\,\text{V}$
$$\Delta L = (2.544\times10^{-5})$$
$$\times\left[\sqrt{0.3653 + 4} - \sqrt{0.3653 + 1.6}\right]$$
$$\Delta L = 1.749\times10^{-5}\,\text{cm} = 0.1749\,\mu\text{m}$$

_____

11.4

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\,\text{V}$$

We find that

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(2\times10^{16})}}$$

$$= 2.544\times10^{-5}\,\text{cm/V}^{1/2}$$

$$V_{DS}(sat) = V_{GS} - V_T = 2.0 - 0.4 = 1.6\,\text{V}$$

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

(a) $\Delta L = (2.544 \times 10^{-5})$
$$\times \left[\sqrt{0.3653 + 3} - \sqrt{0.3653 + 1.6} \right]$$
$\Delta L = 1.10 \times 10^{-5}$ cm $= 0.110 \,\mu$m

Now $\dfrac{\Delta L}{L} = 0.10 = \dfrac{0.110}{L}$

$\Rightarrow L = 1.10 \,\mu$m

(b) $\Delta L = (2.544 \times 10^{-5})$
$$\times \left[\sqrt{0.3653 + 5} - \sqrt{0.3653 + 1.6} \right]$$
$\Delta L = 2.326 \times 10^{-5}$ cm $= 0.2326 \,\mu$m

Now $\dfrac{\Delta L}{L} = 0.10 = \dfrac{0.2326}{L}$

$\Rightarrow L = 2.326 \,\mu$m

_____

11.5

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{120 \times 10^{-8}}$$
$$= 2.876 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$
$$= -0.5 - \frac{(4 \times 10^{10})(1.6 \times 10^{-19})}{2.876 \times 10^{-7}}$$
$$V_{FB} = -0.5223 \text{ V}$$

Now

$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{4 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3832 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3832)}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2}$$
$$= 1.575 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\max)|$$
$$= (1.6 \times 10^{-19})(4 \times 10^{16})(1.575 \times 10^{-5})$$
$$= 1.008 \times 10^{-7} \text{ C/cm}^2$$

So

$$V_T = \frac{1.008 \times 10^{-7}}{2.876 \times 10^{-7}} - 0.5223 + 2(0.3832)$$
$$= 0.595 \text{ V}$$

$$V_{DS}(sat) = V_{GS} - V_T = 1.25 - 0.595 = 0.655 \text{ V}$$

$$\sqrt{\frac{2\epsilon_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(4 \times 10^{16})}}$$
$$= 1.799 \times 10^{-5} \text{ cm/V}^{1/2}$$

(a) $\Delta L = \sqrt{\dfrac{2\epsilon_s}{eN_a}} \left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} \right.$
$$\left. - \sqrt{\phi_{fp} + V_{DS}(sat)} \right]$$

(i) $\Delta L = (1.799 \times 10^{-5})$
$$\times \left[\sqrt{0.3832 + 0.655 + 1} - \sqrt{0.3832 + 0.655} \right]$$
$\Delta L = 7.35 \times 10^{-6}$ cm $= 0.0735 \,\mu$m

(ii) $\Delta L = (1.799 \times 10^{-5})$
$$\times \left[\sqrt{0.3832 + 0.655 + 2} - \sqrt{0.3832 + 0.655} \right]$$
$\Delta L = 1.303 \times 10^{-5}$ cm $= 0.1303 \,\mu$m

(iii) $\Delta L = (1.799 \times 10^{-5})$
$$\times \left[\sqrt{0.3832 + 0.655 + 4} - \sqrt{0.3832 + 0.655} \right]$$
$\Delta L = 2.205 \times 10^{-5}$ cm $= 0.2205 \,\mu$m

(b) $\dfrac{\Delta L}{L} = 0.12 = \dfrac{0.2205}{L}$

$L = 1.84 \,\mu$m

_____

11.6

$$\phi_{fp} = (0.0259)\ln\left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3758 \text{ V}$$

$$\sqrt{\frac{2\epsilon_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(3 \times 10^{16})}}$$
$$= 2.077 \times 10^{-5} \text{ cm/V}^{1/2}$$

(a) Ideal,

$$I_D = \frac{k'_n}{2} \cdot \frac{W}{L} (V_{GS} - V_T)^2$$
$$= \left(\frac{0.05}{2} \right)\left(\frac{15}{0.80} \right)(1.0 - 0.4)^2$$
$$= 0.16875 \text{ mA}$$

(i) $V_{DS}(sat) = 1.0 - 0.4 = 0.6$ V

$\Delta L = (2.077 \times 10^{-5})$
$$\times \left[\sqrt{0.3758 + 2} - \sqrt{0.3758 + 0.6} \right]$$
$$= 1.150 \times 10^{-5} \text{ cm} = 0.115 \,\mu\text{m}$$

$$I'_D = \left(\frac{L}{L - \Delta L} \right) \cdot I_D$$

_____

$$= \left(\frac{0.80}{0.80 - 0.115} \right)(0.16875)$$

$$= 0.19708 \, \text{mA}$$

(ii) $\Delta L = \left(2.077 \times 10^{-5} \right)$

$$\times \left[\sqrt{0.3758 + 4} - \sqrt{0.3758 + 0.6} \right]$$

$$= 2.293 \times 10^{-5} \, \text{cm} = 0.2293 \, \mu\text{m}$$

$$I_D' = \left(\frac{L}{L - \Delta L} \right) \cdot I_D$$

$$= \left(\frac{0.80}{0.80 - 0.2293} \right)(0.16875)$$

$$= 0.23655 \, \text{mA}$$

(b)

$$r_o = \left(\frac{\Delta I_D'}{\Delta V_{DS}} \right)^{-1} = \left[\frac{(0.23655 - 0.19708) \times 10^{-3}}{4 - 2} \right]^{-1}$$

$$\Rightarrow r_o = 5.07 \times 10^4 \, \Omega = 50.7 \, \text{k}\Omega$$

(c) $V_{DS}(sat) = V_{GS} - V_T = 2.0 - 0.4 = 1.6 \, \text{V}$

(i) $\Delta L = \left(2.077 \times 10^{-5} \right)$

$$\times \left[\sqrt{0.3758 + 2} - \sqrt{0.3758 + 1.6} \right]$$

$$= 2.819 \times 10^{-6} \, \text{cm} = 0.02819 \, \mu\text{m}$$

$$I_D' = \left(\frac{L}{L - \Delta L} \right) \cdot I_D$$

$$= \left(\frac{0.80}{0.80 - 0.02819} \right)(0.16875)$$

$$= 0.17491 \, \text{mA}$$

(ii) $\Delta L = \left(2.077 \times 10^{-5} \right)$

$$\times \left[\sqrt{0.3758 + 4} - \sqrt{0.3758 + 0.6} \right]$$

$$= 1.425 \times 10^{-5} \, \text{cm} = 0.1425 \, \mu\text{m}$$

$$I_D' = \left(\frac{L}{L - \Delta L} \right) \cdot I_D$$

$$= \left(\frac{0.80}{0.80 - 0.1425} \right)(0.16875)$$

$$= 0.20532 \, \text{mA}$$

$$r_o = \left(\frac{\Delta I_D'}{\Delta V_{DS}} \right)^{-1}$$

$$= \left[\frac{(0.20532 - 0.17491) \times 10^{-3}}{4 - 2} \right]^{-1}$$

$$r_o = 6.577 \times 10^4 \, \Omega = 65.77 \, \text{k}\Omega$$

_____

11.7

(a)

(i) $I_D = \dfrac{k_n'}{2} \cdot \dfrac{W}{L} \left(V_{GS} - V_T \right)^2$

$$= \left(\frac{0.075}{2} \right)(10)(0.8 - 0.35)^2$$

$$= 0.07594 \, \text{mA} = 75.94 \, \mu\text{A}$$

(ii) $I_D' = I_D \left(1 + \lambda V_{DS} \right)$

$$= (75.9375)\left[1 + (0.02)(1.5) \right]$$

$$= 78.22 \, \mu\text{A}$$

(iii) $r_o = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.02)(75.94)}$

$$= 0.658 \, \text{M}\Omega = 658 \, \text{k}\Omega$$

(b)

(i) $I_D = \left(\dfrac{0.075}{2} \right)(10)(1.25 - 0.35)^2$

$$= 0.30375 \, \text{mA}$$

(ii) $I_D' = (0.30375)\left[1 + (0.02)(1.5) \right]$

$$= 0.3129 \, \text{mA}$$

(iii) $r_o = \dfrac{1}{(0.02)(0.30375)} = 165 \, \text{k}\Omega$

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11.8

Plot

_____

11.9

(a) Assume $V_{DS}(sat) = 1 \, \text{V}$. Then

$$\text{E}_{sat} = \frac{V_{DS}(sat)}{L}$$

We find

L (μm)	E_{sat} (V/cm)
3	3.33×10^3
1	1×10^4
0.5	2×10^4
0.25	4×10^4
0.13	7.69×10^4

(b)

Assume $\mu_n = 500 \, \text{cm}^2/\text{V-s}$, we have

$$\upsilon = \mu_n \text{E}_{sat}$$

Then

For $L = 3 \, \mu\text{m}$, $\upsilon = 1.67 \times 10^6 \, \text{cm/s}$

For $L = 1 \, \mu\text{m}$, $\upsilon = 5 \times 10^6 \, \text{cm/s}$

For $L \leq 0.5 \, \mu\text{m}$, $\upsilon \cong 10^7 \, \text{cm/s}$

_____

11.10

$$k'_n = \mu_n C_{ox} = \frac{\mu_n \in_{ox}}{t_{ox}}$$

$$= \frac{(425)(3.9)(8.85\times10^{-14})}{110\times10^{-8}}$$

$$= 1.334\times10^{-4} \text{ A/V}^2 = 0.1334 \text{ mA/V}^2$$

$$I_D = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$

$$= \left(\frac{0.1334}{2}\right)\left(\frac{20}{1.2}\right)(0.8-0.45)^2$$

$$= 0.1362 \text{ mA}$$

$$\frac{1}{r_o} = \frac{\partial I'_D}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}}\left[\left(\frac{L}{L-\Delta L}\right)\cdot I_D\right]$$

$$= I_D L\cdot\frac{\partial}{\partial V_{DS}}[L-\Delta L]^{-1}$$

$$= I_D L(-1)(L-\Delta L)^{-2}\left(-\frac{\partial\Delta L}{\partial V_{DS}}\right)$$

$$= \frac{I_D L}{(L-\Delta L)^2}\cdot\frac{\partial\Delta L}{\partial V_{DS}}$$

Now

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp}+V_{DS}}-\sqrt{\phi_{fp}+V_{DS}(sat)}\right]$$

$$\frac{\partial\Delta L}{\partial V_{DS}} = \sqrt{\frac{2\in_s}{eN_a}}\cdot\frac{1}{2}\left(\phi_{fp}+V_{DS}\right)^{-1/2}$$

We find

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(3\times10^{16})}}$$

$$= 2.077\times10^{-5} \text{ cm/V}^{1/2}$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758 \text{ V}$$

$$V_{DS} = V_{DS}(sat)+\Delta V_{DS} = V_{GS}-V_T+\Delta V_{DS}$$
$$= 0.8-0.45+2 = 2.35 \text{ V}$$

Now

$$\frac{\partial\Delta L}{\partial V_{DS}} = \frac{(2.077\times10^{-5})}{2\sqrt{0.3758+2.35}} = 6.290\times10^{-6} \text{ cm/V}$$

$$\Delta L = (2.077\times10^{-5})\left[\sqrt{0.3758+2.35}\right.$$
$$\left. -\sqrt{0.3758+0.35}\right]$$

$$= 1.660\times10^{-5} \text{ cm} = 0.166 \,\mu\text{ m}$$

(a)

$$\frac{1}{r_o} = \frac{I_D L}{(L-\Delta L)^2}\cdot\frac{\partial\Delta L}{\partial V_{DS}}$$

$$= \frac{(0.1362\times10^{-3})(1.2\times10^{-4})}{[(1.2-0.166)\times10^{-4}]^2}\cdot(6.290\times10^{-6})$$

$$= 9.615\times10^{-6}$$

$$\Rightarrow r_o = 1.04\times10^5 \,\Omega = 104 \text{ k}\Omega$$

(b)

$$\frac{1}{r_o} = \frac{(0.1362\times10^{-3})(0.8\times10^{-4})}{[(0.8-0.166)\times10^{-4}]^2}\cdot(6.290\times10^{-6})$$

$$= 1.705\times10^{-5}$$

$$\Rightarrow r_o = 5.865\times10^4 \,\Omega = 58.65 \text{ k}\Omega$$

11.11

(a)

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS}-V_T)^2$$

$$= \left(\frac{10}{2}\right)(500)(6.9\times10^{-8})(V_{GS}-1)^2$$

or

$$I_D(sat) = 0.173(V_{GS}-1)^2 \quad\text{(mA)}$$

and

$$\sqrt{I_D(sat)} = \sqrt{0.173}(V_{GS}-1) \quad\text{(mA)}^{1/2}$$

(b)

Let $\mu_{eff} = \mu_O\left(\frac{E_{eff}}{E_C}\right)^{-1/3}$

Where $\mu_O = 1000 \text{ cm}^2/\text{V-s}$ and

$$E_C = 2.5\times10^4 \text{ V/cm}$$

Let $E_{eff} = \frac{V_{GS}}{t_{ox}}$

We find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{6.9\times10^{-8}}$$

or

$$t_{ox} = 500 \,\overset{o}{A}$$

Then

V_{GS}	E_{eff}	μ_{eff}	$\sqrt{I_D(sat)}$
1	--	--	0
2	4×10^5	397	0.370
3	6×10^5	347	0.692
4	8×10^5	315	0.989
5	10×10^5	292	1.27

(c) The slope of the variable mobility curve is not constant, but is continually decreasing.

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11.12

Plot

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11.13

(a) $C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$

$= 1.726 \times 10^{-7} \text{ F/cm}^2$

$K_n = \dfrac{\mu_n C_{ox} W}{2L}$

$= \dfrac{(475)(1.726 \times 10^{-7})(10)}{2(1.0)}$

$= 4.10 \times 10^{-4} \text{ A/V}^2 = 0.410 \text{ mA/V}^2$

For $V_{GS} - V_T = 2 \text{ V}$, $V_{DS}(sat) = 2 \text{ V}$

(i) $I_D = (0.410)\left[2(2)(0.5) - (0.5)^2\right]$

$= 0.7175 \text{ mA}$

(ii) $I_D = (0.410)\left[2(2)(1.0) - (1.0)^2\right]$

$= 1.23 \text{ mA}$

(iii) $I_D = (0.410)\left[2(2)(1.25) - (1.25)^2\right]$

$= 1.409 \text{ mA}$

(iv) $I_D = (0.410)\left[2(2)(2) - (2)^2\right]$

$= 1.64 \text{ mA}$

(b) $I_D = W C_{ox}(V_{GS} - V_T)v_{ds}$

$= (10^{-3})(1.726 \times 10^{-7})(2)v_{ds}$

$= (3.452 \times 10^{-10})v_{ds} \text{ A}$

$= (3.452 \times 10^{-7})v_{ds} \text{ mA}$

(i) For $V_{DS} = 0.5 \text{ V}$, $v_{ds} = \left(\dfrac{0.5}{1.25}\right)(4 \times 10^6)$

$= 1.6 \times 10^6 \text{ cm/s}$

$I_D = (3.452 \times 10^{-7})(1.6 \times 10^6)$

$= 0.552 \text{ mA}$

(ii) For $V_{DS} = 1.0 \text{ V}$, $v_{ds} = \left(\dfrac{1.0}{1.25}\right)(4 \times 10^6)$

$= 3.2 \times 10^6 \text{ cm/s}$

$I_D = (3.452 \times 10^{-7})(3.2 \times 10^6)$

$= 1.10 \text{ mA}$

(iii) For $V_{DS} = 1.25 \text{ V}$, $v_{ds} = 4 \times 10^6 \text{ cm/s}$

$I_D = (3.452 \times 10^{-7})(4 \times 10^6)$

$= 1.38 \text{ mA}$

(iv) For $V_{DS} = 2 \text{ V}$, $v_{ds} = 4 \times 10^6 \text{ cm/s}$

$I_D = 1.38 \text{ mA}$

(c) For part (a), $V_{DS}(sat) = 2 \text{ V}$

For part (b), $V_{DS}(sat) = 1.25 \text{ V}$

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11.14

Plot

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11.15

(a) Non-saturation region

$I_D = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$

We have

$C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} \Rightarrow \dfrac{C_{ox}}{k}$

and

$W \Rightarrow kW$, $L \Rightarrow kL$

also

$V_{GS} \Rightarrow kV_{GS}$, $V_{DS} \Rightarrow kV_{DS}$

So

$I_D = \dfrac{1}{2}\mu_n\left(\dfrac{C_{ox}}{k}\right)\left(\dfrac{kW}{kL}\right)$

$\times\left[2(kV_{GS} - V_T)kV_{DS} - (kV_{DS})^2\right]$

Then

$I_D \Rightarrow \cong kI_D$

In the saturation region,

$I_D = \dfrac{1}{2}\mu_n\left(\dfrac{C_{ox}}{k}\right)\left(\dfrac{kW}{kL}\right)\left[kV_{GS} - V_T\right]^2$

Then

$I_D \Rightarrow \cong kI_D$

(b)

$P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P$

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11.16

$$I_D(sat) = WC_{ox}(V_{GS} - V_T)\upsilon_{sat}$$

$$\Rightarrow (kW)\left(\frac{C_{ox}}{k}\right)(kV_{GS} - V_T)\upsilon_{sat}$$

or

$$I_D(sat) \Rightarrow \cong kI_D(sat)$$

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left[\frac{r_j}{L}\left(\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right)\right]$$

$$= -\frac{(1.6 \times 10^{-19})(5 \times 10^{16})(1.419 \times 10^{-5})}{2.876 \times 10^{-7}}$$

$$\times \left[\frac{0.25}{0.80}\left(\sqrt{1 + \frac{2(0.1419)}{0.25}} - 1\right)\right]$$

$$\Delta V_T = -0.0569 \text{ V}$$

11.17

(a)

(i) $I_D(\max) = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$

$$= \left(\frac{0.15}{2}\right)\left(\frac{6}{1.2}\right)(3 - 0.45)^2$$

$$2.438 \text{ mA}$$

(ii) Scaled device:

$$V_D = V_{GS} = k(3) = (0.65)(3) = 1.95 \text{ V}$$

$$k'_n = \left(\frac{0.15}{k}\right) = \left(\frac{0.15}{0.65}\right) = 0.2308 \text{ mA/V}^2$$

$$L = k(1.2) = (0.65)(1.2) = 0.78 \,\mu\text{m}$$

$$W = k(6) = (0.65)(6) = 3.90 \,\mu\text{m}$$

Then

$$I_D(\max) = \left(\frac{0.2308}{2}\right)\left(\frac{3.9}{0.78}\right)(1.95 - 0.45)^2$$

$$= 1.298 \text{ mA}$$

(b) (i) $P(\max) = I_D(\max)V_D = (2.438)(3)$

$$= 7.314 \text{ mW}$$

(ii) $P(\max) = (1.298)(1.95)$

$$= 2.531 \text{ mW}$$

11.18

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{120 \times 10^{-8}}$$

$$= 2.876 \times 10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{5 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3890 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3890)}{(1.6 \times 10^{-19})(5 \times 10^{16})}\right]^{1/2}$$

$$= 1.419 \times 10^{-5} \text{ cm}$$

11.19

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{80 \times 10^{-8}}$$

$$= 4.314 \times 10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3653 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3653)}{(1.6 \times 10^{-19})(2 \times 10^{16})}\right]^{1/2}$$

$$= 2.174 \times 10^{-5} \text{ cm}$$

$$\Delta V_T = -\frac{(1.6 \times 10^{-19})(2 \times 10^{16})(2.174 \times 10^{-5})}{4.314 \times 10^{-7}}$$

$$\times \left[\frac{0.30}{0.70}\left(\sqrt{1 + \frac{2(0.2174)}{0.30}} - 1\right)\right]$$

$$\Delta V_T = -0.0391 \text{ V}$$

$$V_T = V_{TO} + \Delta V_T$$

$$0.35 = V_{TO} - 0.0391$$

$$\Rightarrow V_{TO} = 0.389 \text{ V}$$

11.20

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$$

$$= 1.726 \times 10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3758 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3758)}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$$

$$= 1.80 \times 10^{-5} \text{ cm}$$

_____

$$\Delta V_T = -0.15$$

$$= -\frac{\left(1.6\times10^{-19}\right)\left(3\times10^{16}\right)\left(1.80\times10^{-5}\right)}{1.726\times10^{-7}}$$

$$\times\left[\frac{0.30}{L}\left(\sqrt{1+\frac{2(0.18)}{0.30}}-1\right)\right]$$

$$0.15 = (0.5006)\left(\frac{0.30}{L}\right)(0.4832)$$

$$\Rightarrow L = 0.484\,\mu\text{m}$$

_____

11.21

We have

$$L' = L-(a+b)$$

and from the geometry

$$(1)\quad (a+r_j)^2 + x_{dT}^2 = (r_j + x_{dS})^2$$

and

$$(2)\quad (b+r_j)^2 + x_{dT}^2 = (r_j + x_{dD})^2$$

From (1)

$$(a+r_j)^2 = (r_j + x_{dS})^2 - x_{dT}^2$$

so that

$$a = \sqrt{(r_j + x_{dS})^2 - x_{dT}^2} - r_j$$

which can be written as

$$a = r_j\left[\sqrt{\left(1+\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

or

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\left(\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

Define

$$\alpha^2 = \frac{x_{dS}^2 - x_{dT}^2}{r_j^2}$$

We can then write

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2} - 1\right]$$

Similarly from (2), we will have

$$b = r_j\left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2} - 1\right]$$

where

$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is

$$|Q_B'|\cdot L = eN_a x_{dT}\left(\frac{L+L'}{2}\right)$$

or

$$|Q_B'| = eN_a x_{dT}\left(\frac{L+L'}{2L}\right)$$

We can write

$$\frac{L+L'}{2L} = \frac{1}{2} + \frac{L'}{2L} = \frac{1}{2} + \frac{1}{2L}\left[L-(a+b)\right]$$

which is

$$\frac{L+L'}{2L} = 1 - \frac{a+b}{2L}$$

Now, $|Q_B'|$ replaces $|Q_{SD}'(\max)|$ in the threshold equation. Then

$$\Delta V_T = \frac{|Q_B'|}{C_{ox}} - \frac{|Q_{SD}'(\max)|}{C_{ox}}$$

$$= \frac{eN_a x_{dT}}{C_{ox}}\left[1-\frac{a+b}{2L}\right] - \frac{eN_a x_{dT}}{C_{ox}}$$

or

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{(a+b)}{2L}$$

Then substituting, we obtain

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{r_j}{2L}\left\{\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right]\right.$$

$$\left.+\left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]\right\}$$

Note that if $x_{dS} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

_____

11.22

We have $L' = 0$, so Equation (11.27) becomes

$$\frac{L+L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2}$$

$$= \left\{1-\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

or

$$\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right] = \frac{1}{2}$$

Then Equation (11.28) is

$$|Q'_B| = eN_a x_{dT}\left(\frac{1}{2}\right)$$

Then change in the threshold voltage is

$$\Delta V_T = \frac{|Q'_B|}{C_{oc}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

which becomes

$$\Delta V_T = -\frac{1}{2} \cdot \frac{eN_a x_{dT}}{C_{ox}}$$

_____

11.23
   Plot

_____

11.24
   Plot

_____

11.25

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left\{\frac{kr_j}{kL}\left[\sqrt{1+\frac{2kx_{dT}}{kr_j}}-1\right]\right\}$$

or

$$\Delta V_T \Rightarrow k\Delta V_T$$

_____

11.26

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{80\times10^{-8}}$$

$$= 4.314\times10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3758)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$$= 1.80\times10^{-5} \text{ cm}$$

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

$$= \frac{(1.6\times10^{-19})(3\times10^{16})(1.80\times10^{-5})^2\left(\frac{\pi}{2}\right)}{(4.314\times10^{-7})(2.2\times10^{-4})}$$

$$\Delta V_T = +0.0257 \text{ V}$$

_____

11.27

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}}$$

$$= 2.876\times10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.3473 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3473)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

$$= 3.0\times10^{-5} \text{ cm}$$

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

In this case, $\xi = 1$
So

$$0.045 = \frac{(1.6\times10^{-19})(10^{16})(1.0)(3\times10^{-5})^2}{(2.876\times10^{-7})(W)}$$

$$\Rightarrow W = 1.11 \mu\text{m}$$

_____

11.28
   Plot

_____

11.29

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

Assume that ξ is a constant, then

$$\Delta V_T \Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left(\frac{\xi kx_{dT}}{kW}\right)$$

or

$$\Delta V_T \Rightarrow k\Delta V_T$$

_____

11.30

  (a)

    (i) $E_{ox} \cong \dfrac{V_G}{t_{ox}}$

        $6 \times 10^6 = \dfrac{V_G}{200 \times 10^{-8}}$

        $\Rightarrow V_G = 12 \text{ V}$

    (ii) $V_G = \dfrac{12}{3} = 4 \text{ V}$

  (b)

    (i) $6 \times 10^6 = \dfrac{V_G}{80 \times 10^{-8}}$

        $\Rightarrow V_G = 4.8 \text{ V}$

    (ii) $V_G = \dfrac{4.8}{3} = 1.6 \text{ V}$

11.31

  (a) $V_G = (8)(3) = 24 = E_{ox} t_{ox} = (6 \times 10^6)(t_{ox})$

      $t_{ox} = 4 \times 10^{-6} \text{ cm}$

    or $t_{ox} = 40 \text{ nm} = 400 \overset{o}{A}$

  (b) $V_G = (12)(3) = 36 = E_{ox} t_{ox} = (6 \times 10^6)(t_{ox})$

      $t_{ox} = 6 \times 10^{-6} \text{ cm}$

    or $t_{ox} = 60 \text{ nm} = 600 \overset{o}{A}$

11.32

  Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18)\log_{10}\left(\frac{I_o}{3 \times 10^{-9}}\right)$$

  and

$$M = \frac{1}{1 - \left(\dfrac{V_{CE}}{V_{BO}}\right)^m}$$

  Let $V_{BO} = 15 \text{ V}$ and $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1 - \left(\dfrac{V_{CE}}{15}\right)^3}$$

  we can write this as

$$1 - \left(\frac{V_{CE}}{15}\right)^3 = \alpha \Rightarrow V_{CE} = 15 \cdot \sqrt[3]{1-\alpha}$$

Now

I_D	α	V_{CE}
10^{-8}	0.0941	14.5
10^{-7}	0.274	13.5
10^{-6}	0.454	12.3
10^{-5}	0.634	10.7
10^{-4}	0.814	8.6
10^{-3}	0.994	2.7

11.33

  One Debye length is

$$L_D = \left[\frac{\in_s (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85 \times 10^{-14})(0.0259)}{(1.6 \times 10^{-19})(10^{16})}\right]^{1/2}$$

  or

    $L_D = 4.09 \times 10^{-6} \text{ cm}$

Six Debye lengths is then

    $6L_D = 0.246 \times 10^{-4} \text{ cm} = 0.246 \, \mu\text{m}$

From Example 11.5, we have

$x_{dO} = 0.336 \, \mu\text{m}$, which is the zero-biased
source-substrate junction width.
At near punch-through, we will have

    $x_{dO} + 6L_D + x_d = L$

where x_d is the reverse-biased drain-
substrate junction width. Now

    $0.336 + 0.246 + x_d = 1.2$

or

    $x_d = 0.618 \, \mu\text{m}$

Then, at near punch-through we have

$$x_d = \left[\frac{2\in_s (V_{bi} + V_{DS})}{eN_a}\right]^{1/2}$$

or

$$V_{bi} + V_{DS} = \frac{x_d^2 eN_a}{2 \in_s}$$

$$= \frac{(0.618 \times 10^{-4})^2 (1.6 \times 10^{-19})(10^{16})}{2(11.7)(8.85 \times 10^{-14})}$$

which yields

    $V_{bi} + V_{DS} = 2.95 \text{ V}$

From Example 11.5, we have $V_{bi} = 0.874$ V, so

$$V_{DS} = 2.08 \text{ V}$$

which is the near punch-through voltage. The ideal punch-through voltage was

$$V_{DS} = 4.9 \text{ V}$$

11.34

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(3\times10^{16})}{(1.5\times10^{10})^2}\right] = 0.902 \text{ V}$$

The zero-biased source-substrate junction width is given by

$$x_{dO} = \left[\frac{2\in_s V_{bi}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$$x_{dO} = 0.197\times10^{-4} \text{ cm} = 0.197 \,\mu\text{m}$$

The Debye length is

$$L_D = \left[\frac{\in_s (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85\times10^{-14})(0.0259)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$$L_D = 2.36\times10^{-6} \text{ cm}$$

so that

$$6L_D = 0.142\times10^{-4} \text{ cm} = 0.142 \,\mu\text{m}$$

Now

$$x_{dO} + 6L_D + x_d = L$$

We have for $V_{DS} = 5$ V,

$$x_d = \left[\frac{2\in_s (V_{bi}+V_{DS})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+5)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$$x_d = 0.505\times10^{-4} \text{ cm} = 0.505 \,\mu\text{m}$$

Then

$$L = 0.197 + 0.142 + 0.505$$

or

$$L = 0.844 \,\mu\text{m}$$

11.35

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2\in_s (V_{bi}+V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+2)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$$x_{dO} = 0.354\times10^{-4} \text{ cm} = 0.354 \,\mu\text{m}$$

We have $6L_D = 0.142 \,\mu\text{m}$ from the previous problem.

Now

$$x_d = \left[\frac{2\in_s (V_{bi}+V_{DS}+V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+5+2)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$$x_d = 0.584\times10^{-4} \text{ cm} = 0.584 \,\mu\text{m}$$

Then

$$L = x_{dO} + 6L_D + x_d$$

$$= 0.354 + 0.142 + 0.584$$

or

$$L = 1.08 \,\mu\text{m}$$

11.36

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}} = 2.876\times10^{-7} \text{ F/cm}^2$$

$$|\Delta V_T| = \frac{eD_I}{C_{ox}}$$

Implant acceptor ions for a positive threshold voltage shift.

$$D_I = \frac{\Delta V_T (C_{ox})}{e} = \frac{(0.80)(2.876\times10^{-7})}{1.6\times10^{-19}}$$

$$= 1.438\times10^{12} \text{ cm}^{-2}$$

11.37

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$

$$= 1.9175\times10^{-7} \text{ F/cm}^2$$

$$|\Delta V_T| = \frac{eD_I}{C_{ox}}$$

Implant donor ions for a negative threshold voltage shift.

$$D_I = \frac{|\Delta V_T| C_{ox}}{e} = \frac{(0.60)(1.9175 \times 10^{-7})}{1.6 \times 10^{-19}}$$

$$= 7.19 \times 10^{11} \text{ cm}^{-2}$$

_____

11.38

(a) $\phi_{ms} \cong -1.08$ V

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{150 \times 10^{-8}}$$

$$= 2.301 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

$$= -1.08 - \frac{(5 \times 10^{10})(1.6 \times 10^{-19})}{2.301 \times 10^{-7}}$$

$$= -1.115 \text{ V}$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{6 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.3341 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3341)}{(1.6 \times 10^{-19})(6 \times 10^{15})}\right]^{1/2}$$

$$= 3.797 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\text{max})|$$

$$= (1.6 \times 10^{-19})(6 \times 10^{15})(3.797 \times 10^{-5})$$

$$= 3.645 \times 10^{-8} \text{ C/cm}^2$$

$$V_{TO} = \frac{|Q'_{SD}(\text{max})|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

$$= \frac{3.645 \times 10^{-8}}{2.301 \times 10^{-7}} - 1.115 + 2(0.3341)$$

$$V_{TO} = -0.2884 \text{ V}$$

(b)   For a positive threshold voltage shift, add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = 0.50 - (-0.2884)$$

$$= 0.788 \text{ V}$$

Then

$$D_I = \frac{(\Delta V_T) C_{ox}}{e} = \frac{(0.788)(2.301 \times 10^{-7})}{1.6 \times 10^{-19}}$$

$$= 1.13 \times 10^{12} \text{ cm}^{-2}$$

_____

11.39

(a) $\phi_{ms} \cong +1.08$ V

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8}}$$

$$= 1.9175 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

$$= +1.08 - \frac{(10^{11})(1.6 \times 10^{-19})}{1.9175 \times 10^{-7}}$$

$$= +0.9966 \text{ V}$$

$$\phi_{fn} = (0.0259)\ln\left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3653 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3653)}{(1.6 \times 10^{-19})(2 \times 10^{16})}\right]^{1/2}$$

$$= 2.1744 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\text{max})|$$

$$= (1.6 \times 10^{-19})(2 \times 10^{16})(2.1744 \times 10^{-5})$$

$$= 6.958 \times 10^{-8} \text{ C/cm}^2$$

$$V_{TO} = -\frac{|Q'_{SD}(\text{max})|}{C_{ox}} + V_{FB} - 2\phi_{fn}$$

$$= -\frac{6.958 \times 10^{-8}}{1.9175 \times 10^{-7}} + 0.9966 - 2(0.3653)$$

$$V_{TO} = -0.0969 \text{ V}$$

(b)   For a negative threshold voltage shift, add donor ions.

$$\Delta V_T = V_T - V_{TO} = -0.40 - (-0.0969)$$

$$= -0.3031 \text{ V}$$

Then $$D_I = \frac{|\Delta V_T| C_{ox}}{e}$$

$$= \frac{(0.3031)(1.9175 \times 10^{-7})}{1.6 \times 10^{-19}}$$

$$= 3.63 \times 10^{11} \text{ cm}^{-2}$$

_____

11.40

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{4 \times 10^{15}}{1.5 \times 10^{10}}\right) = 0.3236 \text{ V}$

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{80 \times 10^{-8}}$$

$$= 4.314 \times 10^{-7} \text{ F/cm}^2$$

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.3236)}{(1.6 \times 10^{-19})(4 \times 10^{15})}\right]^{1/2}$$

$$= 4.576 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\text{max})|$$

$$= (1.6 \times 10^{-19})(4 \times 10^{15})(4.576 \times 10^{-5})$$

$$= 2.929 \times 10^{-8} \text{ C/cm}^2$$

_____

$$V_{TO} = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

$$= \frac{2.929 \times 10^{-8}}{4.314 \times 10^{-7}} - 1.25 + 2(0.3236)$$

$$= -0.5349 \text{ V}$$

(b) For a positive threshold voltage shift, add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = 0.40 - (-0.5349)$$

$$= 0.9349 \text{ V}$$

Then $D_I = \dfrac{(\Delta V_T)C_{ox}}{e}$

$$= \frac{(0.9349)(4.314 \times 10^{-7})}{1.6 \times 10^{-19}}$$

$$= 2.52 \times 10^{12} \text{ cm}^{-2}$$

(c) Add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = -0.40 - (-0.5349)$$

$$= 0.1349 \text{ V}$$

Then $D_I = \dfrac{(0.1349)(4.314 \times 10^{-7})}{1.6 \times 10^{-19}}$

$$= 3.64 \times 10^{11} \text{ cm}^{-2}$$

_____

11.41

The total space charge width is greater than x_i, so from Chapter 10

$$\Delta V_T = \frac{\sqrt{2e \in_s N_a}}{C_{ox}} \left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

Now

$$\phi_{fp} = (0.0259) \ln \left(\frac{10^{14}}{1.5 \times 10^{10}} \right) = 0.228 \text{ V}$$

and

$$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}}$$

$$= 6.90 \times 10^{-8} \text{ F/cm}^2$$

Then

ΔV_T

$$= \frac{\left[2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{14}) \right]^{1/2}}{6.90 \times 10^{-8}}$$

$$\times \left[\sqrt{2(0.228) + V_{SB}} - \sqrt{2(0.228)} \right]$$

or

$$\Delta V_T = 0.0834 \left[\sqrt{0.456 + V_{SB}} - \sqrt{0.456} \right]$$

Then

V_{SB} (V)	ΔV_T (V)
1	0.0443
3	0.0987
5	0.1385

_____

11.42

(a)

$$\phi_{fn} = (0.0259) \ln \left(\frac{10^{17}}{1.5 \times 10^{10}} \right) = 0.407 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85 \times 10^{-14})(0.407)}{(1.6 \times 10^{-19})(10^{17})} \right]^{1/2}$$

$$= 1.026 \times 10^{-5} \text{ cm}$$

n^+ poly on n-type $\phi_{ms} = -0.32$ V

We have

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{17})(1.026 \times 10^{-5})$$

$$= 1.64 \times 10^{-7} \text{ C/cm}^2$$

Now

$$V_{TP} = \left[-1.64 \times 10^{-7} - (1.6 \times 10^{-19})(5 \times 10^{10}) \right]$$

$$\times \frac{80 \times 10^{-8}}{(3.9)(8.85 \times 10^{-14})} - 0.32 - 2(0.407)$$

or

$V_{TP} = -1.53$ V (Enhancement PMOS)

(b) For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions.

$$\Delta V_T = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_I = \frac{(1.53)(3.9)(8.85 \times 10^{-14})}{(80 \times 10^{-8})(1.6 \times 10^{-19})}$$

or

$$D_I = 4.13 \times 10^{12} \text{ cm}^{-2}$$

_____

_____

11.43

The areal density of generated holes is

$$= \left(8\times10^{12}\right)\left(10^5\right)\left(750\times10^{-8}\right) = 6\times10^{12}\ \text{cm}^{-2}$$

The equivalent surface charge trapped is

$$= (0.10)\left(6\times10^{12}\right) = 6\times10^{11}\ \text{cm}^{-2}$$

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}}$$

$$= -\frac{\left(6\times10^{11}\right)\left(1.6\times10^{-19}\right)\left(750\times10^{-8}\right)}{(3.9)\left(8.85\times10^{-14}\right)}$$

or

$$\Delta V_T = -2.09\ \text{V}$$

_____

11.44

The areal density of generated holes is

$6\times10^{12}\ \text{cm}^{-2}$. Now

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85\times10^{-14}\right)}{750\times10^{-8}}$$

$$= 4.6\times10^{-8}\ \text{F/cm}^2$$

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}} = -\frac{\left(6\times10^{12}\right)(x)\left(1.6\times10^{-19}\right)}{4.6\times10^{-8}}$$

where x is the fraction of holes that may be trapped. For $\Delta V_T = -0.50\ \text{V}$ we find

$$x = 0.024 \Rightarrow x = 2.4\%$$

_____

11.45

We have the areal density of generated holes as

$$= (g)(\gamma)(t_{ox})$$

where g is the generation rate and γ is the radiation dose. The equivalent charge trapped is

$$= xg\gamma t_{ox}$$

where x is the fraction of generated holes trapped.

Then

$$\Delta V_T = -\frac{Q'_{ss}}{C_{ox}} = -\frac{exg\gamma t_{ox}}{\left(\epsilon_{ox}/t_{ox}\right)} = -\left(\frac{exg\gamma}{\epsilon_{ox}}\right)\left(t_{ox}\right)^2$$

or

$$\Delta V_T \propto -\left(t_{ox}\right)^2$$

_____

Chapter 12

12.1

Sketch

_____

12.2

Sketch

_____

12.3

(a) $I_S = \dfrac{eD_n A_{BE} n_{B0}}{x_B}$

$= \dfrac{(1.6 \times 10^{-19})(18)(5 \times 10^{-5})(4 \times 10^3)}{0.80 \times 10^{-4}}$

$= 7.2 \times 10^{-15}$ A

(b) $I_C = I_S \exp\left(\dfrac{\upsilon_{BE}}{V_t}\right)$

(i) $I_C = (7.2 \times 10^{-15}) \exp\left(\dfrac{0.58}{0.0259}\right)$

$= 3.827 \times 10^{-5}$ A $= 38.27 \, \mu$ A

(ii) $I_C = (7.2 \times 10^{-15}) \exp\left(\dfrac{0.65}{0.0259}\right)$

$= 5.710 \times 10^{-4}$ A $= 0.571$ mA

(iii) $I_C = (7.2 \times 10^{-15}) \exp\left(\dfrac{0.72}{0.0259}\right)$

$= 8.519 \times 10^{-3}$ A $= 8.519$ mA

_____

12.4

(a) $|i_C| = \dfrac{eD_n A_{BE}}{x_B} \cdot n_{B0} \cdot \exp\left(\dfrac{\upsilon_{BE}}{V_t}\right)$

$2 \times 10^{-3} = \dfrac{(1.6 \times 10^{-19})(22) A_{BE}}{0.80 \times 10^{-4}}$

$\times (2 \times 10^4) \exp\left(\dfrac{0.60}{0.0259}\right)$

$\Rightarrow A_{BE} = 1.975 \times 10^{-4}$ cm^2

(b) $5 \times 10^{-3} = \dfrac{(1.6 \times 10^{-19})(22)(1.975 \times 10^{-4})}{0.80 \times 10^{-4}}$

$\times (2 \times 10^4) \exp\left(\dfrac{\upsilon_{BE}}{0.0259}\right)$

$5 \times 10^{-3} = (1.738 \times 10^{-13}) \exp\left(\dfrac{\upsilon_{BE}}{0.0259}\right)$

Then

$\upsilon_{BE} = (0.0259) \ln\left(\dfrac{5 \times 10^{-3}}{1.738 \times 10^{-13}}\right)$

$\Rightarrow \upsilon_{BE} = 0.6237$ V

_____

12.5

(a) $\beta = \dfrac{\alpha}{1 - \alpha} = \dfrac{0.9850}{1 - 0.9850} = 65.7$

(b)
(i) For $I_C = 38.27 \, \mu$ A,

$I_B = \dfrac{I_C}{\beta} = \dfrac{38.27}{65.67} = 0.5828 \, \mu$ A

$I_E = \dfrac{I_C}{\alpha} = \dfrac{38.27}{0.9850} = 38.85 \, \mu$ A

(ii) For $I_C = 0.571$ mA,

$I_B = \dfrac{0.571}{65.67} = 0.008695$ mA

$= 8.695 \, \mu$ A

$I_E = \dfrac{0.571}{0.9850} = 0.5797$ mA

(iii) For $I_C = 8.519$ mA,

$I_B = \dfrac{8.519}{65.67} = 0.1297$ mA

$I_E = \dfrac{8.519}{0.9850} = 8.649$ mA

(c) $\beta = \dfrac{0.9940}{1 - 0.9940} = 165.7$

(i) For $I_C = 38.27 \, \mu$ A,

$I_B = \dfrac{38.27}{165.7} = 0.2310 \, \mu$ A

$I_E = \dfrac{38.27}{0.9940} = 38.50 \, \mu$ A

(ii) For $I_C = 0.571$ mA,

$I_B = \dfrac{0.571}{165.7} = 0.003446$ mA

$= 3.446 \, \mu$ A

$I_E = \dfrac{0.571}{0.9940} = 0.5744$ mA

_____

(iii) For $I_C = 8.519$ mA,

$$I_B = \frac{8.519}{165.7} = 0.05141 \text{ mA}$$

$$= 51.41 \, \mu A$$

$$I_E = \frac{8.519}{0.9940} = 8.570 \text{ mA}$$

_____

12.6

(a) $\beta = \dfrac{I_C}{I_B} = \dfrac{0.625}{0.0042} = 148.8$

$\alpha = \dfrac{\beta}{1+\beta} = \dfrac{148.8}{149.8} = 0.9933$

$I_E = \dfrac{I_C}{\alpha} = \dfrac{0.625}{0.9933} = 0.6292 \text{ mA}$

(b) $\alpha = \dfrac{I_C}{I_E} = \dfrac{1.254}{1.273} = 0.9851$

$\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9851}{1-0.9851} = 66.0$

$I_B = \dfrac{I_C}{\beta} = \dfrac{1.254}{66} = 0.0190 \text{ mA}$

$= 19.0 \, \mu A$

(c) $\alpha = \dfrac{150}{151} = 0.99338$

$I_C = \beta I_B = (150)(0.065) = 9.75 \, \mu A$

$I_E = (1+\beta)I_B = (151)(0.065) = 9.815 \, \mu A$

_____

12.7

(c) For $i_B = 0.05$ mA,

$i_C = \beta i_B = (100)(0.05)$

or

$i_C = 5$ mA

We have

$\upsilon_{CE} = V_{CC} - i_C R = 10 - (5)(1)$

or

$\upsilon_{CE} = 5$ V

_____

12.8

(a) For $V_{CE} = 3$ V, $I_C = 0$

(i) $V_{CE} = V_{CC} - I_C R_C$

$0.2 = 3 - I_C(25), \ I_C = 0.112 \text{ mA}$

(ii) For $V_{CB} = 0 \Rightarrow V_{CE} = V_{BE} = 0.65$ V

$0.65 = 3 - I_C(25), \ I_C = 0.094 \text{ mA}$

(b) For $V_{CE} = 3$ V, $I_C = 0$

(i) $V_{CE} = V_{CC} - I_C R_C$

$0.2 = 3 - I_C(10), \ I_C = 0.28 \text{ mA}$

(ii) For $V_{CB} = 0 \Rightarrow V_{CE} = V_{BE} = 0.65$ V

$0.65 = 3 - I_C(10), \ I_C = 0.235 \text{ mA}$

_____

12.9

(a) $p_{E0} = \dfrac{n_i^2}{N_E} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{8 \times 10^{17}}$

$= 2.8125 \times 10^2 \text{ cm}^{-3}$

$n_{B0} = \dfrac{n_i^2}{N_B} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{2 \times 10^{16}}$

$= 1.125 \times 10^4 \text{ cm}^{-3}$

$p_{C0} = \dfrac{n_i^2}{N_C} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{10^{15}}$

$= 2.25 \times 10^5 \text{ cm}^{-3}$

(b) $n_B(0) = n_{B0} \exp\left(\dfrac{V_{BE}}{V_t}\right)$

$= \left(1.125 \times 10^4\right)\exp\left(\dfrac{0.640}{0.0259}\right)$

$= 6.064 \times 10^{14} \text{ cm}^{-3}$

$p_E(0) = p_{E0} \exp\left(\dfrac{V_{BE}}{V_t}\right)$

$= \left(2.8125 \times 10^2\right)\exp\left(\dfrac{0.640}{0.0259}\right)$

$= 1.516 \times 10^{13} \text{ cm}^{-3}$

_____

12.10

(a) $n_{E0} = \dfrac{n_i^2}{N_E} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{5 \times 10^{17}}$

$= 4.5 \times 10^2 \text{ cm}^{-3}$

$p_{B0} = \dfrac{n_i^2}{N_B} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{10^{16}}$

$= 2.25 \times 10^4 \text{ cm}^{-3}$

$n_{C0} = \dfrac{n_i^2}{N_C} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{10^{15}}$

$= 2.25 \times 10^5 \text{ cm}^{-3}$

_____

(b) $p_B(0) = p_{B0} \exp\left(\dfrac{V_{EB}}{V_t}\right)$

$\qquad = \left(2.25 \times 10^4\right) \exp\left(\dfrac{0.615}{0.0259}\right)$

$\qquad = 4.62 \times 10^{14}\,\text{cm}^{-3}$

$n_E(0) = n_{E0} \exp\left(\dfrac{V_{EB}}{V_t}\right)$

$\qquad = \left(4.5 \times 10^2\right) \exp\left(\dfrac{0.615}{0.0259}\right)$

$\qquad = 9.24 \times 10^{12}\,\text{cm}^{-3}$

_____

12.11

(a) $n_{B0} = \dfrac{n_i^2}{N_B} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{2 \times 10^{16}}$

$\qquad = 1.125 \times 10^4\,\text{cm}^{-3}$

Now

$\qquad n_B(0) = (0.1)(N_B) = 2 \times 10^{15}\,\text{cm}^{-3}$

$\qquad = n_{B0} \exp\left(\dfrac{V_{BE}}{V_t}\right)$

Then $V_{BE} = (0.0259) \ln\left(\dfrac{2 \times 10^{15}}{1.125 \times 10^4}\right)$

$\qquad = 0.6709\,\text{V}$

(b) $p_{E0} = \dfrac{n_i^2}{N_E} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{8 \times 10^{17}}$

$\qquad = 2.8125 \times 10^2\,\text{cm}^{-3}$

$p_E(0) = p_{E0} \exp\left(\dfrac{V_{BE}}{V_t}\right) +$

$\qquad = \left(2.8125 \times 10^2\right) \exp\left(\dfrac{0.6709}{0.0259}\right)$

$\qquad = 5.0 \times 10^{13}\,\text{cm}^{-3}$

_____

12.12

We have

$\dfrac{d(\delta n_B)}{dx} = \dfrac{n_{BO}}{\sinh\left(\dfrac{x_B}{L_B}\right)} \left\{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right]\right.$

$\left. \times \left(\dfrac{-1}{L_B}\right) \cosh\left(\dfrac{x_B - x}{L_B}\right) - \dfrac{1}{L_B} \cosh\left(\dfrac{x}{L_B}\right)\right\}$

At $x = 0$,

$\left.\dfrac{d(\delta n_B)}{dx}\right|_{x=0} = \dfrac{-n_{BO}}{L_B \sinh\left(\dfrac{x_B}{L_B}\right)} \left\{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right]\right.$

$\left. \times \cosh\left(\dfrac{x_B}{L_B}\right) + 1\right\}$

At $x = x_B$,

$\left.\dfrac{d(\delta n_B)}{dx}\right|_{x=x_B} = \dfrac{-n_{BO}}{L_B \sinh\left(\dfrac{x_B}{L_B}\right)} \left\{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right]\right.$

$\left. + \cosh\left(\dfrac{x_B}{L_B}\right)\right\}$

Taking the ratio

$\dfrac{\left.\dfrac{d(\delta n_B)}{dx}\right|_{x=x_B}}{\left.\dfrac{d(\delta n_B)}{dx}\right|_{x=0}}$

$= \dfrac{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\dfrac{x_B}{L_B}\right)}{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right] \cdot \cosh\left(\dfrac{x_B}{L_B}\right) + 1}$

$\cong \dfrac{1}{\cosh\left(\dfrac{x_B}{L_B}\right)}$

(a) For $\dfrac{x_B}{L_B} = 0.1 \Rightarrow$ Ratio $= 0.9950$

(b) For $\dfrac{x_B}{L_B} = 1.0 \Rightarrow$ Ratio $= 0.648$

(c) For $\dfrac{x_B}{L_B} = 10 \Rightarrow$ Ratio $= 9.08 \times 10^{-5}$

_____

12.13

In the base of the transistor, we have

$D_B \dfrac{d^2(\delta n_B(x))}{dx^2} - \dfrac{\delta n_B(x)}{\tau_{BO}} = 0$

or

$\dfrac{d^2(\delta n_B(x))}{dx^2} - \dfrac{\delta n_B(x)}{L_B^2} = 0$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution to the differential equation is of the form

$$\delta n_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

Also

$$\delta n_B(x_B) = A \exp\left(\frac{x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right)$$

$$= -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary condition, we find

$$B\left[\exp\left(\frac{x_B}{L_B}\right) - \exp\left(\frac{-x_B}{L_B}\right)\right]$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] \cdot \exp\left(\frac{x_B}{L_B}\right) + n_{BO}$$

Solving for B, we find

$$B = \frac{n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] \cdot \exp\left(\frac{x_B}{L_B}\right) + n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A = \frac{-n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] \cdot \exp\left(\frac{-x_B}{L_B}\right) - n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

_____

12.14

In the base of the pnp transistor, we have

$$D_B \frac{d^2(\delta p_B(x))}{dx^2} - \frac{\delta p_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta p_B(x))}{dx} - \frac{\delta p_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution is of the form

$$\delta p_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\delta p_B(0) = A + B = p_B(0) - p_{BO}$$

$$= p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]$$

Also

$$\delta p_B(x_B) = A \exp\left(\frac{x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right)$$

$$= -p_{BO}$$

From the first boundary condition equation, we find

$$A = p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary equation, we obtain

$$B = \frac{p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] \cdot \exp\left(\frac{x_B}{L_B}\right) + p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A = \frac{-p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] \cdot \exp\left(\frac{-x_B}{L_B}\right) - p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for A and B into the general solution and collecting terms, we obtain

$$\delta p_B(x) = \frac{p_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\right.$$

$$\left. \times \sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}$$

_____

12.15

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x) = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] \cdot \left(\frac{x_B - x}{x_B}\right)$$

_____

The excess carrier concentration is

$$\delta n_B(x) = n_B(x) - n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B - x}{x_B}\right) - 1\right\}$$

At $x = \frac{1}{2}x_B$, we have

$$\delta n_{BO}\left(\frac{x_B}{2}\right) = n_{BO}\left\{\frac{1}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] - 1\right\}$$

For the actual case, we have

$$\delta n_B\left(\frac{x_B}{2}\right) = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\sinh\left(\frac{x_B}{2L_B}\right) - \sinh\left(\frac{x_B}{2L_B}\right)\right\}$$

(a) For $\frac{x_B}{L_B} = 0.10$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\frac{\delta n_{BO}\left(\frac{x_B}{2}\right) - \delta n_B\left(\frac{x_B}{2}\right)}{\delta n_{BO}\left(\frac{x_B}{2}\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right](0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) \gg 1$, then we find that the ratio is

$$= \frac{0.00063}{0.50} = 0.00126 \Rightarrow 0.126\%$$

(b) For $\frac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\frac{\delta n_{BO}\left(\frac{x_B}{2}\right) - \delta n_B\left(\frac{x_B}{2}\right)}{\delta n_{BO}\left(\frac{x_B}{2}\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BR}}{V_t}\right)\right](0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Again assume that $\exp\left(\frac{V_{BE}}{V_t}\right) \gg 1$. Then the ratio becomes

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow 11.32\%$$

_____

12.16

(a) $\delta p_B(x_B) = -5\times10^3 = -p_{B0}$

$\Rightarrow p_{B0} = 5\times10^3\ \text{cm}^{-3}$

$$p_{B0} = \frac{n_i^2}{N_B}$$

$$\Rightarrow N_B = \frac{n_i^2}{p_{B0}} = \frac{(1.5\times10^{10})^2}{5\times10^3}$$

$$= 4.5\times10^{16}\ \text{cm}^{-3}$$

$$\delta p_B(0) \cong p_{B0}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$\Rightarrow V_{EB} = V_t\ln\left(\frac{\delta p_B(0)}{p_{B0}}\right)$$

$$= (0.0259)\ln\left(\frac{10^{15}}{5\times10^3}\right)$$

$$= 0.6740\ \text{V}$$

(b) Using the linear approximation,

$$|J| = eD_B\frac{d(\delta p_B(x))}{dx} \cong \frac{eD_B p_{B0}}{x_B}\exp\left(\frac{V_{EB}}{V_t}\right)$$

Since $x_B \ll L_B$, $|J|\big|_{x=0} \cong |J|\big|_{x=x_B}$

Then

$$|J| = \frac{(1.6 \times 10^{-19})(10)(5 \times 10^3)}{0.8 \times 10^{-4}} \exp\left(\frac{0.674}{0.0259}\right)$$

$$= 20.0 \,\text{A/cm}^2$$

(c) Using Equation (12.15a),

$$\frac{d(\delta p_B(0))}{dx} = \frac{-p_{B0}}{L_B \cdot \sinh\left(\dfrac{x_B}{L_B}\right)}$$

$$\times \left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cosh\left(\frac{x_B - x}{L_B}\right) + \cosh\left(\frac{x}{L_B}\right)\right\}$$

Now

$$|J| = eD_B \frac{d(\delta p_B(x))}{dx} = \frac{eD_B p_{B0}}{L_B \sinh\left(\dfrac{x_B}{L_B}\right)}$$

$$\times \left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cosh\left(\frac{x_B - x}{L_B}\right) + \cosh\left(\frac{x}{L_B}\right)\right\}$$

For $x = 0$, $\sinh(1) = 1.1752$, $\cosh(1) = 1.5431$

$$\cosh(0) = 1.0$$

Then

$$|J|_{x=0} = \frac{(1.6 \times 10^{-19})(10)(5 \times 10^3)}{(10 \times 10^{-4})(1.1752)}$$

$$\times \left\{\left[\exp\left(\frac{0.6740}{0.0259}\right) - 1\right](1.5431) + (1.0)\right\}$$

$$|J|_{x=0} = 2.1042 \,\text{A/cm}^2$$

For $x = x_B$,

$$|J|_{x=x_B} = \frac{(1.6 \times 10^{-19})(10)(5 \times 10^3)}{(10 \times 10^{-4})(1.1752)}$$

$$\times \left\{\left[\exp\left(\frac{0.6740}{0.0259}\right) - 1\right](1.0) + (1.5431)\right\}$$

$$|J|_{x=x_B} = 1.3636 \,\text{A/cm}^2$$

(d) For part (b),

$$\frac{J\Big|_{x=x_B}}{J\Big|_{x=0}} = 1.0$$

For part (c),

$$\frac{J\Big|_{x=x_B}}{J\Big|_{x=0}} = \frac{1.3636}{2.1042} = 0.648$$

12.17

(a) For an npn transistor biased in saturation, the excess minority carrier electron concentration in the base is found from

$$D_B \frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B \tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A \exp\left(\frac{x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

If $x_B \ll L_B$, then also $x \ll L_B$, so that

$$\delta n_B(x) \cong A\left(1 + \frac{x}{L_B}\right) + B\left(1 - \frac{x}{L_B}\right)$$

$$= (A + B) + (A - B)\left(\frac{x}{L_B}\right)$$

which can be written as

$$\delta n_B(x) = C + D\left(\frac{x}{L_B}\right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

and

$$\delta n_B(x_B) = C + D\left(\frac{x_B}{L_B}\right) = n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]$$

The coefficient D can be written as

$$D = \left(\frac{L_B}{x_B}\right)(n_{BO})\left\{\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\right.$$

$$\left. - \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(1 - \frac{x}{x_B}\right)\right.$$

$$\left. + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{x}{x_B}\right)\right\}$$

(b) The electron diffusion current density is

$$J_n = eD_B \frac{d(\delta n_B(x))}{dx}$$

$$= eD_B n_{BO} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \left(\frac{-1}{x_B}\right) \right.$$

$$\left. + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] \cdot \left(\frac{1}{x_B}\right) \right\}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B} \left\{ \exp\left(\frac{V_{BE}}{V_t}\right) - \exp\left(\frac{V_{BC}}{V_t}\right) \right\}$$

(c) The total excess charge in the base region is

$$Q_{nB} = -e \int_0^{x_B} \delta n_B(x)dx$$

$$= -e n_{BO} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \cdot \left(x - \frac{x^2}{2x_B}\right) \right.$$

$$\left. + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] \cdot \left(\frac{x^2}{2x_B}\right) \right\} \Big|_0^{x_B}$$

which yields

$$Q_{nB} = \frac{-e n_{BO} x_B}{2} \left\{ \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right] \right.$$

$$\left. + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] \right\}$$

_____

12.18

(a) Using the linear approximation, we can write

$$|J_n| = \frac{eD_B n_{BO}}{x_B} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - \exp\left(\frac{V_{BC}}{V_t}\right) \right]$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}}$$

$$= 4.5 \times 10^3 \text{ cm}^{-3}$$

Then

$$125 = \frac{(1.6 \times 10^{-19})(25)(4.5 \times 10^3)}{0.7 \times 10^{-4}}$$

$$\times \left[\exp\left(\frac{0.70}{0.0259}\right) - \exp\left(\frac{V_{BC}}{V_t}\right) \right]$$

$$4.8611 \times 10^{11} = 5.4662 \times 10^{11} - \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$V_{BC} = (0.0259)\ln(6.051 \times 10^{10})$$

$$= 0.6430 \text{ V}$$

(b) $V_{CE}(sat) = V_{BE} - V_{BC} = 0.70 - 0.6430$

$$= 0.057 \text{ V}$$

(c) We have

$$\frac{|Q_n|}{e} = \frac{n_{BO} x_B}{2} \left[\exp\left(\frac{V_{BE}}{V_t}\right) + \exp\left(\frac{V_{BC}}{V_t}\right) \right]$$

$$= \frac{(4.5 \times 10^3)(0.7 \times 10^{-4})}{2}$$

$$\times \left[\exp\left(\frac{0.70}{0.0259}\right) + \exp\left(\frac{0.643}{0.0259}\right) \right]$$

$$= (0.1575)[5.466 \times 10^{11} + 6.052 \times 10^{10}]$$

$$\frac{|Q_n|}{e} = 9.56 \times 10^{10} \text{ cm}^{-2}$$

(d) In the collector,

$$\delta p_C(x'') \cong p_{C0} \left[\exp\left(\frac{V_{BC}}{V_t}\right) \right] \cdot \left[\exp\left(\frac{-x''}{L_C}\right) \right]$$

Now

$$\frac{|Q_p|}{e} = \int_0^\infty \delta p_C(x'')dx''$$

$$= p_{C0} \left[\exp\left(\frac{V_{BC}}{V_t}\right) \right](-L_C)\left[\exp\left(\frac{-x''}{L_C}\right) \right]_0^\infty$$

$$= p_{C0} L_C \exp\left(\frac{V_{BC}}{V_t}\right)$$

We find

$$p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5 \times 10^{10})^2}{10^{15}}$$

$$= 2.25 \times 10^5 \text{ cm}^{-3}$$

Then

$$\frac{|Q_p|}{e} = (2.25 \times 10^5)(35 \times 10^{-4})\exp\left(\frac{0.643}{0.0259}\right)$$

$$= 4.77 \times 10^{13} \text{ cm}^{-2}$$

_____

12.19

(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

and

_____

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5\times10^{10}\right)^2}{7\times10^{15}} = 3.21\times10^4 \text{ cm}^{-3}$$

At $x = x_B$,

$$n_B(x_B) = n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(2.25\times10^3\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or $n_B(x_B) = 6.7\times10^{12} \text{ cm}^{-3}$

At $x'' = 0$,

$$p_C(0) = p_{CO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(3.21\times10^4\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or $p_C(0) = 9.56\times10^{13} \text{ cm}^{-3}$

(c) From the B-C space charge region,

$$V_{bi1} = (0.0259)\ln\left[\frac{\left(10^{17}\right)\left(7\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.745 \text{ V}$$

Then

$$x_{p1} = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.745-0.565)}{1.6\times10^{-19}}\right.$$

$$\left. \times\left(\frac{7\times10^{15}}{10^{17}}\right)\left(\frac{1}{7\times10^{15}+10^{17}}\right)\right]^{1/2}$$

or $x_{p1} = 1.23\times10^{-6} \text{ cm}$

From the B-E space charge region

$$V_{bi2} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(10^{17}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.933 \text{ V}$$

Then

$$x_{p2} = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.933+2)}{1.6\times10^{-19}}\right.$$

$$\left. \times\left(\frac{10^{19}}{10^{17}}\right)\left(\frac{1}{10^{19}+10^{17}}\right)\right]^{1/2}$$

or

$$x_{p2} = 1.94\times10^{-5} \text{ cm}$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2}$$

$$= 1.20 - 0.0123 - 0.194$$

or

$$x_B = 0.994 \,\mu\text{m}$$

_____

12.20

Low-injection limit is reached when

$$p_C(0) = (0.10)N_C = (0.10)\left(5\times10^{14}\right)$$

or

$$p_C(0) = 5\times10^{13} \text{ cm}^{-3}$$

We have

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{14}} = 4.5\times10^5 \text{ cm}^{-3}$$

Also

$$p_C(0) = p_{CO} \exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$V_{CB} = V_t \ln\left(\frac{p_C(0)}{p_{CO}}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{13}}{4.5\times10^5}\right)$$

or

$$V_{CB} = 0.48 \text{ V}$$

_____

12.21

(a)

(i) $\gamma = \dfrac{1}{1+\dfrac{I_{pE}}{I_{nE}}} = \dfrac{1}{1+\dfrac{0.0035}{0.50}} = 0.99305$

(ii) $\alpha_T = \dfrac{I_{nC}}{I_{nE}} = \dfrac{0.495}{0.50} = 0.990$

(iii) $\delta = \dfrac{I_{nE}+I_{pE}}{I_{nE}+I_R+I_{pE}}$

$$= \frac{0.50+0.0035}{0.50+0.005+0.0035} = 0.990167$$

(iv) $\alpha = \delta\alpha_T\delta = (0.99305)(0.990)(0.990167)$

$$= 0.97345$$

(v) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.97345}{1-0.97345} = 36.7$

(b) For $\beta = 120 \Rightarrow \alpha = \dfrac{\beta}{1+\beta} = \dfrac{120}{121}$

$$\alpha = 0.991736$$

Then $\gamma = \alpha_T = \delta = 0.997238$

$$\alpha_T = 0.997238 = \frac{I_{nC}}{I_{nE}} = \frac{I_{nC}}{0.50}$$

$$\Rightarrow I_{nC} = 0.4986 \text{ mA}$$

$$\gamma = 0.997238 = \frac{1}{1+\dfrac{I_{pE}}{I_{nE}}} = \frac{1}{1+\dfrac{I_{pE}}{0.50}}$$

$$\Rightarrow I_{pE} = 0.00138\ \text{mA} = 1.38\ \mu\text{A}$$

$$\delta = \frac{I_{nE} + I_{pE}}{I_{nE} + I_R + I_{pE}}$$

$$0.997238 = \frac{0.50 + 0.00138}{0.50 + I_R + 0.00138}$$

$$\Rightarrow I_R = 0.00139\ \text{mA} = 1.39\ \mu\text{A}$$

_____

12.22

(a) Using Equation (12.37)

$$I_{nC} = \frac{eD_B p_{B0} A_{BE}}{L_B}$$

$$\times \left\{ \frac{\left[\exp\left(\dfrac{V_{EB}}{V_t}\right)-1\right]}{\sinh\left(\dfrac{x_B}{L_B}\right)} + \frac{1}{\tanh\left(\dfrac{x_B}{L_B}\right)} \right\}$$

Now

$$p_{B0} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}}$$

$$= 2.25\times10^4\ \text{cm}^{-3}$$

$$L_B = \sqrt{D_B \tau_{B0}} = \sqrt{(10)\left(5\times10^{-7}\right)}$$

$$= 2.236\times10^{-3}\ \text{cm}$$

We find

$$\sinh\left(\frac{x_B}{L_B}\right) = \sinh\left(\frac{0.70\times10^{-4}}{2.236\times10^{-3}}\right)$$

$$= 0.03131$$

$$\tanh\left(\frac{x_B}{L_B}\right) = \tanh\left(\frac{0.70\times10^{-4}}{2.236\times10^{-3}}\right)$$

$$= 0.03130$$

Then

$$I_{nC} = \frac{\left(1.6\times10^{-19}\right)(10)\left(2.25\times10^4\right)\left(5\times10^{-4}\right)}{2.236\times10^{-3}}$$

$$\times \left\{ \frac{\left[\exp\left(\dfrac{0.550}{0.0259}\right)-1\right]}{0.03131} + \frac{1}{0.03130} \right\}$$

$$I_{nC} = 4.29\times10^{-4}\ \text{A} = 0.429\ \text{mA}$$

(b) $$\gamma = \frac{1}{1 + \dfrac{N_B}{N_E}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{x_B}{x_E}}$$

$$= \frac{1}{1+\left(\dfrac{10^{16}}{5\times10^{17}}\right)\left(\dfrac{15}{10}\right)\left(\dfrac{0.7}{0.5}\right)} = 0.95969$$

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\dfrac{0.70\times10^{-4}}{2.236\times10^{-3}}\right)}$$

$$= 0.99951$$

$$\alpha = \gamma\alpha_T\delta = (0.95969)(0.99951)(0.995)$$

$$= 0.95442$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.95442}{1-0.95442} = 20.94$$

Then

$$I_C = \beta I_B = (20.94)(0.80) = 16.75\ \mu\text{A}$$

(c) $$I_C = \alpha I_E = (0.95442)(125) = 119.3\ \mu\text{A}$$

_____

12.23

(a) We have

$$J_{nE} = \frac{eD_B n_{BO}}{L_B} \left\{ \frac{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right)-1\right]}{\tanh\left(\dfrac{x_B}{L_B}\right)} + \frac{1}{\sinh\left(\dfrac{x_B}{L_B}\right)} \right\}$$

We find that

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)\left(5\times10^{-8}\right)}$$

$$= 8.660\times10^{-4}\ \text{cm}$$

Then

$$J_{nE} = \frac{\left(1.6\times10^{-19}\right)(15)\left(4.5\times10^3\right)}{8.66\times10^{-4}}$$

$$\times \left\{ \frac{\exp\left(\dfrac{0.60}{0.0259}\right)}{\tanh\left(\dfrac{0.70}{8.66}\right)} + \frac{1}{\sinh\left(\dfrac{0.70}{8.66}\right)} \right\}$$

or

$$J_{nE} = 1.779\ \text{A/cm}^2$$

We also have

_____

$$J_{pE} = \frac{eD_E p_{EO}}{L_E}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\frac{1}{\tanh\left(\dfrac{x_E}{L_E}\right)}$$

Now

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{\left(1.5\times10^{10}\right)^2}{10^{18}} = 2.25\times10^2\ \text{cm}^{-3}$$

and

$$L_E = \sqrt{D_E \tau_{EO}} = \sqrt{(8)(10^{-8})}$$
$$= 2.828\times10^{-4}\ \text{cm}$$

Then

$$J_{pE} = \frac{\left(1.6\times10^{-19}\right)(8)\left(2.25\times10^2\right)}{2.828\times10^{-4}}$$
$$\times\left[\exp\left(\frac{0.60}{0.0259}\right)-1\right]\cdot\frac{1}{\tanh\left(\dfrac{0.8}{2.828}\right)}$$

or

$$J_{pE} = 0.04251\ \text{A/cm}^2$$

We can find

$$J_{nC} = \frac{eD_B n_{BO}}{L_B}$$

$$\times\left\{\frac{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right)-1\right]}{\sinh\left(\dfrac{x_B}{L_B}\right)}+\frac{1}{\tanh\left(\dfrac{x_B}{L_B}\right)}\right\}$$

$$= \frac{\left(1.6\times10^{-19}\right)(15)\left(4.5\times10^3\right)}{8.66\times10^{-4}}$$

$$\times\left\{\frac{\left[\exp\left(\dfrac{0.60}{0.0259}\right)-1\right]}{\sinh\left(\dfrac{0.7}{8.66}\right)}+\frac{1}{\tanh\left(\dfrac{0.7}{8.66}\right)}\right\}$$

or

$$J_{nC} = 1.773\ \text{A/cm}^2$$

The recombination current density is

$$J_R = J_{ro}\exp\left(\frac{V_{BE}}{2V_t}\right)$$
$$= \left(3\times10^{-8}\right)\exp\left[\frac{0.60}{2(0.0259)}\right]$$

or

$$J_R = 3.218\times10^{-3}\ \text{A/cm}^2$$

(b) Using the calculated current densities, we find

$$\gamma = \frac{J_{nE}}{J_{nE}+J_{pE}} = \frac{1.779}{1.779+0.04251}$$

or

$$\gamma = 0.9767$$

We also find

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.773}{1.779}$$

or

$$\alpha_T = 0.9966$$

Also

$$\delta = \frac{J_{nE}+J_{pE}}{J_{nE}+J_R+J_{pE}}$$
$$= \frac{1.779+0.04251}{1.779+0.003218+0.04251}$$

or

$$\delta = 0.9982$$

Then

$$\alpha = \gamma\alpha_T\delta = (0.9767)(0.9966)(0.9982)$$

or

$$\alpha = 0.9716$$

Now

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9716}{1-0.9716}$$

or

$$\beta = 34.2$$

_____

12.24

(a) We have

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{x_B}{x_E}} \cong 1-\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}$$

or

$$\gamma \cong 1-K\cdot\frac{N_B}{N_E}$$

(i) Now

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1-\dfrac{2N_{BO}}{N_E}\cdot K}{1-\dfrac{N_{BO}}{N_E}\cdot K}$$

$$\cong \left(1-\frac{2N_{BO}}{N_E}\cdot K\right)\left(1+\frac{N_{BO}}{N_E}\cdot K\right)$$

$$\cong 1-\frac{2N_{BO}}{N_e}\cdot K+\frac{N_{BO}}{N_E}\cdot K$$

or finally

$$\frac{\gamma(B)}{\gamma(A)} = 1 - \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)

We have

$$\gamma \cong 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} = 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K' \cdot \dfrac{x_{BO}}{2x_E}}{1 - K' \cdot \dfrac{x_{BO}}{x_E}}$$

$$\cong \left(1 - K' \cdot \frac{x_{BO}}{2x_E}\right)\left(1 + K' \cdot \frac{x_{BO}}{x_E}\right)$$

$$\cong 1 - K' \cdot \frac{x_{BO}}{2x_E} + K' \cdot \frac{x_{BO}}{x_E}$$

$$= 1 + K' \cdot \frac{x_{BO}}{2x_E}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 + \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_{BO}}{2x_E}$$

(b) (i) We find
$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = \frac{\left(1 - \dfrac{1}{2} \cdot \left[\dfrac{(x_{BO}/2)}{L_B}\right]^2\right)}{\left(1 - \dfrac{1}{2} \cdot \left[\dfrac{x_{BO}}{L_B}\right]^2\right)}$$

$$= \frac{\left(1 - \dfrac{1}{8}\left[\dfrac{x_{BO}}{L_B}\right]^2\right)}{\left(1 - \dfrac{1}{2}\left[\dfrac{x_{BO}}{L_B}\right]^2\right)}$$

$$\cong \left(1 - \frac{1}{8}\left[\frac{x_{BO}}{L_B}\right]^2\right)\left(1 + \frac{1}{2}\left[\frac{x_{BO}}{L_B}\right]^2\right)$$

$$\cong 1 - \frac{1}{8}\left[\frac{x_{BO}}{L_B}\right]^2 + \frac{1}{2}\left[\frac{x_{BO}}{L_B}\right]^2$$

or finally

$$\frac{\alpha_T(C)}{\alpha_T(A)} \cong 1 + \frac{3}{8}\left[\frac{x_{BO}}{L_B}\right]^2$$

(c) Neglect any change in space charge width.

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{sO}} \cdot \exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$\cong 1 - \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right) = 1 - \frac{K}{J_{sO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \dfrac{K}{J_{sOB}}}{1 - \dfrac{K}{J_{sOA}}} \cong \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$

$$\cong 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now

$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so

$$\frac{\delta(B)}{\delta(A)} \cong 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then finally

$$\frac{\delta(B)}{\delta(A)} = 1 - \frac{J_{rO}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}{\left(\dfrac{eD_B n_{BO}}{x_B}\right)}$$

(ii) We find

$$\frac{\delta(C)}{\delta(A)} \cong 1 + \frac{J_{rO}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}{\left(\dfrac{eD_B n_{BO}}{x_B}\right)}$$

(d) Device C has the largest β. The emitter injection efficiency, base transport, and recombination factors all increase.

_____

12.25
(a) We have

$$\gamma = \frac{1}{1 + \dfrac{N_B}{N_E} \cdot \dfrac{D_E}{D_B} \cdot \dfrac{x_B}{x_E}} = \frac{1}{1 + K \cdot \dfrac{N_B}{N_E}}$$

or

$$\gamma \cong 1 - K \cdot \frac{N_B}{N_E}$$

(i)   Then

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - K \cdot \dfrac{N_B}{2N_{EO}}}{1 - K \cdot \dfrac{N_B}{N_{EO}}}$$

$$\cong \left(1 - K \cdot \frac{N_B}{2N_{EO}}\right)\left(1 + K \cdot \frac{N_B}{N_{EO}}\right)$$

$$\cong 1 - K \cdot \frac{N_B}{2N_{EO}} + K \cdot \frac{N_B}{N_{EO}}$$

$$= 1 + K \cdot \frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)} = 1 + \frac{N_B}{2N_{EO}} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii) Now

$$\gamma = \frac{1}{1 + K' \cdot \dfrac{x_B}{x_E}} \cong 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K' \cdot \dfrac{x_B}{(x_{EO}/2)}}{1 - K' \cdot \dfrac{x_B}{x_{EO}}}$$

$$\cong \left(1 - K' \cdot \frac{2x_B}{x_{EO}}\right)\left(1 + K' \cdot \frac{x_B}{x_{EO}}\right)$$

$$\cong 1 - 2K' \cdot \frac{x_B}{x_{EO}} + K' \cdot \frac{x_B}{x_{EO}}$$

$$= 1 - K' \cdot \frac{x_B}{x_{EO}}$$

or finally

$$\frac{\gamma(C)}{\gamma(A)} = 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_{EO}}$$

(b) We have

$$\alpha_T = 1 - \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2$$

(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = 1$$

(c) Neglect any change in space charge width.

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{sO}} \exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \dfrac{K}{J_{sO}}} \cong 1 - \frac{K}{J_{sO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \dfrac{K}{J_{sOB}}}{1 - \dfrac{K}{J_{sOA}}} \cong \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$

$$\cong 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now

$$J_{sO} \propto \frac{1}{N_E x_E}$$

so

$$\frac{\delta(B)}{\delta(A)} = 1 - K'(2N_{EO}) + K'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)} = 1 - K' \cdot N_{EO}$$

Recombination factor decreases
(ii) We have

$$\frac{\delta(C)}{\delta(A)} = 1 - K''\left(\frac{x_{EO}}{2}\right) + K''(x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)} = 1 + \frac{1}{2}K'' \cdot x_{EO}$$

Recombination factor increases

_____

12.26
  (b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{17}} = 2.25 \times 10^3 \, \text{cm}^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(2.25 \times 10^3\right) \exp\left(\frac{0.6}{0.0259}\right)$$

$$= 2.59 \times 10^{13} \, \text{cm}^{-3}$$

Now

_____

$$J_{nC} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6\times10^{-19})(20)(2.59\times10^{13})}{10^{-4}}$$

or

$$J_{nC} = 0.828 \text{ A/cm}^2$$

Assuming a long collector

$$J_{pC} = \frac{eD_C p_{nO}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{cO} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$$

and

$$L_C = \sqrt{D_C \tau_{CO}} = \sqrt{(15)(2\times10^{-7})}$$

$$= 1.732\times10^{-3} \text{ cm}$$

Then

$$J_{pC} = \frac{(1.6\times10^{-19})(15)(2.25\times10^4)}{1.732\times10^{-3}}$$

$$\times \exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359 \text{ A/cm}^2$$

The collector current is

$$I_C = (J_{nC} + J_{pC})\cdot A$$

$$= (0.828 + 0.359)(10^{-3})$$

or

$$I_C = 1.19 \text{ mA}$$

The emitter current is

$$I_E = J_{nC} \cdot A = (0.828)(10^{-3})$$

or

$$I_E = 0.828 \text{ mA}$$

_____

12.27

(a)

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} \quad \text{and} \quad \beta = \frac{\alpha_T}{1-\alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	$\cong 0$

(b) For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1+(p_{EO}/n_{BO})} = \frac{1}{1+(N_B/N_E)}$$

and

$$\beta = \frac{\gamma}{1-\gamma}$$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c) For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E << 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

_____

12.28

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B \tanh(x_B/L_B)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3 \text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)(10^{-7})}$$

$$= 15.8\times10^{-4} \text{ cm}$$

Then

$$J_{sO} = \frac{(1.6\times10^{-19})(25)(2.25\times10^3)}{(15.8\times10^{-4})\tanh(0.7/15.8)}$$

or

$$J_{sO} = 1.287\times10^{-10} \text{ A/cm}^2$$

Now

_____

$$\delta = \frac{1}{1+\dfrac{J_{rO}}{J_{sO}}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1+\dfrac{2\times10^{-9}}{1.287\times10^{-10}}\exp\left[\dfrac{-V_{BE}}{2(0.0259)}\right]}$$

or

(a)

$$\delta = \frac{1}{1+(15.54)\exp\left(\dfrac{-V_{BE}}{0.0518}\right)}$$

and

(b)

$$\beta = \frac{\delta}{1-\delta}$$

Now

V_{BE}	δ	β
0.20	0.7535	3.06
0.40	0.99316	145
0.60	0.999855	6,902

(c) If $V_{BE} < 0.4$ V, the recombination factor is likely the limiting factor in the current gain.

_____

12.29

$$\alpha = \frac{\beta}{1+\beta} = \frac{150}{151} = 0.993377$$

$$\alpha = \gamma\alpha_T\delta$$

$$0.993377 = (\gamma\alpha_T)(0.9975)$$

$$\Rightarrow \gamma\alpha_T = 0.995867$$

Let $x_B = 0.80\,\mu$m

$$L_B = \sqrt{D_B\tau_{B0}} = \sqrt{(23)(2\times10^{-7})}$$

$$= 2.145\times10^{-3}\,\text{cm}$$

Then

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\dfrac{0.80\times10^{-4}}{2.145\times10^{-3}}\right)}$$

$$= 0.99930$$

Now

$$\gamma = \frac{\alpha}{\alpha_T\delta} = \frac{0.993377}{(0.99930)(0.9975)} = 0.99656$$

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{x_B}{x_E}}$$

$$0.99656 = \frac{1}{1+\left(\dfrac{2\times10^{16}}{N_E}\right)\left(\dfrac{8}{23}\right)\left(\dfrac{0.80}{0.35}\right)}$$

$$\Rightarrow N_E = 4.61\times10^{18}\,\text{cm}^{-3}$$

_____

12.30

(a) We have $J_{rO} = 5\times10^{-8}\,\text{A/cm}^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 4.5\times10^3\,\text{cm}^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})}$$

$$= 15.8\times10^{-4}\,\text{cm}$$

Then

$$J_{sO} = \frac{eD_Bn_{BO}}{L_B\,\tanh(x_B/L_B)}$$

$$= \frac{(1.6\times10^{-19})(25)(4.5\times10^3)}{(15.8\times10^{-4})\tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.139\times10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1+\dfrac{J_{rO}}{J_{sO}}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

For $T = 300$ K and $V_{BE} = 0.55$ V.

$$\delta = 0.995$$

$$= \frac{1}{1+\left(\dfrac{5\times10^{-8}}{1.139\times10^{-11}}\right)\cdot\tanh\left(\dfrac{x_B}{L_B}\right)\cdot\exp\left(\dfrac{-0.55}{0.0518}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.0468$$

or

$$x_B = (0.0468)(15.8) = 0.739\,\mu\text{m}$$

(b) For $T = 400$ K and $J_{rO} = 5\times10^{-8}\,\text{A/cm}^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3\cdot\frac{\exp\left[\dfrac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\dfrac{-E_g}{0.0259}\right]}$$

For $E_g = 1.12$ eV,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.175 \times 10^5$$

or

$$n_{BO}(400) = (1.175 \times 10^5)(4.5 \times 10^3)$$
$$= 5.29 \times 10^8 \text{ cm}^{-3}$$

Then

$$J_{sO} = \frac{(1.6 \times 10^{-19})(25)(5.29 \times 10^8)}{(15.8 \times 10^{-4})\tanh(0.739/15.8)}$$

or

$$J_{sO} = 2.865 \times 10^{-5} \text{ A/cm}^2$$

Finally

$$\delta = \frac{1}{1 + \frac{5 \times 10^{-8}}{2.865 \times 10^{-5}} \cdot \exp\left[\frac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\delta = 0.9999994$$

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12.31

Plot

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12.32

Plot

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12.33

Plot

_____

12.34

Plot

_____

12.35

(a) $I_C = \frac{1}{r_o}(V_{CE} + V_A) \Rightarrow r_o = \frac{(V_{CE} + V_A)}{I_C}$

(i) $r_o = \frac{2 + 120}{1.2} = 101.67 \text{ k}\Omega$

(ii) $g_o = \frac{1}{r_o} = \frac{1}{101.67} = 0.00984 \text{ (k}\Omega)^{-1}$
$$= 9.84 \times 10^{-6} (\Omega)^{-1}$$

(iii) $I_C = \frac{4 + 120}{101.667} = 1.22 \text{ mA}$

(b)

(i) $r_o = \frac{V_{CE} + V_A}{I_C} = \frac{2 + 160}{0.25} = 648 \text{ k}\Omega$

(ii) $g_o = \frac{1}{r_o} = \frac{1}{648} = 0.00154 \text{ (k}\Omega)^{-1}$
$$= 1.54 \times 10^{-6} (\Omega)^{-1}$$

(iii) $I_C = \frac{4 + 160}{648} = 0.253 \text{ mA}$

_____

12.36

$$r_o = \frac{\Delta V_{EC}}{\Delta I_C} \Rightarrow \Delta I_C = \frac{\Delta V_{EC}}{r_o} = \frac{5 - 2}{180}$$
$$\Delta I_C = 0.01667 \text{ mA} = 16.67 \mu\text{A}$$

_____

12.37

$$x_{dB} = \left\{\frac{2 \in_s (V_{bi} + V_{CB})}{e}\left[\frac{N_C}{N_B} \cdot \frac{1}{(N_B + N_C)}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left[\frac{2 \times 10^{15}}{2 \times 10^{16}} \cdot \frac{1}{(2 \times 10^{15} + 2 \times 10^{16})}\right]\right\}^{1/2}$$

$$= \left\{(5.8832 \times 10^{-11})(V_{bi} + V_{CB})\right\}^{1/2}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_B N_C}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(2 \times 10^{15})(2 \times 10^{16})}{(1.5 \times 10^{10})^2}\right]$$

$$= 0.6709 \text{ V}$$

(i) For $V_{CB} = 4$ V, $x_{dB} = 0.1658 \mu\text{m}$

(ii) For $V_{CB} = 8$ V, $x_{dB} = 0.2259 \mu\text{m}$

(iii) For $V_{CB} = 12$ V, $x_{dB} = 0.2730 \mu\text{m}$

Neglecting the B-E space charge width,

(i) For $V_{CB} = 4$ V,
$$x_B = 0.85 - 0.1658 = 0.6842 \mu\text{m}$$

(ii) For $V_{CB} = 8$ V,
$$x_B = 0.85 - 0.2259 = 0.6241 \mu\text{m}$$

(iii) For $V_{CB} = 12$ V,
$$x_B = 0.85 - 0.2730 = 0.5770 \mu\text{m}$$

Now

$$J_C = \frac{eD_B n_{B0}}{x_B} \exp\left(\frac{V_{BE}}{V_t}\right)$$

where

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{16}}$$

$$= 1.125 \times 10^4 \text{ cm}^{-3}$$

so

$$J_C = \frac{(1.6 \times 10^{-19})(25)(1.125 \times 10^4)}{x_B} \exp\left(\frac{0.650}{0.0259}\right)$$

$$= \frac{3.5686 \times 10^{-3}}{x_B} \text{ A/cm}^2$$

(i) For $V_{CB} = 4$ V, $J_C = 52.16$ A/cm^2

(ii) For $V_{CB} = 8$ V, $J_C = 57.18$ A/cm^2

(iii) For $V_{CB} = 12$ V, $J_C = 61.85$ A/cm^2

(b) $\quad \dfrac{\Delta J_C}{\Delta V_{CE}} = \dfrac{J_C}{V_{CE} + V_A}$

$$\frac{61.85 - 52.16}{12 - 4} = \frac{52.16}{4 + 0.650 + V_A}$$

$$\Rightarrow V_A = 38.4 \text{ V}$$

_____

12.38

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{16}} = 7.5 \times 10^3 \text{ cm}^{-3}$$

and

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (7.5 \times 10^3) \exp\left(\frac{0.7}{0.0259}\right)$$

or

$$n_B(0) = 4.10 \times 10^{15} \text{ cm}^{-3}$$

We have

$$J = eD_B \frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6 \times 10^{-19})(20)(4.10 \times 10^{15})}{x_B}$$

or

$$J = \frac{1.312 \times 10^{-2}}{x_B} \text{ A/cm}^2$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259) \ln\left[\frac{(3 \times 10^{16})(5 \times 10^{15})}{(1.5 \times 10^{10})^2}\right]$$

or

$$V_{bi} = 0.705 \text{ V}$$

Also

$$x_p = \left[\frac{2 \in_s}{e}(V_{bi} + V_{CB})\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_B + N_C}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{3 \times 10^{16}}\right)\left(\frac{1}{5 \times 10^{15} + 3 \times 10^{16}}\right)\right]^{1/2}$$

or

$$x_p = \left[(6.163 \times 10^{-11})(V_{bi} + V_{CB})\right]^{1/2}$$

For $V_{CB} = 5$ V, $x_p = 0.1875 \, \mu$m

For $V_{CB} = 10$ V, $x_p = 0.2569 \, \mu$m

(a) For $x_{BO} = 1.0 \, \mu$m

For $V_{CB} = 5$ V,

$$x_B = 1.0 - 0.1875 = 0.8125 \, \mu\text{m}$$

Then

$$J = \frac{1.312 \times 10^{-2}}{0.8125 \times 10^{-4}} = 161.5 \text{ A/cm}^2$$

For $V_{CB} = 10$ V,

$$x_B = 1.0 - 0.2569 = 0.7431 \, \mu\text{m}$$

and

$$J = \frac{1.312 \times 10^{-2}}{0.7431 \times 10^{-4}} = 176.6 \text{ A/cm}^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

where

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6 - 161.5}{10 - 5}$$

$$= 3.02 \text{ A/cm}^2/\text{V}$$

Then

$$161.5 = (3.02)(5.7 + V_A)$$

which yields

$$V_A = 47.8 \text{ V}$$

(b) For $x_{BO} = 0.80\,\mu\text{m}$

For $V_{CB} = 5$ V,

$x_B = 0.80 - 0.1875 = 0.6125\,\mu$ m

Then

$$J = \frac{1.312 \times 10^{-2}}{0.6125 \times 10^{-4}} = 214.2\ \text{A/cm}^2$$

For $V_{CB} = 10$ V,

$x_B = 0.80 - 0.2569 = 0.5431\,\mu$ m

and

$$J = \frac{1.312 \times 10^{-2}}{0.5431 \times 10^{-4}} = 241.6\ \text{A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{10 - 5}$$

$$= 5.48\ \text{A/cm}^2/\text{V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

or

$$214.2 = (5.48)(5.7 + V_A)$$

which yields

$V_A = 33.4$ V

(c) For $x_{BO} = 0.60\,\mu\text{m}$

For $V_{CB} = 5$ V,

$x_B = 0.60 - 0.1875 = 0.4125\,\mu$ m

Then

$$J = \frac{1.312 \times 10^{-2}}{0.4125 \times 10^{-4}} = 318.1\ \text{A/cm}^2$$

For $V_{CB} = 10$ V,

$x_B = 0.60 - 0.2569 = 0.3431\,\mu$ m

and

$$J = \frac{1.312 \times 10^{-2}}{0.3431 \times 10^{-4}} = 382.4\ \text{A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{10 - 5}$$

$$= 12.86\ \text{A/cm}^2/\text{V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

or

$$318.1 = (12.86)(5.7 + V_A)$$

which yields

$V_A = 19.0$ V

_____

12.39

(a)

$$x_{dB} = \left\{ \frac{2 \in_s}{e}\left(V_{bi} + V_{BC}\right)\left[\frac{N_C}{N_B} \cdot \frac{1}{\left(N_B + N_C\right)}\right]\right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)\left(8.85 \times 10^{-14}\right)\left(V_{bi} + V_{BC}\right)}{1.6 \times 10^{-19}} \right.$$

$$\left. \times\left[\frac{10^{15}}{10^{16}} \cdot \frac{1}{\left(10^{15} + 10^{16}\right)}\right]\right\}^{1/2}$$

$$= \left\{\left(1.1766 \times 10^{-10}\right)\left(V_{bi} + V_{BC}\right)\right\}^{1/2}$$

Now

$$V_{bi} = V_t\ln\left(\frac{N_B N_C}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(10^{15}\right)\left(10^{16}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$$

$$= 0.6350\ \text{V}$$

For $V_{BC} = 1$ V, $x_{dB} = 0.1387\,\mu$ m

For $V_{BC} = 5$ V, $x_{dB} = 0.2575\,\mu$ m

Then $\Delta x_{dB} = 0.2575 - 0.1387 = 0.1188\,\mu$ m

(b) $I_C = \dfrac{eD_B p_{B0} A_{BE}}{x_B}\exp\left(\dfrac{V_{EB}}{V_t}\right)$

We find

$$p_{B0} = \frac{n_i^2}{N_B} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{16}}$$

$$= 2.25 \times 10^4\ \text{cm}^{-3}$$

Then

$$I_C = \frac{\left(1.6 \times 10^{-19}\right)(10)\left(2.25 \times 10^4\right)\left(10^{-4}\right)}{x_B}$$

$$\times\exp\left(\frac{0.625}{0.0259}\right)$$

$$= \frac{1.0874 \times 10^{-7}}{x_B}\ \text{A}$$

For $V_{BC} = 1$ V, $I_C = \dfrac{1.0874 \times 10^{-7}}{(0.70 - 0.1387) \times 10^{-4}}$

$$= 1.937 \times 10^{-3}\ \text{A} = 1.937\ \text{mA}$$

For $V_{BC} = 5$ V, $I_C = \dfrac{1.0874 \times 10^{-7}}{(0.70 - 0.2575) \times 10^{-4}}$

$$= 2.456 \times 10^{-3}\ \text{A} = 2.456\ \text{mA}$$

Then

$\Delta I_C = 2.456 - 1.937 = 0.519$ mA

_____

(c) $\dfrac{\Delta I_C}{\Delta V_{BC}} = \dfrac{I_C}{V_{EC} + V_A}$

$\dfrac{0.519 \times 10^{-3}}{5 - 1} = \dfrac{1.937 \times 10^{-3}}{1 + 0.625 + V_A}$

$V_A = 13.3 \text{ V}$

(d) $r_o = \dfrac{V_{EC} + V_A}{I_C} = \dfrac{1.625 + 13.3}{1.937 \times 10^{-3}}$

$= 7.705 \times 10^3 \ \Omega = 7.705 \text{ k}\Omega$

_____

12.40

Let $x_B = x_E$, $L_B = L_E$, $D_B = D_E$

Then the emitter injection efficiency is

$\gamma = \dfrac{1}{1 + \dfrac{p_{EO}}{n_{BO}}} = \dfrac{1}{1 + \dfrac{n_{iE}^2}{N_E} \cdot \dfrac{N_B}{n_{iB}^2}}$

where $n_{iB}^2 = n_i^2$.

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing,

$$n_{iE}^2 = n_i^2 \exp\left(\dfrac{\Delta E_g}{kT}\right)$$

Then

$\gamma = \dfrac{1}{1 + \dfrac{N_B}{N_E} \exp\left(\dfrac{\Delta E_g}{kT}\right)}$

(a) No bandgap narrowing, so $\Delta E_g = 0$

$\alpha = \gamma \alpha_T \delta = \gamma (0.995)^2$. We find

N_E	γ	α	β
10^{17}	0.5	0.495	0.980
10^{18}	0.909	0.8999	8.99
10^{19}	0.990	0.980	49.3
10^{20}	0.9990	0.989	90.2

(b) Taking into account bandgap narrowing, we find

N_E	ΔE_g (meV)	γ	α	β
10^{17}	0	0.5	0.495	0.98
10^{18}	25	0.792	0.784	3.63
10^{19}	80	0.820	0.812	4.32
10^{20}	230	0.122	0.121	0.14

_____

12.41

(a) We have

$\gamma = \dfrac{1}{1 + \dfrac{p_{EO} D_E L_B}{n_{BO} D_B L_E} \cdot \dfrac{\tanh(x_B / L_B)}{\tanh(x_E / L_E)}}$

For $x_B = x_E$, $L_B = L_E$, $D_B = D_E$, we obtain

$\gamma = \dfrac{1}{1 + \dfrac{N_B}{N_E} \exp\left(\dfrac{\Delta E_g}{kT}\right)}$

For $N_E = 10^{19} \text{ cm}^{-3}$, we have $\Delta E_g = 80 \text{ meV}$
Then

$0.996 = \dfrac{1}{1 + \dfrac{N_B}{10^{19}} \exp\left(\dfrac{0.080}{0.0259}\right)}$

which yields

$N_B = 1.83 \times 10^{15} \text{ cm}^{-3}$

(b) Neglecting bandgap narrowing, we would have

$\gamma = \dfrac{1}{1 + \dfrac{N_B}{N_E}} \Rightarrow 0.996 = \dfrac{1}{1 + \dfrac{N_B}{10^{19}}}$

which yields

$N_B = 4.02 \times 10^{16} \text{ cm}^{-3}$

_____

12.42

(a)

(i) $R = \dfrac{(\rho)(Length)}{Area} = \dfrac{(S/2)}{\sigma(x_B L)}$

$= \dfrac{(S/2)}{(e \mu_p N_B)(x_B L)}$

$= \dfrac{1}{(1.6 \times 10^{-19})(250)(2 \times 10^{16})}$

$\times \dfrac{5 \times 10^{-4}}{(0.65 \times 10^{-4})(25 \times 10^{-4})}$

$R = 3.846 \times 10^3 \ \Omega = 3.846 \text{ k}\Omega$

(ii) $\Delta V = (I_B / 2) R = (5 \times 10^{-6})(3.846 \times 10^3)$

$= 0.01923 \text{ V}$

_____

(iii) $\dfrac{\delta n_B(x=S/2)}{\delta n_B(x=0)} = \dfrac{\exp\left[\dfrac{V_{BE}(x=S/2)}{0.0259}\right]}{\exp\left[\dfrac{V_{BE}(x=0)}{0.0259}\right]}$

$= \dfrac{\exp\left[\dfrac{0.60}{0.0259}\right]}{\exp\left[\dfrac{0.60+\Delta V}{0.0259}\right]}$

$= \exp\left[\dfrac{-\Delta V}{0.0259}\right]$

$= \exp\left[\dfrac{-0.01923}{0.0259}\right]$

Then

$\dfrac{\delta n_B(x=S/2)}{\delta n_B(x=0)} = 0.476$

(b)

(i) $R = \dfrac{1}{(1.6\times10^{-19})(250)(2\times10^{16})}$

$\times \dfrac{1.5\times10^{-4}}{(0.65\times10^{-4})(25\times10^{-4})}$

$= 1.154\times10^3\ \Omega = 1.154\ k\Omega$

(ii) $\Delta V = (I_B/2)R = (5\times10^{-6})(1.154\times10^3)$

$= 0.005769\ V$

(iii) $\dfrac{\delta n_B(x=S/2)}{\delta n_B(x=0)} = \exp\left[\dfrac{-\Delta V}{V_t}\right]$

$= \exp\left[\dfrac{-0.005769}{0.0259}\right]$

Then

$\dfrac{\delta n_B(x=S/2)}{\delta n_B(x=0)} = 0.80$

_____

12.43

$\dfrac{\delta n_B(x=S/2)}{\delta n_B(x=0)} = 0.90 = \exp\left[\dfrac{-\Delta V}{V_t}\right]$

Then

$\Delta V = V_t\ \ln\left(\dfrac{1}{0.90}\right) = (0.0259)\ln\left(\dfrac{1}{0.90}\right)$

$\Delta V = 0.002729\ V = (I_B/2)R$

$= (5\times10^{-6})R$

$R = 545.8\ \Omega$

Now

$R = \dfrac{1}{e\mu_p N_B} \cdot \dfrac{(S/2)}{x_B L}$

$545.8 = \dfrac{1}{(1.6\times10^{-19})(250)(2\times10^{16})}$

$\times \dfrac{(S/2)}{(0.65\times10^{-4})(25\times10^{-4})}$

$\Rightarrow S = 1.42\times10^{-4}\ cm = 1.42\ \mu\ m$

_____

12.44

(a)

$N_B = N_B(0)\exp\left(\dfrac{-ax}{x_B}\right)$

where

$a = \ln\left[\dfrac{N_B(0)}{N_B(x_B)}\right] > 0$

and is a constant. In thermal equilibrium

$J_p = e\mu_p N_B E - eD_p\ \dfrac{dN_B}{dx} = 0$

so that

$E = \dfrac{D_p}{\mu_p} \cdot \dfrac{1}{N_B} \cdot \dfrac{dN_B}{dx} = \left(\dfrac{kT}{e}\right) \cdot \dfrac{1}{N_B} \cdot \dfrac{dN_B}{dx}$

which becomes

$E = \left(\dfrac{kT}{e}\right) \cdot \dfrac{1}{N_B} \cdot N_B(0) \cdot \left(\dfrac{-a}{x_B}\right) \cdot \exp\left(\dfrac{-ax}{x_B}\right)$

$= \left(\dfrac{kT}{e}\right) \cdot \left(\dfrac{-a}{x_B}\right) \cdot \dfrac{1}{N_B} \cdot N_B$

or

$E = -\left(\dfrac{a}{x_B}\right)\left(\dfrac{kT}{e}\right)$

which is a constant.

(b) The electric field is in the negative
x-direction which will aid the flow of
minority carrier electrons across the base.

(c)

$J_n = e\mu_n n E + eD_n\ \dfrac{dn}{dx}$

Assuming no recombination in the base, J_n
will be a constant across the base. Then

$\dfrac{dn}{dx} + \left(\dfrac{\mu_n}{D_n}\right) \cdot nE = \dfrac{J_n}{eD_n} = \dfrac{dn}{dx} + n\left(\dfrac{E}{V_t}\right)$

where $V_t = \left(\dfrac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \dfrac{\text{E}}{V_t}$

The solution is of the form

$$n_H = n_H(0)\exp(-Ax)$$

The particular solution is found from

$$n_P \cdot A = B$$

where $B = \dfrac{J_n}{eD_n}$

The particular solution is then

$$n_P = \frac{B}{A} = \frac{\left(\dfrac{J_n}{eD_n}\right)}{\left(\dfrac{\text{E}}{V_t}\right)} = \frac{J_n V_t}{eD_n \text{E}} = \frac{J_n}{e\mu_n \text{E}}$$

The total solution is then

$$n = \frac{J_n}{e\mu_n \text{E}} + n_H(0)\exp(-Ax)$$

and

$$n(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right)$$

Then

$$n_H(0) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n \text{E}}$$

_____

12.45

(a) For $N_C = 2\times10^{15}\,\text{cm}^{-3}$,
$$BV_{BC0} \cong 180\,\text{V}$$

(b) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9930}{1-0.9930} = 141.86$

$$BV_{EC0} = \frac{BV_{BC0}}{\sqrt[n]{\beta}} = \frac{180}{\sqrt[3]{141.86}} = 34.5\,\text{V}$$

(c) For $N_B = 5\times10^{16}\,\text{cm}^{-3}$,
$$BV_{EB} \cong 19\,\text{V}$$

_____

12.46

We want $BV_{CEO} = 60\,\text{V}$

Then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221\,\text{V}$$

For this breakdown voltage, we need

$$N_C \cong 1.5\times10^{15}\,\text{cm}^{-3}$$

The depletion width into the collector at this voltage is

$$x_C = x_n$$

$$= \left[\frac{2\in_s (V_{bi}+V_{BC})}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right]^{1/2}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(1.5\times10^{15})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.646\,\text{V}$$

and

$$V_{BC} \cong BV_{CEO} = 60\,\text{V}$$

so that

$$x_C = \left[\frac{2(11.7)(8.85\times10^{-14})(0.646+60)}{1.6\times10^{-19}}\right.$$

$$\left. \times\left(\frac{10^{16}}{1.5\times10^{15}}\right)\left(\frac{1}{10^{16}+1.5\times10^{15}}\right)\right]^{1/2}$$

or

$$x_C = 6.75\times10^{-4}\,\text{cm} = 6.75\,\mu\text{m}$$

_____

12.47

(a) For $N_C = 8\times10^{15}\,\text{cm}^{-3}$,
$$BV_{CB0} \cong 64\,\text{V}$$

(b) $V_{pt} = \dfrac{ex_{B0}^2}{2\in_s} \cdot \dfrac{N_B(N_C+N_B)}{N_C}$

$$= \frac{(1.6\times10^{-19})(0.50\times10^{-4})^2}{2(11.7)(8.85\times10^{-14})}$$

$$\times\frac{(5\times10^{16})(8\times10^{15}+5\times10^{16})}{8\times10^{15}}$$

$$V_{pt} = 70.0\,\text{V}$$

_____

12.48

(a) $V_{pt} = \dfrac{ex_{B0}^2}{2 \in_s} \cdot \dfrac{N_B(N_C + N_B)}{N_C}$

$= \dfrac{(1.6 \times 10^{-19})(0.65 \times 10^{-4})^2}{2(11.7)(8.85 \times 10^{-14})}$

$\times \dfrac{(2 \times 10^{16})(5 \times 10^{15} + 2 \times 10^{16})}{5 \times 15}$

$V_{pt} = 32.6\,\text{V}$

(b) From Chapter 7,

$|E_{max}| \cong \left\{ \dfrac{2eV_{pt}}{\in_s} \left(\dfrac{N_B N_C}{N_B + N_C} \right) \right\}^{1/2}$

$= \left\{ \dfrac{2(1.6 \times 10^{-19})(32.6)}{(11.7)(8.85 \times 10^{-14})} \right.$

$\left. \times \left[\dfrac{(5 \times 10^{15})(2 \times 10^{16})}{5 \times 10^{15} + 2 \times 10^{16}} \right] \right\}^{1/2}$

$|E_{max}| = 2.01 \times 10^5\,\text{V/cm}$

_____

12.49

$V_{pt} = \dfrac{ex_{B0}^2}{2 \in_s} \cdot \dfrac{N_B(N_C + N_B)}{N_C}$

$15 = \dfrac{(1.6 \times 10^{-19})(x_{B0}^2)}{2(11.7)(8.85 \times 10^{-14})}$

$\times \dfrac{(5 \times 10^{16})(3 \times 10^{15} + 5 \times 10^{16})}{3 \times 10^{15}}$

$\Rightarrow x_{B0} = 1.483 \times 10^{-5}\,\text{cm} = 0.1483\,\mu\text{m}$

_____

12.50

We have

$V_{CE}(sat) = V_t \cdot \ln\left[\dfrac{I_C(1 - \alpha_R) + I_B}{\alpha_F I_B - I_C(1 - \alpha_F)} \cdot \dfrac{\alpha_F}{\alpha_R} \right]$

We can write

$\exp\left[\dfrac{V_{CE}(sat)}{0.0259} \right] = \dfrac{(1)(1 - 0.2) + I_B}{(0.99)I_B - (1)(1 - 0.99)} \left(\dfrac{0.99}{0.20} \right)$

or

$\exp\left[\dfrac{V_{CE}(sat)}{0.0259} \right] = \left(\dfrac{0.8 + I_B}{0.99 I_B - 0.01} \right)(4.95)$

(a) For $V_{CE}(sat) = 0.30\,\text{V}$, we find

$\exp\left[\dfrac{0.30}{0.0259} \right] = 1.0726 \times 10^5$

$= \left(\dfrac{0.8 + I_B}{0.99 I_B - 0.01} \right)(4.95)$

We find

$I_B = 0.01014\,\text{mA} = 10.14\,\mu\text{A}$

(b) For $V_{CE}(sat) = 0.20\,\text{V}$, we find

$I_B = 0.0119\,\text{mA} = 11.9\,\mu\text{A}$

(c) For $V_{CE}(sat) = 0.10\,\text{V}$, we find

$I_B = 0.105\,\text{mA} = 105\,\mu\text{A}$

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12.51

For an npn transistor biased in the active mode, we have $V_{BC} < 0$, so that

$\exp\left(\dfrac{V_{BC}}{V_t} \right) \cong 0$. Now

$I_E + I_B + I_C = 0 \Rightarrow I_B = -(I_C + I_E)$

Then we have

$I_B = -\left\{ \alpha_F I_{ES} \left[\exp\left(\dfrac{V_{BE}}{V_t} \right) - 1 \right] + I_{CS} \right\}$

$-\left\{ -\alpha_R I_{CS} - I_{ES} \left[\exp\left(\dfrac{V_{BE}}{V_t} \right) - 1 \right] \right\}$

or

$I_B = (1 - \alpha_F) I_{ES} \left[\exp\left(\dfrac{V_{BE}}{V_t} \right) - 1 \right]$

$-(1 - \alpha_R) I_{CS}$

_____

12.52

We can write

$I_{ES} \left[\exp\left(\dfrac{V_{BE}}{V_t} \right) - 1 \right]$

$= \alpha_R I_{CS} \left[\exp\left(\dfrac{V_{BC}}{V_t} \right) - 1 \right] - I_E$

Substituting, we find

$I_C = \alpha_F \left\{ \alpha_R I_{CS} \left[\exp\left(\dfrac{V_{BC}}{V_t} \right) - 1 \right] - I_E \right\}$

$- I_{CS} \left[\exp\left(\dfrac{V_{BC}}{V_t} \right) - 1 \right]$

From the definition of currents, we have
$I_E = -I_C$ for the case of $I_B = 0$. Then

$$I_C = \alpha_F \alpha_R I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right] + \alpha_F I_C$$

$$- I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

When a C-E voltage is applied, then the B-C

becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right) \cong 0$.

Then
$$I_C = -\alpha_F \alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$
Finally, we find
$$I_C = I_{CEO} = \frac{I_{CS}(1 - \alpha_F \alpha_R)}{1 - \alpha_F}$$

12.53

(a) $I_C = \alpha_F I_{ES} \left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1 \right]$

$$- I_{CS} \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1 \right]$$

For $V_{BE} = 0.2$ V,

$$I_C = (0.992)(5 \times 10^{-14}) \left[\exp\left(\frac{0.20}{0.0259}\right) - 1 \right]$$

$$- (10^{-13}) \left[\exp\left(\frac{V_{BC}}{0.0259}\right) - 1 \right]$$

$$= 1.1197 \times 10^{-10}$$

$$- (10^{-13}) \left[\exp\left(\frac{V_{BC}}{0.0259}\right) - 1 \right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 1.1197 \times 10^{-10} - 2.4214 \times 10^{-5}$$

$$= -2.421 \times 10^{-5} \text{ A} = -24.21\,\mu\text{A}$$

For $V_{CB} = -V_{BC} = -0.25$ V

$$I_C = 1.1197 \times 10^{-10} - 1.5561 \times 10^{-9}$$

$$= -1.44 \times 10^{-9} \text{ A}$$

For $V_{CB} = -V_{BC} \geq 0$ V

$$I_C = 1.1197 \times 10^{-10} \text{ A}$$

(b) For $V_{BE} = 0.4$ V,

$$I_C = 2.5277 \times 10^{-7}$$

$$- (10^{-13}) \left[\exp\left(\frac{V_{BC}}{0.0259}\right) - 1 \right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 2.5277 \times 10^{-7} - 2.4214 \times 10^{-5}$$

$$= -2.396 \times 10^{-5} \text{ A} \cong -24\,\mu\text{A}$$

For $V_{CB} = -V_{BC} = -0.25$ V

$$I_C = 2.5277 \times 10^{-7} - 1.5561 \times 10^{-9}$$

$$= 2.51 \times 10^{-7} \text{ A} = +0.251\,\mu\text{A}$$

For $V_{CB} = -V_{BC} \geq 0$ V

$$I_C = 2.5277 \times 10^{-7} \text{ A} = 0.2528\,\mu\text{A}$$

(c) For $V_{BE} = 0.6$ V,

$$I_C = 5.7063 \times 10^{-4}$$

$$- (10^{-13}) \left[\exp\left(\frac{V_{BC}}{0.0259}\right) - 1 \right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 5.7063 \times 10^{-4} - 2.4214 \times 10^{-5}$$

$$= 5.464 \times 10^{-4} \text{ A} = 0.5464 \text{ mA}$$

For $V_{CB} = -V_{BC} \geq -0.25$ V

$$I_C = 5.7063 \times 10^{-4} \text{ A} = 0.5706 \text{ mA}$$

12.54

$$V_{CE}(sat) = V_t \ln\left[\frac{I_C(1 - \alpha_R) + I_B}{\alpha_F I_B - (1 - \alpha_F)I_C} \cdot \frac{\alpha_F}{\alpha_R} \right]$$

$$= (0.0259) \ln\left[\frac{(5)(1 - 0.15) + I_B}{(0.975)I_B - (1 - 0.975)(5)} \left(\frac{0.975}{0.150}\right) \right]$$

$$= (0.0259) \ln\left[\frac{4.25 + I_B}{(0.975)I_B - 0.125}(6.5) \right]$$

$I_B = 0.15$ A, $V_{CE}(sat) = 0.187$ V
$I_B = 0.25$ A, $V_{CE}(sat) = 0.143$ V
$I_B = 0.50$ A, $V_{CE}(sat) = 0.115$ V
$I_B = 1.0$ A, $V_{CE}(sat) = 0.0956$ V

12.55

(a) (i) $r'_e = \dfrac{V_t}{I_E} = \dfrac{0.0259}{0.25} = 0.1036\,\text{k}\Omega$

$$\tau_e = r'_e C_{je} = (103.6)(0.35 \times 10^{-12})$$

$$= 3.626 \times 10^{-11} \text{ s} = 36.26 \text{ ps}$$

(ii) $\tau_b = \dfrac{x_B^2}{2D_n} = \dfrac{(0.65 \times 10^{-4})^2}{2(25)}$

$$= 8.45 \times 10^{-11} \text{ s} = 84.5 \text{ ps}$$

(iii) $\tau_d = \dfrac{x_{dc}}{\upsilon_s} = \dfrac{2.2\times10^{-4}}{10^7}$

$= 2.2\times10^{-11}\,\text{s} = 22\,\text{ps}$

(iv) $\tau_c = r_c\left(C_\mu + C_s\right)$

$= (18)(0.020+0.020)\times10^{-12}$

$= 7.2\times10^{-13}\,\text{s} = 0.72\,\text{ps}$

(b) $\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$

$= 36.26 + 84.5 + 22 + 0.72 = 143.48\,\text{ps}$

(c) $f_T = \dfrac{1}{2\pi\tau_{ec}} = \dfrac{1}{2\pi\left(143.48\times10^{-12}\right)}$

$= 1.109\times10^9\,\text{Hz} = 1.109\,\text{GHz}$

(d) $f_\beta = \dfrac{f_T}{\beta} = \dfrac{1.109\times10^9}{125}$

$= 8.87\times10^6\,\text{Hz} = 8.87\,\text{MHz}$

12.56

$\tau_b = \dfrac{x_B^2}{2D_B} = \dfrac{\left(0.5\times10^{-4}\right)^2}{2(20)} = 6.25\times10^{-11}\,\text{s}$

We have

$\tau_b = 0.2\tau_{ec}$

so that

$\tau_{ec} = 3.125\times10^{-10}\,\text{s}$

Then

$f_T = \dfrac{1}{2\pi\tau_{ec}} = \dfrac{1}{2\pi\left(3.125\times10^{-10}\right)}$

or

$f_T = 5.09\times10^8\,\text{Hz} = 509\,\text{MHz}$

12.57

We have

$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$

We are given

$\tau_b = 100\,\text{ps}$ and $\tau_e = 25\,\text{ps}$

We find

$\tau_d = \dfrac{x_d}{\upsilon_s} = \dfrac{1.2\times10^{-4}}{10^7} = 1.2\times10^{-11}\,\text{s}$

or

$\tau_d = 12\,\text{ps}$

Also

$\tau_c = r_c C_c = (10)\left(0.1\times10^{-12}\right) = 10^{-12}\,\text{s}$

or

$\tau_c = 1\,\text{ps}$

Then

$\tau_{ec} = 25 + 100 + 12 + 1 = 138\,\text{ps}$

We obtain

$f_T = \dfrac{1}{2\pi\tau_{ec}} = \dfrac{1}{2\pi\left(138\times10^{-12}\right)}$

$= 1.15\times10^9\,\text{Hz}$

or

$f_T = 1.15\,\text{GHz}$

Chapter 13

13.1

Sketch

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13.2

Sketch

_____

13.3

(a) $V_{pO} = \dfrac{ea^2 N_d}{2 \in_s}$

(i) $V_{pO} = \dfrac{(1.6 \times 10^{-19})(0.40 \times 10^{-4})^2 (3 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$

$= 3.312\,\text{V}$

(ii) $V_{bi} = (0.0259)\ln\left[\dfrac{(3 \times 10^{16})(2 \times 10^{18})}{(1.8 \times 10^6)^2}\right]$

$= 1.328\,\text{V}$

$V_p = V_{bi} - V_{pO} = 1.328 - 3.312$

$= -1.984\,\text{V}$

(b) $h_2 = \left[\dfrac{2 \in_s (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$

(i) h_2

$= \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(1.328 + 0 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 2.97 \times 10^{-5}\,\text{cm} = 0.297\,\mu\text{m}$

$a - h_2 = 0.40 - 0.297 = 0.103\,\mu\text{m}$

(ii) h_2

$= \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(1.328 + 0.5 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 3.35 \times 10^{-5}\,\text{cm} = 0.335\,\mu\text{m}$

$a - h_2 = 0.40 - 0.335 = 0.065\,\mu\text{m}$

(iii) h_2

$= \left[\dfrac{2(13.1)(8.85 \times 10^{-14})(1.328 + 2.5 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 4.57 \times 10^{-5}\,\text{cm} = 0.457\,\mu\text{m}$

$h_2 > a \Rightarrow a - h_2 = 0$

(c) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

(i) $V_{DS}(sat) = 3.312 - (1.328 - 0)$

$= 1.984\,\text{V}$

(ii) $V_{DS}(sat) = 3.312 - (1.328 - (-1.0))$

$= 0.984\,\text{V}$

_____

13.4

(a) $V_{pO} = \dfrac{ea^2 N_d}{2 \in_s}$

(i) $V_{pO} = \dfrac{(1.6 \times 10^{-19})(0.40 \times 10^{-4})^2 (3 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$

$= 3.709\,\text{V}$

(ii) $V_{bi} = (0.0259)\ln\left[\dfrac{(3 \times 10^{16})(2 \times 10^{18})}{(1.5 \times 10^{10})^2}\right]$

$= 0.860\,\text{V}$

$V_p = V_{bi} - V_{pO} = 0.860 - 3.709$

$= -2.849\,\text{V}$

(b) $h_2 = \left[\dfrac{2 \in_s (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$

(i) h_2

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(0.860 + 0 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 2.42 \times 10^{-5}\,\text{cm} = 0.242\,\mu\text{m}$

$a - h_2 = 0.40 - 0.242 = 0.158\,\mu\text{m}$

(ii) h_2

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(0.860 + 0.5 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 2.83 \times 10^{-5}\,\text{cm} = 0.283\,\mu\text{m}$

$a - h_2 = 0.40 - 0.283 = 0.117\,\mu\text{m}$

(iii) h_2

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(0.860 + 2.5 - (-0.5))}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$h_2 = 4.08 \times 10^{-5}\,\text{cm} = 0.408\,\mu\text{m}$

$h_2 > a \Rightarrow a - h_2 = 0$

_____

(c) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

  (i) $V_{DS}(sat) = 3.705 - (0.860 - 0)$
$$= 2.845 \text{ V}$$

  (ii) $V_{DS}(sat) = 3.705 - (0.860 - (-1.0))$
$$= 1.845 \text{ V}$$

_____

13.5

(a) $V_{pO} = \dfrac{ea^2 N_a}{2 \in_s} \Rightarrow N_a = \dfrac{2 \in_s V_{pO}}{ea^2}$

$$N_a = \frac{2(13.1)(8.85 \times 10^{-14})(2.75)}{(1.6 \times 10^{-19})(0.65 \times 10^{-4})^2}$$

$$= 9.433 \times 10^{15} \text{ cm}^{-3}$$

(b) $V_{bi} = (0.0259) \ln \left[\dfrac{(9.433 \times 10^{15})(10^{18})}{(1.8 \times 10^6)^2} \right]$

$$= 1.280 \text{ V}$$

$V_p = V_{pO} - V_{bi} = 2.75 - 1.280$
$$= 1.47 \text{ V}$$

(c) $a - h_2 = 0.15 = 0.65 - h_2$

$h_2 = 0.50 \, \mu \text{m}$

$$h_2 = \left[\frac{2 \in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a} \right]^{1/2}$$

$(0.50 \times 10^{-4})^2$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.28 + 0 + V_{GS})}{(1.6 \times 10^{-19})(9.433 \times 10^{15})} \right]$$

$2.5 \times 10^{-9} = (1.5363 \times 10^{-9})(1.28 + V_{GS})$

$\Rightarrow V_{GS} = 0.347 \text{ V}$

(d) $h_2 = \left[\dfrac{2 \in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a} \right]^{1/2}$

$(0.65 \times 10^{-4})^2$

$$= \left[\frac{2(13.1)(8.85 \times 10^{-14})(1.28 + V_{SD})}{(1.6 \times 10^{-19})(9.433 \times 10^{15})} \right]$$

$(0.65 \times 10^{-4})^2 = (1.5363 \times 10^{-9})(1.28 + V_{SD})$

$\Rightarrow V_{SD} = 1.47 \text{ V}$

_____

13.6

(a) $N_a = \dfrac{2 \in_s V_{pO}}{ea^2}$

$$= \frac{2(11.7)(8.85 \times 10^{-14})(2.75)}{(1.6 \times 10^{-19})(0.65 \times 10^{-4})^2}$$

or
$$N_a = 8.425 \times 10^{15} \text{ cm}^{-3}$$

(b) $V_{bi} = (0.0259) \ln \left[\dfrac{(8.425 \times 10^{15})(10^{18})}{(1.5 \times 10^{10})^2} \right]$

$$= 0.8095 \text{ V}$$

$V_p = V_{pO} - V_{bi} = 2.75 - 0.8095$
$$= 1.9405 \text{ V}$$

(c) $a - h_2 = 0.15 = 0.65 - h_2$

$\Rightarrow h_2 = 0.50 \, \mu \text{m}$

$$h_2 = \left[\frac{2 \in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a} \right]^{1/2}$$

$(0.50 \times 10^{-4})^2$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.8095 + 0 + V_{GS})}{(1.6 \times 10^{-19})(8.425 \times 10^{15})} \right]$$

$2.5 \times 10^{-9} = (1.536 \times 10^{-9})(0.8095 + V_{GS})$

$\Rightarrow V_{GS} = 0.8178 \text{ V}$

(d) $h_2 = \left[\dfrac{2 \in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a} \right]^{1/2}$

$(0.65 \times 10^{-4})^2$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.8095 + V_{SD})}{(1.6 \times 10^{-19})(8.425 \times 10^{15})} \right]$$

$(0.65 \times 10^{-4})^2 = (1.536 \times 10^{-9})(0.8095 + V_{SD})$

$\Rightarrow V_{SD} = 1.94 \text{ V}$

_____

13.7

(a) $V_{bi} = V_t \ln \left(\dfrac{N_a N_d}{n_i^2} \right)$

$$= (0.0259) \ln \left[\frac{(2 \times 10^{16})(3 \times 10^{18})}{(1.5 \times 10^{10})^2} \right]$$

$$= 0.860 \text{ V}$$

$V_p = V_{pO} - V_{bi}$

$3.0 = V_{pO} - 0.860 \Rightarrow V_{pO} = 3.86 \text{ V}$

Now $a = \left[\dfrac{2 \in_s V_{pO}}{eN_a} \right]^{1/2}$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(3.86)}{(1.6 \times 10^{-19})(2 \times 10^{16})} \right]^{1/2}$$

$$\cong 5.0 \times 10^{-5} \text{ cm} = 0.50 \, \mu \text{m}$$

(b) $V_{pO} = 3.86$ V

(c) $V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

(i) $V_{SD}(sat) = 3.86 - 0.86 = 3.0$ V

(ii) $V_{SD}(sat) = 3.86 - (0.86 + 1.5) = 1.5$ V

13.8

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2\times10^{16})(3\times10^{18})}{(1.8\times10^6)^2}\right]$$

$$= 1.328 \text{ V}$$

$V_p = V_{pO} - V_{bi}$

$3.0 = V_{pO} - 1.328 \Rightarrow V_{pO} = 4.328$ V

$a = \left[\dfrac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(4.328)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$

$$= 5.60\times10^{-5} \text{ cm} = 0.560 \,\mu\text{m}$$

(b) $V_{pO} = 4.328$ V

(c) $V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

(i) $V_{SD}(sat) = 4.328 - (1.328 + 0) = 3.0$ V

(ii) $V_{SD}(sat) = 4.328 - (1.328 + 1.5) = 1.5$ V

13.9

(a) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(4\times10^{18})}{(1.5\times10^{10})^2}\right]$$

$$= 0.886 \text{ V}$$

We find

$5 = V_{pO} - 0.886 \Rightarrow V_{pO} = 5.886$ V

$a = \left[\dfrac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(5.886)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

$$= 4.36\times10^{-5} \text{ cm} = 0.436 \,\mu\text{m}$$

(b) (i) $V_{pO} = 5.886$ V

(ii) $V_p = V_{bi} - V_{pO} = 0.886 - 5.886 = -5.0$ V

13.10

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{(5\times10^{15})(10^{18})}{(1.8\times10^6)^2}\right]$

$$= 1.264 \text{ V}$$

$V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

$3.5 = V_{pO} - (1.264 + 1.0) \Rightarrow V_{pO} = 5.764$ V

$a = \left[\dfrac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(5.764)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$$

$$= 1.293\times10^{-4} \text{ cm} = 1.293 \,\mu\text{m}$$

(b)

(i) $V_{pO} = 5.764$ V

(ii) $V_p = V_{pO} - V_{bi} = 5.764 - 1.264$

$$= 4.5 \text{ V}$$

13.11

(a)

$I_{P1} = \dfrac{\mu_n (eN_d)^2 W a^3}{6\in_s L}$

$$= \frac{(1000)[(1.6\times10^{-19})(10^{16})]^2}{6(11.7)(8.85\times10^{-14})}$$

$$\times \frac{(400\times10^{-4})(0.5\times10^{-4})^3}{20\times10^{-4}}$$

or

$I_{P1} = 1.03$ mA

(b)

$V_{PO} = \dfrac{ea^2 N_d}{2\in_s}$

$$= \left[\frac{(1.6\times10^{-19})(0.5\times10^{-4})^2(10^{16})}{2(11.7)(8.85\times10^{-14})}\right]$$

or

$V_{PO} = 1.93$ V

Also

$V_{bi} = (0.0259)\ln\left[\dfrac{(10^{19})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.874$ V

Now

$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$

$$= 1.93 - 0.874 + V_{GS}$$

or

$V_{DS}(sat) = 1.056 + V_{GS}$

_____

We have
$$V_P = V_{bi} - V_{PO} = 0.874 - 1.93 = -1.056 \text{ V}$$
Then
(i) For $V_{GS} = 0$, $V_{DS}(sat) = 1.06 \text{ V}$

(ii) For $V_{GS} = \frac{1}{4}V_P = -0.264 \text{ V}$,
$$V_{DS}(sat) = 0.792 \text{ V}$$

(iii) For $V_{GS} = \frac{1}{2}V_P = -0.528 \text{ V}$,
$$V_{DS}(sat) = 0.528 \text{ V}$$

(iv) For $V_P = \frac{3}{4}V_P = -0.792 \text{ V}$,
$$V_{DS}(sat) = 0.264 \text{ V}$$

(c)
$$I_{D1}(sat) = I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\right.$$
$$\left.\times\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$
$$= (1.03)\left[1 - 3\left(\frac{0.874 - V_{GS}}{1.93}\right)\right.$$
$$\left.\times\left(1 - \frac{2}{3}\sqrt{\frac{0.874 - V_{GS}}{1.93}}\right)\right]$$

(i) For $V_{GS} = 0$, $I_{D1}(sat) = 0.258 \text{ mA}$
(ii) For $V_{GS} = -0.264 \text{ V}$,
$$I_{D1}(sat) = 0.141 \text{ mA}$$
(iii) For $V_{GS} = -0.528 \text{ V}$,
$$I_{D1}(sat) = 0.0608 \text{ mA}$$
(iv) For $V_{GS} = -0.792 \text{ V}$,
$$I_{D1}(sat) = 0.0148 \text{ mA}$$

_____

13.12
$$g_d = G_{O1}\left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)^{1/2}\right]$$

where
$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03 \times 10^{-3})}{1.93}$$
or
$$G_{O1} = 1.60 \times 10^{-3} \text{ S} = 1.60 \text{ mS}$$
Then

V_{GS}	$(V_{bi} - V_{GS})/V_{PO}$	g_d (mS)
0	0.453	0.523
-0.264	0.590	0.371
-0.528	0.726	0.237
-0.792	0.863	0.114
-1.056	1.0	0

_____

13.13
n-channel JFET - GaAs
(a)
$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$
$$= \frac{(1.6 \times 10^{-19})(8000)(2 \times 10^{16})}{10 \times 10^{-4}}$$
$$\times (30 \times 10^{-4})(0.35 \times 10^{-4})$$
or
$$G_{O1} = 2.69 \times 10^{-3} \text{ S}$$
(b)
$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
We have
$$V_{PO} = \frac{ea^2 N_d}{2 \in_s}$$
$$= \frac{(1.6 \times 10^{-19})(0.35 \times 10^{-4})^2(2 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$
or
$$V_{PO} = 1.69 \text{ V}$$
We find
$$V_{bi} = (0.0259)\ln\left[\frac{(5 \times 10^{18})(2 \times 10^{16})}{(1.8 \times 10^6)^2}\right]$$
or
$$V_{bi} = 1.34 \text{ V}$$
Then
$$V_P = V_{bi} - V_{PO} = 1.34 - 1.69 = -0.35 \text{ V}$$
We then obtain
$$V_{DS}(sat) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$$
For $V_{GS} = 0$, $V_{DS}(sat) = 0.35 \text{ V}$

For $V_{GS} = \frac{1}{2}V_P = -0.175 \text{ V}$,
$$V_{DS}(sat) = 0.175 \text{ V}$$

(c)
$$I_{D1}(sat) = I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\right.$$
$$\left.\times\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

_____

where

$$I_{P1} = \frac{\mu_n (eN_d)^2 W a^3}{6 \in_s L}$$

$$= \frac{(8000)[(1.6 \times 10^{-19})(2 \times 10^{16})]^2}{6(13.1)(8.85 \times 10^{-14})}$$

$$\times \frac{(30 \times 10^{-4})(0.35 \times 10^{-4})^3}{10 \times 10^{-4}}$$

or

$$I_{P1} = 1.515 \,\text{mA}$$

Then

$$I_{D1}(sat) = (1.515)\left[1 - 3\left(\frac{1.34 - V_{GS}}{1.69}\right)\right.$$

$$\left. \times \left(1 - \frac{2}{3}\sqrt{\frac{1.34 - V_{GS}}{1.69}}\right)\right] \quad (\text{mA})$$

For $V_{GS} = 0$, $I_{D1}(sat) = 0.0506$ mA

and

For $V_{GS} = -0.175$ V,

$$I_{D1}(sat) = 0.0124 \text{ mA}$$

_____

13.14

$$g_{mS} = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)$$

We have

$I_{P1} = 1.03 \,\text{mA}$, $V_{PO} = 1.93$ V, $V_{bi} = 0.874$ V

The maximum transconductance occurs when $V_{GS} = 0$. Then

$$g_{mS}(\text{max}) = \frac{3(1.03)}{1.93}\left(1 - \sqrt{\frac{0.874}{1.93}}\right)$$

or

$$g_{mS}(\text{max}) = 0.524 \text{ mS}$$

For $W = 400 \,\mu$m, we have

$$g_{mS}(\text{max}) = \frac{0.524}{400 \times 10^{-4}}$$

or

$$g_{mS}(\text{max}) = 13.1 \,\text{mS/cm} = 1.31 \text{ mS/mm}$$

_____

13.15

The maximum transconductance occurs for $V_{GS} = 0$, so we have

(a) $\quad g_{mS}(\text{max}) = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$

which can be written as

$$g_{mS}(\text{max}) = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

We found

$G_{O1} = 2.69$ mS, $V_{bi} = 1.34$ V, $V_{PO} = 1.69$ V

Then

$$g_{mS}(\text{max}) = (2.69)\left(1 - \sqrt{\frac{1.34}{1.69}}\right)$$

or

$$g_{mS}(\text{max}) = 0.295 \text{ mS}$$

This is for a channel length of $L = 10 \,\mu$ m.

(b) If the channel length is reduced to $L = 2 \,\mu$ m, then

$$g_{mS}(\text{max}) = (0.2947)\left(\frac{10}{2}\right) = 1.47 \text{ mS}$$

_____

13.16

n-channel MESFET - GaAs

(a)

$$V_{PO} = \frac{ea^2 N_d}{2 \in_s}$$

$$= \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2(1.5 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$$V_{PO} = 2.59 \text{ V}$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right) = (0.0259)\ln\left(\frac{4.7 \times 10^{17}}{1.5 \times 10^{16}}\right)$$

or

$$\phi_n = 0.0892 \text{ V}$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811 \text{ V}$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$V_T = -1.78 \text{ V}$$

(b) If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

_____

13.17

n-channel MESFET - GaAs

(a) We want $V_T = +0.10\,V$

Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_c}{N_d}\right) - \frac{ea^2 N_d}{2 \in_s}$$

which can be written as

$$(0.0259)\ln\left(\frac{4.7\times10^{17}}{N_d}\right)$$

$$+\frac{\left(1.6\times10^{-19}\right)\left(0.35\times10^{-4}\right)^2 N_d}{2(13.1)\left(8.85\times10^{-14}\right)} = 0.89 - 0.10$$

or

$$(0.0259)\ln\left(\frac{4.7\times10^{17}}{N_d}\right)$$

$$+\left(8.453\times10^{-17}\right)N_d = 0.79$$

By trial and error,

$$N_d = 8.1\times10^{15}\,cm^{-3}$$

(b) At $T = 400\,K$

$$\frac{N_c(400)}{N_c(300)} = \left(\frac{400}{300}\right)^{3/2} = 1.54$$

Then

$$N_c(400) = \left(4.7\times10^{17}\right)(1.54)$$

$$= 7.24\times10^{17}\,cm^{-3}$$

Also

$$V_t = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453)\ln\left(\frac{7.24\times10^{17}}{8.1\times10^{15}}\right)$$

$$-\left(8.453\times10^{-17}\right)\left(8.1\times10^{15}\right)$$

which becomes

$$V_T = +0.050\,V$$

13.18

(a) $a = \left[\frac{2 \in_s V_{pO}}{eN_d}\right]^{1/2}$

$$= \left[\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.5)}{\left(1.6\times10^{19}\right)\left(2\times10^{16}\right)}\right]^{1/2}$$

$$= 3.30\times10^{-5}\,cm = 0.330\,\mu m$$

(b) $V_T = V_{bi} - V_{pO}$

We find

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right) = (0.0259)\ln\left(\frac{4.7\times10^{17}}{2\times10^{16}}\right)$$

$$= 0.0818\,V$$

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.0818 = 0.788\,V$$

Then

$$V_T = 0.788 - 1.5 = -0.712\,V$$

(c) $h_2 = \left[\frac{2 \in_s \left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$

$$= \left[\frac{2(13.1)\left(8.85\times10^{-14}\right)\left(V_{bi} + V_{DS} - V_{GS}\right)}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}\right]^{1/2}$$

(i) $h_2 = \left[\left(7.246\times10^{-10}\right)(0.788 + 0 - 0.4)\right]^{1/2}$

$$= 1.677\times10^{-5}\,cm = 0.1677\,\mu m$$

$$a - h_2 = 0.330 - 0.1677 = 0.1623\,\mu m$$

(ii)

$$h_2 = \left[\left(7.246\times10^{-10}\right)(0.788 + 1.0 - 0.4)\right]^{1/2}$$

$$= 3.171\times10^{-5}\,cm = 0.3171\,\mu m$$

$$a - h_2 = 0.330 - 0.3171 = 0.0129\,\mu m$$

(iii)

$$h_2 = \left[\left(7.246\times10^{-10}\right)(0.788 + 4.0 - 0.4)\right]^{1/2}$$

$$= 5.64\times10^{-5}\,cm = 0.564\,\mu m$$

$$h_2 > a \Rightarrow a - h_2 = 0$$

13.19

(a) $V_{pO} = \frac{ea^2 N_d}{2 \in_s}$

$$= \frac{\left(1.6\times10^{-19}\right)\left(0.50\times10^{-4}\right)^2\left(5\times10^{15}\right)}{2(13.1)\left(8.85\times10^{-14}\right)}$$

$$= 0.8626\,V$$

We find

$$\phi_n = (0.0259)\ln\left(\frac{4.7\times10^{17}}{5\times10^{15}}\right)$$

$$= 0.1177\,V$$

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.1177$$

$$= 0.7523\,V$$

$$V_T = V_{bi} - V_{pO} = 0.7523 - 0.8626$$

$$= -0.1103\,V$$

_____

(b) $\phi_n = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{3\times10^{16}}\right) = 0.0713\,\text{V}$

$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.0713$
$\qquad = 0.7987\,\text{V}$

$V_T = V_{bi} - V_{pO}$

or $\;V_{pO} = V_{bi} - V_T = 0.7987 - (-0.1103)$
$\qquad\qquad = 0.909\,\text{V}$

Then

$a = \left[\dfrac{2\in_s V_{pO}}{eN_d}\right]^{1/2}$

$\quad = \left[\dfrac{2(13.1)(8.85\times10^{-14})(0.909)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$\quad = 2.095\times10^{-5}\,\text{cm} = 0.2095\,\mu\text{m}$

_____

13.20

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want $V_T = 0.5\,\text{V}$, so

$0.5 = 0.85 - \phi_n - V_{PO}$

Now

$\phi_n = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{N_d}\right)$

and

$V_{PO} = \dfrac{ea^2 N_d}{2\in_s}$

$\qquad = \dfrac{(1.6\times10^{-19})(0.25\times10^{-4})^2 N_d}{2(13.1)(8.85\times10^{-14})}$

or

$V_{PO} = (4.31\times10^{-17})N_d$

Then

$0.5 = 0.85 - (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{N_d}\right)$
$\qquad\qquad\qquad - (4.31\times10^{-17})N_d$

By trial and error

$N_d = 5.45\times10^{15}\,\text{cm}^{-3}$

_____

13.21

  n-channel MESFET - silicon
  (a) For a gold contact, $\phi_{Bn} = 0.82\,\text{V}$.
  We find

$\phi_n = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{10^{16}}\right) = 0.206\,\text{V}$

and

$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614\,\text{V}$

With $V_{DS} = 0$ and $V_{GS} = 0.35\,\text{V}$, we find

$a - h = 0.075\times10^{-4}$

$\qquad = a - \left[\dfrac{2\in_s (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$

so that

$a = 0.075\times10^{-4}$

$\qquad + \left[\dfrac{2(11.7)(8.85\times10^{-14})(0.614 - 0.35)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$

or

$a = 0.26\times10^{-4}\,\text{cm} = 0.26\,\mu\text{m}$

Now

$V_T = V_{bi} - V_{PO} = 0.614 - \dfrac{ea^2 N_d}{2\in_s}$

or

$V_T = 0.614 - \dfrac{(1.6\times10^{-19})(0.26\times10^{-4})^2(10^{16})}{2(11.7)(8.85\times10^{-14})}$

We obtain

$V_T = 0.092\,\text{V}$

(b)

$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$
$\qquad\qquad = (V_{bi} - V_T) - (V_{bi} - V_{GS})$

or

$V_{DS}(sat) = V_{GS} - V_T = 0.35 - 0.092$

which yields

$V_{DS}(sat) = 0.258\,\text{V}$

_____

13.22

(a) $\phi_n = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{2\times10^{16}}\right)$

$\qquad = 0.0818\,\text{V}$

(i) $V_{bi} = \phi_{Bn} - \phi_n = 0.90 - 0.0818$
$\qquad = 0.818\,\text{V}$

(ii) $V_{pO} = \dfrac{ea^2 N_d}{2\in_s}$

$\qquad = \dfrac{(1.6\times10^{-19})(0.65\times10^{-4})^2(2\times10^{16})}{2(13.1)(8.85\times10^{-14})}$

$\qquad = 5.83\,\text{V}$

(iii) $V_T = V_{bi} - V_{pO} = 0.818 - 5.83$
$\qquad = -5.012\,\text{V}$

_____

(b) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

 (i) $V_{DS}(sat) = 5.83 - (0.818 - (-1.0))$

$= 4.01 \, \text{V}$

 (ii) $V_{DS}(sat) = 5.83 - (0.818 - (-2.0))$

$= 3.01 \, \text{V}$

 (iii) $V_{DS}(sat) = 5.83 - (0.818 - (-3.0))$

$= 2.01 \, \text{V}$

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13.23

(a) $k_n = \dfrac{\mu_n \in_s W}{2aL}$

$= \dfrac{(6500)(13.1)(8.85 \times 10^{-14})(12 \times 10^{-4})}{2(0.25 \times 10^{-4})(1.5 \times 10^{-4})}$

$= 1.206 \times 10^{-3} \, \text{A/V}^2 = 1.206 \, \text{mA/V}^2$

(b) $I_{D1}(sat) = k_n(V_{GS} - V_T)^2$

 (i) $I_{D1}(sat) = (1.206)(0.25 - 0.15)^2$

$= 0.01206 \, \text{mA} = 12.06 \, \mu\text{A}$

 (ii) $I_{D1}(sat) = (1.206)(0.45 - 0.15)^2$

$= 0.1085 \, \text{mA}$

(c) $V_{DS}(sat) = V_{GS} - V_T$

 (i) $V_{DS}(sat) = 0.25 - 0.15 = 0.10 \, \text{V}$

 (ii) $V_{DS}(sat) = 0.45 - 0.15 = 0.30 \, \text{V}$

_____

13.24

(a) $g_{ms} = \dfrac{\partial I_D}{\partial V_{GS}} = \dfrac{\partial}{\partial V_{GS}}\left[k_n(V_{GS} - V_T)^2\right]$

$= 2k_n(V_{GS} - V_T)$

$1.25 = 2k_n(0.45 - 0.15)$

$\Rightarrow k_n = 2.083 \, \text{mA/V}^2$

$k_n = \dfrac{\mu_n \in_s W}{2aL}$

$2.083 \times 10^{-3} = \dfrac{(6500)(13.1)(8.85 \times 10^{-14})W}{2(0.25 \times 10^{-4})(1.5 \times 10^{-4})}$

$\Rightarrow W = 2.073 \times 10^{-3} \, \text{cm} = 20.73 \, \mu\text{m}$

(b) $I_{D1}(sat) = k_n(V_{GS} - V_T)^2$

 (i) $I_{D1}(sat) = (2.083)(0.25 - 0.15)^2$

$= 0.02083 \, \text{mA} = 20.83 \, \mu\text{A}$

 (ii) $I_{D1}(sat) = (2.083)(0.45 - 0.15)^2$

$= 0.1875 \, \text{mA}$

_____

13.25

Plot

_____

13.26

Plot

_____

13.27

$V_{bi} = (0.0259)\ln\left[\dfrac{(10^{18})(3 \times 10^{16})}{(1.5 \times 10^{10})^2}\right]$

$= 0.8424 \, \text{V}$

$V_{pO} = \dfrac{ea^2 N_d}{2 \in_s}$

$= \dfrac{(1.6 \times 10^{-19})(0.50 \times 10^{-4})^2(3 \times 10^{16})}{2(11.7)(8.85 \times 10^{-14})}$

$= 5.795 \, \text{V}$

(a) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

$= 5.795 - 0.8424 = 4.953 \, \text{V}$

$\Delta L = \left[\dfrac{2 \in_s (V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(10 - 4.953)}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$\Delta L = 4.666 \times 10^{-5} \, \text{cm}$

Now

$\dfrac{L'}{L} = 1 - \dfrac{(1/2)\Delta L}{L} = 0.90$

$L = \dfrac{\Delta L}{2(0.10)} = \dfrac{4.666 \times 10^{-5}}{2(0.10)}$

$L = 2.333 \times 10^{-4} \, \text{cm} = 2.333 \, \mu\text{m}$

(b) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

$= 5.795 - (0.8424 + 3)$

$= 1.953 \, \text{V}$

$\Delta L = \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(10 - 1.953)}{(1.6 \times 10^{-19})(3 \times 10^{16})}\right]^{1/2}$

$= 5.892 \times 10^{-5} \, \text{cm}$

Then

$L = \dfrac{\Delta L}{2(0.10)} = \dfrac{5.892 \times 10^{-5}}{2(0.10)}$

$= 2.946 \times 10^{-4} \, \text{cm} = 2.946 \, \mu\text{m}$

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13.28

We have that

$$I'_{D1} = I_{D1}\left(\frac{L}{L-(1/2)\Delta L}\right)$$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$.

We can write

$$I'_{D1}(sat) = I_{D1}(sat) \cdot \frac{1}{1-\frac{1}{2}\cdot\frac{\Delta L}{L}}$$

If $\Delta L << L$, then

$$I'_{D1}(sat) \cong I_{D1}(sat)\left[1+\frac{1}{2}\cdot\frac{\Delta L}{L}\right]$$

We have that

$$\Delta L = \left[\frac{2\in_s (V_{DS}-V_{DS}(sat))}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2\in_s V_{DS}}{eN_d}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

which can be written as

$$\Delta L = V_{DS}\left[\frac{2\in_s}{eN_d V_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat)(1+\lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L}\left[\frac{2\in_s}{eN_d V_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

The parameter λ is not independent of V_{DS}.
Define

$$x \equiv \frac{V_{DS}}{V_{DS}(sat)}$$

and consider the function

$$f = \frac{1}{x}\left(1-\frac{1}{x}\right)$$

which is directly proportional to λ. Then

x	$f(x)$
1.5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

_____

13.29

(a) Saturation occurs when $E = 1\times10^4$ V/cm.
As a first approximation, let

$$E = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = E \cdot L = (10^4)(2\times10^{-4}) = 2 \text{ V}$$

(b) We have that

$$h_2 = h_{sat} = \left[\frac{2\in_s (V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{18})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.8915 \text{ V}$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8915+2)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

or

$$h_{sat} = 0.306\times10^{-4}\text{ cm} = 0.306\ \mu\text{m}$$

(c) We then find

$$I_{D1}(sat) = eN_d \upsilon_{sat}(a-h_{sat})W$$

$$= (1.6\times10^{-19})(4\times10^{16})(10^7)$$

$$\times(0.50-0.306)(10^{-4})(30\times10^{-4})$$

or

$$I_{D1}(sat) = 3.72 \text{ mA}$$

(d) For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1}\left[1-3\left(\frac{V_{bi}}{V_{PO}}\right)\left(1-\frac{2}{3}\sqrt{\frac{V_{bi}}{V_{PO}}}\right)\right]$$

Now

$$I_{P1} = \frac{\mu_n (eN_d)^2 Wa^3}{6\in_s L}$$

$$= \frac{(1000)[(1.6\times10^{-19})(4\times10^{16})]^2}{6(11.7)(8.85\times10^{-14})}$$

$$\times\frac{(30\times10^{-4})(0.5\times10^{-4})^3}{(2\times10^{-4})}$$

or

$$I_{P1} = 12.36 \text{ mA}$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2 \in_s}$$

$$= \frac{\left(1.6 \times 10^{-19}\right)\left(0.5 \times 10^{-4}\right)^2 \left(4 \times 10^{16}\right)}{2(11.7)\left(8.85 \times 10^{-14}\right)}$$

or

$$V_{PO} = 7.726 \text{ V}$$

Then

$$I_{D1}(sat) = (12.36)\left[1 - 3\left(\frac{0.8915}{7.726}\right)\right.$$
$$\left. \times \left(1 - \frac{2}{3}\sqrt{\frac{0.8915}{7.726}}\right)\right]$$

or

$$I_{D1}(sat) = 9.05 \text{ mA}$$

_____

13.30

(a) If $L = 1\,\mu$ m, then saturation will occur when

$$V_{DS} = E \cdot L = \left(10^4\right)\left(1 \times 10^{-4}\right) = 1 \text{ V}$$

We find

$$h_2 = h_{sat} = \left[\frac{2 \in_s \left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

We have $V_{bi} = 0.8915$ V and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.8915 + 1)}{\left(1.6 \times 10^{-19}\right)\left(4 \times 10^{16}\right)}\right]^{1/2}$$

or

$$h_{sat} = 0.247 \times 10^{-4} \text{ cm} = 0.247\,\mu\text{ m}$$

Then

$$I_{D1}(sat) = eN_d \upsilon_{sat} (a - h_{sat})W$$
$$= \left(1.6 \times 10^{-19}\right)\left(4 \times 10^{16}\right)\left(10^7\right)$$
$$\times (0.50 - 0.247)\left(10^{-4}\right)\left(30 \times 10^{-4}\right)$$

or

$$I_{D1}(sat) = 4.86 \text{ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(sat) = (9.05)\left(\frac{2}{1}\right) = 18.1 \text{mA}$$

(b) If velocity saturation occurs, then the relation $I_{D1}(sat) \propto (1/L)$ does not apply.

_____

13.31

(a)

$$\upsilon = \mu_n E = (8000)\left(5 \times 10^3\right) = 4 \times 10^7 \text{ cm/s}$$

Then

$$t_d = \frac{L}{\upsilon} = \frac{2 \times 10^{-4}}{4 \times 10^7} = 5 \times 10^{-12} \text{ s}$$

or

$$t_d = 5 \text{ ps}$$

(b) Assume $\upsilon = \upsilon_{sat} = 10^7$ cm/s

Then

$$t_d = \frac{L}{\upsilon_{sat}} = \frac{2 \times 10^{-4}}{10^7} = 2 \times 10^{-11} \text{ s}$$

or

$$t_d = 20 \text{ ps}$$

_____

13.32

(a)

$$\upsilon = \mu_n E = (1000)\left(10^4\right) = 10^7 \text{ cm/s}$$

Then

$$t_d = \frac{L}{\upsilon} = \frac{2 \times 10^{-4}}{10^7} = 2 \times 10^{-11} \text{ s}$$

or

$$t_d = 20 \text{ ps}$$

(b) For $\upsilon = \upsilon_{sat} = 10^7$ cm/s

$$t_d = \frac{L}{\upsilon_{sat}} = \frac{2 \times 10^{-4}}{10^7} = 2 \times 10^{-11} \text{ s}$$

or

$$t_d = 20 \text{ ps}$$

_____

13.33

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5 \times 10^{18}\right)\left(3 \times 10^{16}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.884 \text{ V}$$

Also $V_{PO} = \frac{ea^2 N_d}{2 \in_s}$

$$= \left[\frac{\left(1.6 \times 10^{-19}\right)\left(0.3 \times 10^{-4}\right)^2 \left(3 \times 10^{16}\right)}{2(11.7)\left(8.85 \times 10^{-14}\right)}\right]$$

_____

or

$\quad V_{PO} = 2.086$ V

Then

$\quad V_P = 0.884 - 2.086 = -1.20$ V

Let $V_{GS} = -1.20$ V

Now

$$x_n = \left[\frac{2 \in_s (V_{bi} + V_{DS} - V_{GS})}{e N_d} \right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})} \right.$$

$$\left. \times \frac{(0.884 + V_{DS} - (-1.20))}{(3 \times 10^{16})} \right]^{1/2}$$

or

$$x_n = \left[(4.314 \times 10^{-10})(2.084 + V_{DS}) \right]^{1/2}$$

(a) For $V_{DS} = 0$, $x_n = 0.30 \, \mu$m

(b) For $V_{DS} = 1$ V, $x_n = 0.365 \, \mu$m

(c) For $V_{DS} = 5$ V, $x_n = 0.553 \, \mu$m

The depletion region volume at the drain is

$$Vol = \left(a\right)\left(\frac{L}{2}\right)(W) + (x_n)(2a)(W)$$

$$= (0.3 \times 10^{-4})\left(\frac{2.4 \times 10^{-4}}{2}\right)(30 \times 10^{-4})$$

$$+ (x_n)(0.6 \times 10^{-4})(30 \times 10^{-4})$$

or

$$Vol = 10.8 \times 10^{-12} + x_n (18 \times 10^{-8})$$

(a) For $V_{DS} = 0$, $Vol = 1.62 \times 10^{-11}$ cm^3

(b) For $V_{DS} = 1$ V, $Vol = 1.737 \times 10^{-11}$ cm^3

(c) For $V_{DS} = 5$ V, $Vol = 2.075 \times 10^{-11}$ cm^3

The generation current at the drain is

$$I_{DG} = e\left(\frac{n_i}{2\tau_O}\right) \cdot Vol$$

$$= (1.6 \times 10^{-19})\left[\frac{1.5 \times 10^{10}}{2(5 \times 10^{-8})}\right] \cdot Vol$$

or

$$I_{DG} = (2.4 \times 10^{-2}) \cdot Vol$$

(a) For $V_{DS} = 0$, $I_{DG} = 0.39$ pA

(b) For $V_{DS} = 1$ V, $I_{DG} = 0.42$ pA

(c) For $V_{DS} = 5$ V, $I_{DG} = 0.50$ pA

_____

13.34

(a) The ideal transconductance for $V_{GS} = 0$ is

$$g_{mS} = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

where

$$G_{O1} = \frac{e \mu_n N_d W a}{L}$$

$$= \frac{(1.6 \times 10^{-19})(4500)(7 \times 10^{16})}{1.5 \times 10^{-4}}$$

$$\times (5 \times 10^{-4})(0.3 \times 10^{-4})$$

or

$\quad G_{O1} = 5.04$ mS

We find

$$V_{PO} = \frac{e a^2 N_d}{2 \in_s}$$

$$= \frac{(1.6 \times 10^{-19})(0.3 \times 10^{-4})^2 (7 \times 10^{16})}{2(13.1)(8.85 \times 10^{-14})}$$

or

$\quad V_{PO} = 4.347$ V

We have

$$\phi_n = (0.0259)\ln\left(\frac{4.7 \times 10^{17}}{7 \times 10^{16}}\right) = 0.049$ V

so that

$\quad V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.049 = 0.841$ V

Then

$$g_{mS} = (5.04)\left(1 - \sqrt{\frac{0.841}{4.347}}\right)$$

or

$\quad g_{mS} = 2.82$ mS

(b) With a source resistance

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = \frac{1}{1 + g_m r_s}$$

For

$$\frac{g'_m}{g_m} = 0.80 = \frac{1}{1 + (2.823)r_s}$$

we obtain

$\quad r_s = 88.6 \, \Omega$

(c)

$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e \mu_n n) A}$$

so

$$L = (88.56)(1.6 \times 10^{-19})(4500)(7 \times 10^{16})$$
$$\times (0.3 \times 10^{-4})(5 \times 10^{-4})$$

or

$$L = 0.67 \times 10^{-4} \text{ cm} = 0.67 \, \mu\text{m}$$

_____

13.35

Considering the capacitance charging time, we have

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\epsilon_s WL}{a}$$
$$= \frac{(13.1)(8.85 \times 10^{-14})(5 \times 10^{-4})(1.5 \times 10^{-4})}{0.3 \times 10^{-4}}$$

or

$$C_G = 2.9 \times 10^{-15} \text{ F}$$

We must use g'_m, so we obtain

$$f_T = \frac{(2.82 \times 10^{-3})(0.80)}{2\pi(2.9 \times 10^{-15})} = 1.238 \times 10^{11} \text{ Hz}$$

We can also write

$$f_T = \frac{1}{2\pi \tau_C} \Rightarrow \tau_C = \frac{1}{2\pi f_T}$$

so

$$\tau_C = \frac{1}{2\pi(1.238 \times 10^{11})} = 1.285 \times 10^{-12} \text{ s}$$

The channel transit time is

$$t_t = \frac{1.5 \times 10^{-4}}{10^7} = 1.5 \times 10^{-11} \text{ s}$$

The total time constant is

$$\tau = 1.5 \times 10^{-11} + 1.285 \times 10^{-12}$$
$$= 1.629 \times 10^{-11} \text{ s}$$

Taking into account the channel transit time and the capacitance charging time, we find

$$f_T = \frac{1}{2\pi \tau} = \frac{1}{2\pi(1.629 \times 10^{-11})}$$

or

$$f_T = 9.77 \times 10^9 \text{ Hz} = 9.77 \text{ GHz}$$

_____

13.36

(a) For constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \epsilon_s L^2}$$
$$= \frac{(1.6 \times 10^{-19})(7500)(4 \times 10^{16})(0.30 \times 10^{-4})^2}{2\pi(13.1)(8.85 \times 10^{-14})(1.2 \times 10^{-4})^2}$$
$$f_T = 4.12 \times 10^{11} \text{ Hz} = 412 \text{ GHz}$$

(b) For saturation velocity model

$$f_T = \frac{\upsilon_{sat}}{2\pi L} = \frac{10^7}{2\pi(1.2 \times 10^{-4})}$$
$$f_T = 1.33 \times 10^{10} \text{ Hz} = 13.3 \text{ GHz}$$

_____

13.37

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \epsilon_s L^2}$$
$$= \frac{(1.6 \times 10^{-19})(1000)(2 \times 10^{16})(0.40 \times 10^{-4})^2}{2\pi(11.7)(8.85 \times 10^{-14})L^2}$$
$$f_T = \frac{786.975}{L^2}$$

(a) $f_T = \frac{786.975}{(3 \times 10^{-4})^2}$

$$= 8.74 \times 10^9 \text{ Hz} = 8.74 \text{ GHz}$$

(b) $f_T = \frac{786.975}{(1.5 \times 10^{-4})^2}$

$$= 3.50 \times 10^{10} \text{ Hz} = 35.0 \text{ GHz}$$

_____

13.38

$$f_T = \frac{e\mu_p N_a a^2}{2\pi \epsilon_s L^2}$$

or

$$L = \left[\frac{e\mu_p N_a a^2}{2\pi \epsilon_s f_T} \right]^{1/2}$$
$$= \left[\frac{(1.6 \times 10^{-19})(420)(2 \times 10^{16})(0.40 \times 10^{-4})^2}{2\pi(11.7)(8.85 \times 10^{-14})f_T} \right]^{1/2}$$

$$L = \frac{18.18}{\sqrt{f_T}}$$

_____

(a) $L = \dfrac{18.18}{\sqrt{5 \times 10^9}}$

$= 2.57 \times 10^{-4}$ cm $= 2.57 \, \mu$ m

(b) $L = \dfrac{18.18}{\sqrt{12 \times 10^9}}$

$= 1.66 \times 10^{-4}$ cm $= 1.66 \, \mu$ m

_____

13.39

(a)

$$V_{off} = \phi_B - \frac{\Delta E_c}{e} - V_{P2}$$

where

$$V_{P2} = \frac{e N_d d_d^2}{2 \in_N}$$

$$= \frac{\left(1.6 \times 10^{-19}\right)\left(3 \times 10^{18}\right)\left(350 \times 10^{-8}\right)^2}{2(12.2)\left(8.85 \times 10^{-14}\right)}$$

or

$V_{P2} = 2.72$ V

Then

$V_{off} = 0.89 - 0.24 - 2.72$

or

$V_{off} = -2.07$ V

(b)

$$n_S = \frac{\in_N}{e(d + \Delta d)}\left(V_g - V_{off}\right)$$

For $V_g = 0$, we have

$$n_S = \frac{(12.2)\left(8.85 \times 10^{-14}\right)}{\left(1.6 \times 10^{-19}\right)\left(350 + 80\right)\left(10^{-8}\right)}(2.07)$$

or

$n_S = 3.25 \times 10^{12}$ cm^{-2}

_____

13.40

(a) We have

$$I_D(sat) = \frac{\in_N W}{(d + \Delta d)}\left(V_g - V_{off} - V_O\right)\cdot \upsilon_s$$

We find

$$\left(\frac{g_{mS}}{W}\right) = \frac{\partial}{\partial V_g}\left[\frac{I_D(sat)}{W}\right] = \frac{\in_N \upsilon_s}{(d + \Delta d)}$$

$$= \frac{(12.2)\left(8.85 \times 10^{-14}\right)\left(2 \times 10^7\right)}{(350 + 80)\left(10^{-8}\right)}$$

or

$$\left(\frac{g_{mS}}{W}\right) = 5.02 \text{ S/cm} = 502 \text{ mS/mm}$$

(b) At $V_g = 0$, we obtain

$$\frac{I_D(sat)}{W} = \frac{\in_N}{(d + \Delta d)}\left(-V_{off} - V_O\right)\cdot \upsilon_s$$

$$= \frac{(12.2)\left(8.85 \times 10^{-12}\right)}{(350 + 80)\left(10^{-8}\right)}(2.07 - 1)\left(2 \times 10^7\right)$$

or

$$\frac{I_D(sat)}{W} = 5.37 \text{ A/cm} = 537 \text{ mA/mm}$$

_____

13.41

$$V_{off} = \phi_B - \frac{\Delta E_c}{e} - V_{P2}$$

We want $V_{off} = -0.3$ V, so

$-0.30 = 0.85 - 0.22 - V_{P2}$

or

$V_{P2} = 0.93$ V

We have

$$V_{P2} = \frac{e N_d d_d^2}{2 \in_N}$$

or

$$d_d^2 = \frac{2 \in_N V_{P2}}{e N_d}$$

$$= \frac{2(12.2)\left(8.85 \times 10^{-14}\right)(0.93)}{\left(1.6 \times 10^{-19}\right)\left(2 \times 10^{18}\right)}$$

We then obtain

$$d_d = 2.51 \times 10^{-6} \text{ cm} = 251 \overset{o}{A}$$

_____

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Chapter 14

14.1

$$\lambda_{max} = \frac{1.24}{E_g} \, \mu\text{m}$$

(a) Si: $\lambda_{max} = \frac{1.24}{1.12} = 1.11 \, \mu\text{m}$

(b) Ge: $\lambda_{max} = \frac{1.24}{0.66} = 1.88 \, \mu\text{m}$

(c) GaAs: $\lambda_{max} = \frac{1.24}{1.42} = 0.873 \, \mu\text{m}$

(d) InP: $\lambda_{max} = \frac{1.24}{1.35} = 0.919 \, \mu\text{m}$

_____

14.2

(a) For $\lambda = 480$ nm,

$$E = \frac{1.24}{\lambda} = \frac{1.24}{0.480} = 2.58 \, \text{eV}$$

For $\lambda = 725$ nm,

$$E = \frac{1.24}{0.725} = 1.71 \, \text{eV}$$

(b) For $E = 0.87$ eV,

$$\lambda = \frac{1.24}{E} = \frac{1.24}{0.87} = 1.43 \, \mu\text{m}$$

For $E = 1.32$ eV,

$$\lambda = \frac{1.24}{1.32} = 0.939 \, \mu\text{m}$$

For $E = 1.90$ eV,

$$\lambda = \frac{1.24}{1.90} = 0.653 \, \mu\text{m}$$

_____

14.3

(a) $\lambda = \frac{1.24}{1.65} = 0.752 \, \mu\text{m}$

(i) From Figure 14.4, $\alpha \cong 9 \times 10^3 \, \text{cm}^{-1}$

(ii) $\dfrac{I_v(d)}{I_{v0}} = \exp(-\alpha d)$

$$= \exp\left[-\left(9 \times 10^3\right)\left(1.2 \times 10^{-4}\right)\right]$$
$$= 0.340$$

Fraction absorbed $= 1 - 0.34 = 0.66$

(b) $\lambda = \frac{1.24}{1.90} = 0.653 \, \mu\text{m}$

(i) From Figure 14.4, $\alpha \cong 2.6 \times 10^4 \, \text{cm}^{-1}$

(ii) $\dfrac{I_v(d)}{I_{v0}} = \exp(-\alpha d)$

$$= \exp\left[-\left(2.6 \times 10^4\right)\left(0.80 \times 10^{-4}\right)\right]$$
$$= 0.125$$

Fraction absorbed $= 1 - 0.125 = 0.875$

_____

14.4

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3$ eV, $\lambda = \frac{1.24}{1.3} = 0.95 \, \mu\text{m}$

For silicon: $\alpha \cong 3 \times 10^2 \, \text{cm}^{-1}$

Then for $I(x) = 10^{-2} \, \text{W/cm}^2$, we obtain

$$g' = \frac{\left(3 \times 10^2\right)\left(10^{-2}\right)}{\left(1.6 \times 10^{-19}\right)\left(1.3\right)}$$

or

$$g' = 1.44 \times 10^{19} \, \text{cm}^{-3}\,\text{s}^{-1}$$

The excess concentration is

$$\delta n = g'\tau = \left(1.44 \times 10^{19}\right)\left(10^{-6}\right)$$

or

$$\delta n = 1.44 \times 10^{13} \, \text{cm}^{-3}$$

_____

14.5

(a) $\delta p = g'\tau_{p0} \Rightarrow g' = \dfrac{\delta p}{\tau_{p0}}$

$$g' = \frac{5 \times 10^{15}}{2 \times 10^{-7}} = 2.5 \times 10^{22} \, \text{cm}^{-3}\,\text{s}^{-1}$$

For $h\nu = 1.65$ eV,

$$\Rightarrow \lambda = \frac{1.24}{1.65} = 0.752 \, \mu\text{m}$$

From Figure 14.4, $\alpha \cong 9 \times 10^3 \, \text{cm}^{-1}$

$$I_{v0} = \frac{(g')(h\nu)}{\alpha}$$

$$= \frac{\left(2.5 \times 10^{22}\right)\left(1.6 \times 10^{-19}\right)\left(1.65\right)}{9 \times 10^3}$$

$$= 0.733 \, \text{W/cm}^2$$

(b) $\dfrac{I_v(d)}{I_{v0}} = 0.1 = \exp(-\alpha d)$

$\qquad 0.1 = \exp\left[-\left(9\times10^3\right)(d)\right]$

$\qquad d = \dfrac{1}{9\times10^3}\ln\left(\dfrac{1}{0.1}\right)$

$\qquad\quad = 2.56\times10^{-4}\,\text{cm} = 2.56\,\mu\text{m}$

_____

14.6

$\qquad \lambda = \dfrac{1.24}{E} = \dfrac{1.24}{1.40} = 0.886\,\mu\text{m}$

$\qquad$ From Figure 14.4, $\alpha \cong 4.5\times10^2\,\text{cm}^{-1}$

(a) $\dfrac{I_v(d)}{I_{v0}} = 0.1 = \exp(-\alpha d)$

$\qquad d = \dfrac{1}{\alpha}\ln\left(\dfrac{1}{0.1}\right) = \dfrac{1}{4.5\times10^2}\ln\left(\dfrac{1}{0.1}\right)$

$\qquad\quad = 5.12\times10^{-3}\,\text{cm} = 51.2\,\mu\text{m}$

(b) $d = \dfrac{1}{4.5\times10^2}\ln\left(\dfrac{1}{0.3}\right)$

$\qquad\quad = 2.68\times10^{-3}\,\text{cm} = 26.8\,\mu\text{m}$

_____

14.7

$\quad$ GaAs:

$\quad$ For $x = 1\,\mu\text{m} = 10^{-4}\,\text{cm}$, we have 50%

$\quad$ absorbed or 50% transmitted, then

$\qquad \dfrac{I(x)}{I_o} = 0.50 = \exp(-\alpha x)$

$\quad$ We can write

$\qquad \alpha = \left(\dfrac{1}{x}\right)\cdot\ln\left(\dfrac{1}{0.5}\right) = \left(\dfrac{1}{10^{-4}}\right)\cdot\ln(2)$

$\quad$ or

$\qquad \alpha = 0.69\times10^4\,\text{cm}^{-1}$

$\quad$ This value corresponds to

$\qquad \lambda = 0.75\,\mu\text{m},\quad E = 1.65\,\text{eV}$

_____

14.8

$\quad$ The ambipolar transport equation for minority
$\quad$ carrier holes in steady state is

$\qquad D_p\dfrac{d^2(\delta p_n)}{dx^2} + G_L - \dfrac{\delta p_n}{\tau_p} = 0$

$\quad$ or

$\qquad \dfrac{d^2(\delta p_n)}{dx^2} - \dfrac{\delta p_n}{L_p^2} = -\dfrac{G_L}{D_p}$

where $L_p = \sqrt{D_p\tau_p}$

The photon flux in the semiconductor is

$\qquad \Phi(x) = \Phi_O\exp(-\alpha x)$

and the generation rate is

$\qquad G_L = \alpha\Phi(x) = \alpha\Phi_O\exp(-\alpha x)$

so the differential equation becomes

$\qquad \dfrac{d^2(\delta p_n)}{dx^2} - \dfrac{\delta p_n}{L_p^2} = -\dfrac{\alpha\Phi_O}{D_p}\exp(-\alpha x)$

The general solution is of the form

$\qquad \delta p_n(x) = A\exp\left(\dfrac{-x}{L_p}\right) + B\exp\left(\dfrac{+x}{L_p}\right)$

$$-\dfrac{\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}\cdot\exp(-\alpha x)$$

As $x \to \infty$, $\delta p_n = 0$ so that $B = 0$. Then

$\qquad \delta p_n(x) = A\exp\left(\dfrac{-x}{L_p}\right) - \dfrac{\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}\cdot\exp(-\alpha x)$

At $x = 0$, we have

$\qquad D_p\dfrac{d(\delta p_n)}{dx}\bigg|_{x=0} = s\,\delta p_n\bigg|_{x=0}$

so we can write

$\qquad \delta p_n\bigg|_{x=0} = A - \dfrac{\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}$

and

$\qquad \dfrac{d(\delta p_n)}{dx}\bigg|_{x=0} = -\dfrac{A}{L_p} + \dfrac{\alpha^2\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}$

Then we have

$\qquad -\dfrac{AD_p}{L_p} + \dfrac{\alpha^2\Phi_O\tau_p D_p}{\alpha^2 L_p^2 - 1} = sA - \dfrac{s\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}$

Solving for A, we find

$\qquad A = \dfrac{\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}\cdot\left[\dfrac{s + \alpha D_p}{s + (D_p/L_p)}\right]$

The solution can now be written as

$\qquad \delta p_n(x) = \dfrac{\alpha\Phi_O\tau_p}{\alpha^2 L_p^2 - 1}$

$\qquad \times\left[\dfrac{s + \alpha D_p}{s + (D_p/L_p)}\cdot\exp\left(\dfrac{-x}{L_p}\right) - \exp(-\alpha x)\right]$

_____

_____

14.9

We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n = \sqrt{D_n \tau_n}$

The general solution can be written in the form

$$\delta n_p(x) = A \cosh\left(\frac{x}{L_n}\right) + B \sinh\left(\frac{x}{L_n}\right) + G_L \tau_n$$

For $s = \infty$ at $x = 0$ means $\delta n_p(0) = 0$. Then

$$0 = A + G_L \tau_n \Rightarrow A = -G_L \tau_n$$

At $x = W$,

$$- D_n \frac{d(\delta n_p)}{dx}\bigg|_{x=W} = s_o \delta n_p\bigg|_{x=W}$$

Now

$$\delta n_p(W) = -G_L \tau_n \cosh\left(\frac{W}{L_n}\right)$$

$$+ B \sinh\left(\frac{W}{L_n}\right) + G_L \tau_n$$

and

$$\frac{d(\delta n_p)}{dx}\bigg|_{x=W} = -\frac{G_L \tau_n}{L_n} \sinh\left(\frac{W}{L_n}\right)$$

$$+ \frac{B}{L_n} \cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\frac{G_L \tau_n D_n}{L_n} \sinh\left(\frac{W}{L_n}\right) - \frac{BD_n}{L_n} \cosh\left(\frac{W}{L_n}\right)$$

$$= s_o\left[-G_L \tau_n \cosh\left(\frac{W}{L_n}\right)\right.$$

$$\left. + B \sinh\left(\frac{W}{L_n}\right) + G_L \tau_n\right]$$

Solving for B, we obtain

$$B = \frac{G_L\left\{L_n \sinh\left(\frac{W}{L_n}\right) + s_o \tau_n\left[\cosh\left(\frac{W}{L_n}\right) - 1\right]\right\}}{\frac{D_n}{L_n} \cosh\left(\frac{W}{L_n}\right) + s_o \sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p(x) = G_L \tau_n\left[1 - \cosh\left(\frac{x}{L_n}\right)\right] + B \sinh\left(\frac{x}{L_n}\right)$$

where B was just given.

_____

14.10

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{(25)(10^{-6})} = 5 \times 10^{-3} \text{ cm}$$

$$L_p = \sqrt{D_p \tau_{p0}} = \sqrt{(10)(5 \times 10^{-7})}$$

$$= 2.236 \times 10^{-3} \text{ cm}$$

Now $J_S = en_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$

$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$

$$\times\left[\frac{25}{(5 \times 10^{-3})(10^{16})} + \frac{10}{(2.236 \times 10^{-3})(10^{15})}\right]$$

$$J_S = 1.790 \times 10^{-10} \text{ A/cm}^2$$

$$I_S = AJ_S = (5)(1.79 \times 10^{-10})$$

$$= 8.950 \times 10^{-10} \text{ A}$$

(a) $I_L = eG_L AW$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.6350 \text{ V}$$

$$W = \left\{\frac{2 \in_s V_{bi}}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$W = 9.508 \times 10^{-5} \text{ cm}$$

Then

$$I_L = (1.6 \times 10^{-19})(5 \times 10^{21})(5)(9.508 \times 10^{-5})$$

$$= 0.380 \text{ A} = 380 \text{ mA}$$

(b) $V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_S}\right)$

$$= (0.0259)\ln\left(1 + \frac{0.380}{8.95 \times 10^{-10}}\right)$$

$$V_{oc} = 0.5145 \text{ V}$$

(c) $\dfrac{V_{oc}}{V_{bi}} = \dfrac{0.5145}{0.635} = 0.810$

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14.11

From Problem 14.10, $I_S = 8.95 \times 10^{-10}$ A

(a) $V_{oc} = V_t \ln\left(1 + \dfrac{I_L}{I_S}\right)$

$= (0.0259)\ln\left(1 + \dfrac{120 \times 10^{-3}}{8.95 \times 10^{-10}}\right)$

$= 0.4847$ V

(b) $I = I_L - I_S\left[\exp\left(\dfrac{V}{V_t}\right) - 1\right]$

$100 \times 10^{-3} = 120 \times 10^{-3}$

$\qquad - 8.95 \times 10^{-10}\left[\exp\left(\dfrac{V}{V_t}\right) - 1\right]$

$\Rightarrow V = 0.4383$ V

(c) $\left(1 + \dfrac{V_m}{V_t}\right)\exp\left(\dfrac{V_m}{V_t}\right) = 1 + \dfrac{I_L}{I_S}$

$= 1 + \dfrac{120 \times 10^{-3}}{8.95 \times 10^{-10}}$

$= 1.341 \times 10^8$

By trial and error, $V_m \cong 0.412$ V

Now

$I_m = I_L - I_S\left[\exp\left(\dfrac{V_m}{V_t}\right) - 1\right]$

$= 120 \times 10^{-3}$

$\qquad - \left(8.95 \times 10^{-10}\right)\left[\exp\left(\dfrac{0.412}{0.0259}\right) - 1\right]$

$\Rightarrow I_m = 112.75 \times 10^{-3}$ A $= 112.75$ mA

$P_m = I_m V_m = (112.75)(0.412)$

$= 46.5$ mW

(d) $V_m = I_m R_L \Rightarrow R_L = \dfrac{V_m}{I_m} = \dfrac{0.412}{0.11275}$

$R_L = 3.65\ \Omega$

_____

14.12

From Problem 14.10, $I_S = 8.95 \times 10^{-10}$ A

(a)

(i) $V_{oc} = V_t \ln\left(1 + \dfrac{I_L}{I_S}\right)$

$= (0.0259)\ln\left(1 + \dfrac{10 \times 10^{-3}}{8.95 \times 10^{-10}}\right)$

$= 0.420$ V

(ii) $\left(1 + \dfrac{V_m}{V_t}\right)\exp\left(\dfrac{V_m}{V_t}\right) = 1 + \dfrac{I_L}{I_S}$

$= 1 + \dfrac{10 \times 10^{-3}}{8.95 \times 10^{-10}}$

$= 1.117 \times 10^7$

By trial and error, $V_m \cong 0.351$ V

Now

$I_m = I_L - I_S\left[\exp\left(\dfrac{V_m}{V_t}\right) - 1\right]$

$= 10 \times 10^{-3}$

$\qquad - \left(8.95 \times 10^{-10}\right)\left[\exp\left(\dfrac{0.351}{0.0259}\right) - 1\right]$

$I_m = 9.31 \times 10^{-3}$ A $= 9.31$ mA

Then

$P_m = I_m V_m = (9.31)(0.351) = 3.27$ mW

(b)

(i) $V_{oc} = (0.0259)\ln\left(1 + \dfrac{100 \times 10^{-3}}{8.95 \times 10^{-10}}\right)$

$= 0.480$ V

(ii) $\left(1 + \dfrac{V_m}{V_t}\right)\exp\left(\dfrac{V_m}{V_t}\right) = 1 + \dfrac{I_L}{I_S}$

$= 1 + \dfrac{100 \times 10^{-3}}{8.95 \times 10^{-10}}$

$= 1.117 \times 10^8$

By trial and error, $V_m \cong 0.407$ V

Now

$I_m = I_L - I_S\left[\exp\left(\dfrac{V_m}{V_t}\right) - 1\right]$

$= 100 \times 10^{-3}$

$\qquad - \left(8.95 \times 10^{-10}\right)\left[\exp\left(\dfrac{0.407}{0.0259}\right) - 1\right]$

$I_m = 9.40 \times 10^{-2}$ A $= 94.0$ mA

Then

$P_m = I_m V_m = (94.0)(0.407) = 38.3$ mW

(c) $\dfrac{P_{m2}}{P_{m1}} = \dfrac{38.3}{3.27} = 11.7$

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14.13

$$V_{OC} = V_t \ln\left(1 + \frac{J_L}{J_S}\right)$$

$$= (0.0259)\ln\left(1 + \frac{30 \times 10^{-3}}{J_S}\right)$$

where

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_p}}\right]$$

which becomes

$$J_S = (1.6 \times 10^{-19})(1.8 \times 10^6)^2$$
$$\times\left[\frac{1}{N_a}\sqrt{\frac{225}{5 \times 10^{-8}}} + \frac{1}{10^{19}}\sqrt{\frac{7}{5 \times 10^{-8}}}\right]$$

or

$$J_S = (5.184 \times 10^{-7})\left[\frac{6.708 \times 10^4}{N_a} + 1.183 \times 10^{-15}\right]$$

Then

N_a (cm^{-3})	J_S (A/cm^2)	V_{OC} (V)
10^{15}	3.477×10^{-17}	0.891
10^{16}	3.478×10^{-18}	0.950
10^{17}	3.484×10^{-19}	1.01
10^{18}	3.539×10^{-20}	1.07

_____

14.14
(a)

$$I_L = J_L \cdot A = (25 \times 10^{-3})(2) = 50 \times 10^{-3}\text{ A}$$

We have

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_p}}\right]$$

or

$$J_S = (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$
$$\times\left[\frac{1}{3 \times 10^{16}}\sqrt{\frac{18}{5 \times 10^{-6}}} + \frac{1}{10^{19}}\sqrt{\frac{6}{5 \times 10^{-7}}}\right]$$

which becomes

$$J_S = 2.289 \times 10^{-12}\text{ A/cm}^2$$

or

$$I_S = 4.579 \times 10^{-12}\text{ A}$$

We have

$$I = I_L - I_S\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

or

$$I = 50 \times 10^{-3} - (4.579 \times 10^{-12})\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

We see that when $I = 0$, $V = V_{OC} = 0.599$ V.
We find

V (V)	I (mA)
0	50
0.1	50
0.2	50
0.3	50
0.4	49.98
0.45	49.84
0.50	48.89
0.55	42.36
0.57	33.46
0.59	14.19

(b) The voltage at the maximum power point is found from

$$\left[1 + \frac{V_m}{V_t}\right]\cdot\exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{50 \times 10^{-3}}{4.58 \times 10^{-12}}$$

$$= 1.092 \times 10^{10}$$

By trial and error,
$V_m = 0.520$ V
At this point, we find
$I_m = 47.6$ mA
so the maximum power is
$P_m = I_m V_m = (47.6)(0.520)$
or
$P_m = 24.8$ mW
(c) We have

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{V_m}{I_m} = \frac{0.520}{47.6 \times 10^{-3}}$$

or
$R = 10.9\,\Omega$

_____

14.15

(a) $V_{oc} = (0.0259)\ln\left(1 + \frac{180 \times 10^{-3}}{2 \times 10^{-9}}\right)$

$= 0.474$ V

(b) $\left(1 + \frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$

$= 1 + \frac{180 \times 10^{-3}}{2 \times 10^{-9}} = 9 \times 10^7$

_____

By trial and error, $V_m \cong 0.402$ V

$$I_m \cong I_L - I_S \exp\left(\frac{V_m}{V_t}\right)$$

$$= 180 \times 10^{-3} - \left(2 \times 10^{-9}\right)\exp\left(\frac{0.402}{0.0259}\right)$$

$$= 1.69 \times 10^{-1} \text{ A} = 169 \text{ mA}$$

$$P_m = I_m V_m = (169)(0.402) = 67.9 \text{ mW}$$

(c) $R_L = \dfrac{V_m}{I_m} = \dfrac{0.402}{0.169} = 2.379 \, \Omega$

(d) $R_L \rightarrow (1.5)(2.379) = 3.568 \, \Omega$

Now

$$I = \frac{V}{R_L} = I_L - I_S \exp\left(\frac{V}{V_t}\right)$$

$$\frac{V}{3.568} = 180 \times 10^{-3} - \left(2 \times 10^{-9}\right)\exp\left(\frac{V}{0.0259}\right)$$

By trial and error, $V \cong 0.444$ V

Then $I = \dfrac{V}{R_L} = \dfrac{0.444}{3.568} = 0.1244$ A

$$P = IV = (124.4)(0.444) = 55.2 \text{ mW}$$

_____

14.16

(a) $V_{oc} = (0.0259)\ln\left(1 + \dfrac{100 \times 10^{-3}}{10^{-10}}\right)$

$$= 0.5367 \text{ V}$$

(b) $\left(1 + \dfrac{V_m}{V_t}\right)\exp\left(\dfrac{V_m}{V_t}\right) = 1 + \dfrac{I_L}{I_S}$

$$= 1 + \frac{100 \times 10^{-3}}{10^{-10}}$$

$$= 10^9$$

By trial and error, $V_m \cong 0.461$ V

Then

$$I_m = 100 \times 10^{-3} - \left(10^{-10}\right)\exp\left(\frac{0.461}{0.0259}\right)$$

$$= 9.463 \times 10^{-2} \text{ A} = 94.63 \text{ mA}$$

$$P_m = I_m V_m = (94.63)(0.461) = 43.62 \text{ mW}$$

(c) $n = \dfrac{10}{0.461} = 21.7 \rightarrow n = 22$ cells

(d) Now $V = (22)(0.461) = 10.14$ V

$$P = IV$$

$$5.2 = I(10.14) \Rightarrow I = 0.5128 \text{ A}$$

Then $n' = \dfrac{0.5128}{0.09463} = 5.42 \rightarrow n' = 6$

(e) Then $I = (6)(0.09463) = 0.5678$ A

So $R_L = \dfrac{V}{I} = \dfrac{10.14}{0.5678} = 17.86 \, \Omega$

_____

14.17

Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then in the p-region

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha\Phi_O}{D_n}\exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p(x) = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_n}\right)$$

$$- \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\exp(-\alpha x)$$

As $x \rightarrow \infty$, $\delta n_p = 0$ so that $B = 0$. Then

$$\delta n_p(x) = A\exp\left(\frac{-x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1} \cdot \exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \dfrac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p(x) = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x)\right]$$

where Φ_O is the incident flux at $x = 0$.

_____

14.18

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

Then

_____

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right) \cdot \ln(10)$$

For $h\nu = 1.7$ eV , $\alpha \cong 10^4$ cm^{-1}
Then

$$x = \left(\frac{1}{10^4}\right) \cdot \ln(10) = 2.3 \times 10^{-4} \text{ cm}$$

or

$$x = 2.3 \, \mu \text{m}$$

and for $h\nu = 2.0$ eV, $\alpha \cong 10^5$ cm^{-1}.
Then

$$x = \left(\frac{1}{10^5}\right) \cdot \ln(10) = 0.23 \times 10^{-4} \text{ cm}$$

or

$$x = 0.23 \, \mu \text{m}$$

_____

14.19

(a) $n_o = N_d = 5 \times 10^{15}$ cm^{-3}

$$I = e\mu_n n_o A\text{E}$$
$$= (1.6 \times 10^{-19})(1200)(5 \times 10^{15})$$
$$\times (5 \times 10^{-4})\left(\frac{3}{120 \times 10^{-4}}\right)$$

$$I = 0.12 \text{ A} = 120 \text{ mA}$$

(b) $\delta p = G_L \tau_{p0} = (10^{21})(10^{-7}) = 10^{14}$ cm^{-3}

(c) $\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$
$$= (1.6 \times 10^{-19})(10^{14})(1200 + 400)$$
$$= 2.56 \times 10^{-2} \text{ } (\Omega\text{ -cm})^{-1}$$

(d) $I_L = (\Delta\sigma)A\text{E}$
$$= (2.56 \times 10^{-2})(5 \times 10^{-4})\left(\frac{3}{120 \times 10^{-4}}\right)$$
$$= 3.2 \times 10^{-3} \text{ A} = 3.2 \text{ mA}$$

(e) $\Gamma_{ph} = \frac{I_L}{eG_L AL}$

$$= \frac{3.2 \times 10^{-3}}{(1.6 \times 10^{-19})(10^{21})(5 \times 10^{-4})(120 \times 10^{-4})}$$
$$\Gamma_{ph} = 3.33$$

_____

14.20
n-type, so holes are the minority carrier

(a)
$$\delta p = G_L \tau_p = (10^{21})(10^{-8})$$

or

$$\delta p = \delta n = 10^{13} \text{ cm}^{-3}$$

(b)
$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$
$$= (1.6 \times 10^{-19})(10^{13})(8000 + 250)$$

or

$$\Delta\sigma = 1.32 \times 10^{-2} \text{ } (\Omega\text{ -cm})^{-1}$$

(c)
$$I_L = J_L \cdot A = (\Delta\sigma)A\text{E} = \frac{(\Delta\sigma)AV}{L}$$
$$= \frac{(1.32 \times 10^{-2})(10^{-4})(5)}{100 \times 10^{-4}}$$

or

$$I_L = 0.66 \text{ mA}$$

(d)
$$\Gamma_{ph} = \frac{I_L}{eG_L AL}$$
$$= \frac{0.66 \times 10^{-3}}{(1.6 \times 10^{-19})(10^{21})(10^{-4})(100 \times 10^{-4})}$$

or

$$\Gamma_{ph} = 4.125$$

_____

14.21

$$\Phi(x) = \Phi_O \exp(-\alpha x)$$

The electron-hole generation rate is
$$g' = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

and the excess carrier concentration is
$$\delta p = \tau_p \alpha\Phi(x)$$

Now
$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$

and
$$J_L = (\Delta\sigma)\text{E}$$

The photocurrent is now found from

$$I_L = \iint (\Delta\sigma)\text{E} \cdot dA = \int_0^W dy \int_0^{x_O} (\Delta\sigma)\text{E} \cdot dx$$

$$= We(\mu_n + \mu_p)\text{E} \int_0^{x_O} \delta p \cdot dx$$

Then

_____

$$I_L = We(\mu_n + \mu_p)\mathbb{E}\alpha\Phi_O\tau_p\int_0^{x_O}\exp(-\alpha x)dx$$

$$= We(\mu_n + \mu_p)\mathbb{E}\alpha\Phi_O\tau_p$$

$$\times\left[-\frac{1}{\alpha}\exp(-\alpha x)\right]_0^{x_O}$$

which becomes

$$I_L = We(\mu_n + \mu_p)\mathbb{E}\Phi_O\tau_p[1 - \exp(-\alpha x_O)]$$

Now

$$I_L = (50\times10^{-4})(1.6\times10^{-19})(1200 + 450)(50)$$

$$\times(10^{16})(2\times10^{-7})[1 - \exp(-(5\times10^4)(10^{-4}))]$$

or

$$I_L = 0.131\,\mu A$$

_____

14.22

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.6530\,V$$

$$L_n = \sqrt{D_n\tau_{n0}} = \sqrt{(25)(5\times10^{-7})}$$

$$= 3.536\times10^{-3}\,cm$$

$$L_p = \sqrt{D_p\tau_{p0}} = \sqrt{(10)(10^{-7})}$$

$$= 10^{-3}\,cm$$

$$W = \left\{\frac{2\in_s(V_{bi} + V_R)}{e}\left(\frac{N_a + N_d}{N_aN_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.653 + 5)}{(1.6\times10^{-19})}\right.$$

$$\left.\times\left[\frac{10^{16} + 2\times10^{15}}{(10^{16})(2\times10^{15})}\right]\right\}^{1/2}$$

Then

$$W = 2.095\times10^{-4}\,cm$$

(a) $I_{L1} = eWG_LA$

$$= (1.6\times10^{-19})(2.095\times10^{-4})(10^{21})(10^{-3})$$

$$= 3.352\times10^{-5}\,A = 33.52\,\mu A$$

(b) In n-region,

$$\delta p = G_L\tau_{p0} = (10^{21})(10^{-7}) = 10^{14}\,cm^{-3}$$

In p-region,

$$\delta n = G_L\tau_{n0} = (10^{21})(5\times10^{-7})$$

$$= 5\times10^{14}\,cm^{-3}$$

(c) $I_L = eG_LA(W + L_n + L_p)$

$$= (1.6\times10^{-19})(10^{21})(10^{-3})$$

$$\times(2.095 + 35.36 + 10.0)\times10^{-4}$$

$$I_L = 7.593\times10^{-4}\,A = 0.7593\,mA$$

_____

14.23

In the n-region under steady state and for $E = 0$, we have

$$D_p\frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p = \sqrt{D_p\tau_p}$ and where x' is positive in the negative x direction. The homogeneous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh}(x') = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$-\frac{\delta p_{np}}{L_p^2} = -\frac{G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_LL_p^2}{D_p} = G_L\tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n(x') = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{x'}{L_p}\right) + G_L\tau_p$$

One boundary condition is that δp_n remains finite as $x' \to \infty$ which means that $B = 0$. Then at $x' = 0$, $p_n(0) = 0 = \delta p_n(0) + p_{nO}$, so that $\delta p_n(0) = -p_{nO}$.

We find that

$$A = -(p_{nO} + G_L\tau_p)$$

The solution is then written as

$$\delta p_n(x') = G_L\tau_p - (G_L\tau_p + p_{nO})\exp\left(\frac{-x'}{L_p}\right)$$

_____

The diffusion current density is found as

$$J_p = -eD_p \left. \frac{d(\delta p_n(x'))}{dx} \right|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = -\frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions.
So

$$J_p = +eD_p \left. \frac{d(\delta p_n)}{dx'} \right|_{x'=0}$$

$$= eD_p \left[-(G_L \tau_p + p_{nO}) \right]$$

$$\times \left(\frac{-1}{L_p} \right) \exp\left(\frac{-x'}{L_p} \right) \Big|_{x'=0}$$

Finally

$$J_p = eG_L L_p + \frac{eD_p p_{nO}}{L_p}$$

_____

14.24

(a) $J_L = e\Phi_o \left[1 - \exp(-\alpha W) \right]$

Diode A: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(10^4)(2 \times 10^{-4}) \right] \right\}$$
$$J_L = 6.92 \times 10^{-2} \, \text{A/cm}^2$$

Diode B: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(10^4)(10 \times 10^{-4}) \right] \right\}$$
$$J_L \cong 8.0 \times 10^{-2} \, \text{A/cm}^2$$

Diode C: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(10^4)(80 \times 10^{-4}) \right] \right\}$$
$$J_L = 8.0 \times 10^{-2} \, \text{A/cm}^2$$

(b) $J_L = e\Phi_o \left[1 - \exp(-\alpha W) \right]$

Diode A: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(5 \times 10^2)(2 \times 10^{-4}) \right] \right\}$$
$$J_L = 7.613 \times 10^{-3} \, \text{A/cm}^2$$

Diode B: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(5 \times 10^2)(10 \times 10^{-4}) \right] \right\}$$
$$J_L = 3.148 \times 10^{-2} \, \text{A/cm}^2$$

Diode C: $J_L = (1.6 \times 10^{-19})(5 \times 10^{17})$
$$\times \left\{ 1 - \exp\left[-(5 \times 10^2)(80 \times 10^{-4}) \right] \right\}$$
$$J_L = 7.853 \times 10^{-2} \, \text{A/cm}^2$$

_____

14.25

(a) $G_{L0} = \dfrac{\alpha I_{v0}}{h\nu} = \dfrac{(10^3)(0.080)}{(1.6 \times 10^{-19})(1.5)}$
$$= 3.33 \times 10^{20} \, \text{cm}^{-3} \, \text{s}^{-1}$$

Then
$$G_L(x) = G_{L0} \exp(-\alpha x)$$
$$= (3.33 \times 10^{20}) \exp\left[-(10^3)(x) \right]$$

(b) $J_L = e\Phi_o \left[1 - \exp(-\alpha W) \right]$
$$= \frac{eG_{L0}}{\alpha} \left[1 - \exp(-\alpha W) \right]$$
$$= \frac{(1.6 \times 10^{-19})(3.333 \times 10^{20})}{(10^3)}$$
$$\times \left\{ 1 - \exp\left[-(10^3)(100 \times 10^{-4}) \right] \right\}$$
$$J_L = 5.33 \times 10^{-2} \, \text{A/cm}^2 = 53.3 \, \text{mA/cm}^2$$

_____

14.26

(a) $J_L = eWG_L$
$$= (1.6 \times 10^{-19})(20 \times 10^{-4})(10^{21})$$
$$= 0.32 \, \text{A/cm}^2$$

(b) $J_L = e\Phi_o \left[1 - \exp(-\alpha W) \right]$
$$= \frac{eG_{L0}}{\alpha} \left[1 - \exp(-\alpha W) \right]$$
$$= \frac{(1.6 \times 10^{-19})(10^{21})}{(10^3)}$$
$$\times \left\{ 1 - \exp\left[-(10^3)(20 \times 10^{-4}) \right] \right\}$$
$$J_L = 0.138 \, \text{A/cm}^2$$

_____

14.27

The minimum α occurs when $\lambda = 1 \, \mu\text{m}$
which gives $\alpha = 10^2 \, \text{cm}^{-1}$. We want
$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$
which can be written as
$$\exp(+\alpha x) = \frac{1}{0.10} = 10$$
Then
$$x = \frac{1}{\alpha} \ln(10) = \frac{1}{10^2} \ln(10) = 2.30 \times 10^{-2} \, \text{cm}$$
or
$$x = 230 \, \mu\text{m}$$

_____

14.28

For $Al_x Ga_{1-x} As$ system, a direct bandgap for
$0 \le x \le 0.45$, we have
$$E_g = 1.424 + 1.247x$$
At $x = 0.45$, $E_g = 1.985$ eV, so for the direct
bandgap
$$1.424 \le E_g \le 1.985 \text{ eV}$$
which yields
$$0.625 \le \lambda \le 0.871 \mu m$$

_____

14.29

(a) From Figure 14.24, $E_g \cong 1.64$ eV
$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.64} = 0.756 \mu \text{ m}$$

(b) From Figure 14.24, $E_g \cong 1.78$ eV
$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.78} = 0.697 \mu \text{ m}$$

_____

14.30

$$E_g = \frac{1.24}{\lambda} = \frac{1.24}{0.670} = 1.85 \text{ eV}$$
From Figure 14.23, $x \cong 0.35$

_____

14.31

$$E_g = \frac{1.24}{\lambda} = \frac{1.24}{0.670} = 1.85 \text{ eV}$$
From Figure 14.24, $x \cong 0.38$

_____

14.32

(a) For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.
The critical angle is
$$\theta_C = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.86°$$

The fraction of photons that will not
experience total internal reflection is
$$\frac{2\theta_C}{360} = \frac{2(15.86)}{360} \Rightarrow 8.81\%$$

(b) Fresnel loss:
$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2 = \left(\frac{3.66-1}{3.66+1}\right)^2 = 0.3258$$

The fraction of photons emitted is then
$$(0.0881)(1-0.3258) = 0.0594 \Rightarrow 5.94\%$$

_____

14.33

We can write the external quantum efficiency
as
$$\eta_{ext} = T_1 T_2$$
where $T_1 = 1 - R_1$ and where R_1 is the
reflection coefficient (Fresnel loss), and the
factor T_2 is the fraction of photons that do not
experience total internal reflection. We have
$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$
so that
$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$
which reduces to
$$T_1 = \frac{4\bar{n}_1 \bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}$$
Now consider the solid angle from the source
point. The surface area described by the solid
angle is πp^2. The factor T_1 is given by
$$T_1 = \frac{\pi p^2}{4\pi R^2}$$
From the geometry, we have
$$\sin\left(\frac{\theta_C}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R\sin\left(\frac{\theta_C}{2}\right)$$
Then the area is
$$A = \pi p^2 = 4R^2 \pi \sin^2\left(\frac{\theta_C}{2}\right)$$
Now
$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_C}{2}\right)$$
From a trig identity, we have
$$\sin^2\left(\frac{\theta_C}{2}\right) = \frac{1}{2}(1 - \cos\theta_C)$$
Then
$$T_1 = \frac{1}{2}(1 - \cos\theta_C)$$
The external quantum efficiency is now
$$\eta_{ext} = T_1 T_2 = \frac{4\bar{n}_1 \bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2} \cdot \frac{1}{2}(1 - \cos\theta_C)$$
or
$$\eta_{ext} = \frac{2\bar{n}_1 \bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}(1 - \cos\theta_C)$$

_____

_____

14.34

For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1 \lambda_1}{2} = \frac{(N_1 + 1)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1 \lambda_1}{2} - \frac{(N_1 + 1)\lambda_2}{2} = \frac{N_1 \lambda_1}{2} - \frac{N_1 \lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2}\Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1 \lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2} \cdot \frac{2L}{\lambda}\Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\Delta\lambda = \frac{\lambda^2}{2L}$$

_____

14.35

For GaAs:

$$h\nu = 1.42 \text{ eV} \Rightarrow \lambda = \frac{1.24}{1.42} = 0.873\,\mu\text{m}$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{\left(0.873 \times 10^{-4}\right)^2}{2\left(75 \times 10^{-4}\right)} = 5.08 \times 10^{-7} \text{ cm}$$

or

$$\Delta\lambda = 5.08 \times 10^{-3}\,\mu\text{m}$$

_____

Chapter 15

15.1 See diagrams in Figure 8.29

15.2

$$R = \frac{\Delta V}{\Delta I} = \frac{0.60 - 0.15}{(2 - 20) \times 10^{-3}} = -25\,\Omega$$

15.3

$$f_r = \frac{1}{2\pi R_{\min} C_j} \sqrt{\frac{R_{\min}}{R_p} - 1}$$

$$= \frac{1}{2\pi (10)(2 \times 10^{-9})} \sqrt{\frac{10}{1} - 1}$$

$$= 2.39 \times 10^7 \,\text{Hz} = 23.9\,\text{MHz}$$

15.4

(a) $n_o L = 10^{12}\,\text{cm}^{-2}$

(i) $L = \dfrac{10^{12}}{10^{15}} = 10^{-3}\,\text{cm} = 10\,\mu\text{m}$

(ii) $\tau = \dfrac{L}{\upsilon_d} = \dfrac{10^{-3}}{1.5 \times 10^7} = 6.667 \times 10^{-11}\,\text{s}$

(iii) $f = \dfrac{1}{\tau} = \dfrac{1}{6.667 \times 10^{-11}}$

$$= 1.5 \times 10^{10}\,\text{Hz} = 15\,\text{GHz}$$

(b)

(i) $L = \dfrac{10^{12}}{10^{16}} = 10^{-4}\,\text{cm} = 1\,\mu\text{m}$

(ii) $\tau = \dfrac{L}{\upsilon_d} = \dfrac{10^{-4}}{1.5 \times 10^7} = 6.667 \times 10^{-12}\,\text{s}$

(iii) $f = \dfrac{1}{\tau} = \dfrac{1}{6.667 \times 10^{-12}}$

$$= 1.5 \times 10^{11}\,\text{Hz} = 150\,\text{GHz}$$

15.5

(a) $\text{E} = \dfrac{V}{L} = \dfrac{9}{15 \times 10^{-4}} = 6 \times 10^3\,\text{V/cm}$

(b) $\upsilon_d \cong 1.5 \times 10^7\,\text{cm/s}$

(c) $f = \dfrac{\upsilon_d}{L} = \dfrac{1.5 \times 10^7}{15 \times 10^{-4}} = 1 \times 10^{10}\,\text{Hz}$

$$= 10\,\text{GHz}$$

15.6

$$f = \frac{\upsilon_s}{2L} = \frac{10^7}{2(10 \times 10^{-4})} = 5 \times 10^9\,\text{Hz}$$

$$= 5\,\text{GHz}$$

15.7

(a) $n_{po} = \dfrac{n_i^2}{N_B} = \dfrac{(1.5 \times 10^{10})^2}{8 \times 10^{15}}$

$$= 2.8125 \times 10^4\,\text{cm}^{-3}$$

(i) $\delta n_p(0) \cong n_{po} \exp\left(\dfrac{V_{BE}}{V_t}\right)$

$$\Rightarrow V_{BE} = V_t \ln\left(\frac{\delta n_p(0)}{n_{po}}\right)$$

$$= (0.0259) \ln\left(\frac{10^{14}}{2.8125 \times 10^4}\right)$$

$$= 0.5696\,\text{V}$$

(ii) Neglecting any recombination in the base

$$I_C \cong \frac{e D_B n_{po} A}{x_B} \exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \frac{(1.6 \times 10^{-19})(20)(2.8125 \times 10^4)(0.4)}{2 \times 10^{-4}}$$

$$\times \exp\left(\frac{0.5696}{0.0259}\right)$$

$$I_C = 0.640\,\text{A}$$

(b) $\delta n_p(0) = (0.1) N_B = 8 \times 10^{14}\,\text{cm}^{-3}$

(i) $V_{BE} \cong (0.0259) \ln\left(\dfrac{8 \times 10^{14}}{2.8125 \times 10^4}\right)$

$$= 0.6234\,\text{V}$$

(ii) $I_C = \dfrac{(1.6 \times 10^{-19})(20)(2.8125 \times 10^4)(0.4)}{2 \times 10^{-4}}$

$$\times \exp\left(\frac{0.6234}{0.0259}\right)$$

$$I_C = 5.12\,\text{A}$$

_____

15.8

(a) From Figure 7.15, $BV_{BC} \cong 450$ V

(b) $V_{pt} = \dfrac{ex_B^2}{2 \in_s} \cdot \dfrac{N_B (N_C + N_B)}{N_C}$

$= \dfrac{(1.6 \times 10^{-19})(2 \times 10^{-4})^2}{2(11.7)(8.85 \times 10^{-14})}$

$\times \dfrac{(8 \times 10^{15})(6 \times 10^{14} + 8 \times 10^{15})}{6 \times 10^{14}}$

$V_{pt} = 354.4$ V

(c) From Figure 7.15, $BV_{BE} \cong 65$ V

_____

15.9

From the junction breakdown curve, for $BV_{CBO} = 1000$ V, we need the collector doping concentration to be $N_C \cong 2 \times 10^{14}$ cm^{-3}.

Depletion width into the base (neglect V_{bi}).

$x_p = \left[\dfrac{2 \in_s V_{BC}}{e} \left(\dfrac{N_C}{N_B} \right) \left(\dfrac{1}{N_B + N_C} \right) \right]^{1/2}$

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(1000)}{(1.6 \times 10^{-19})} \right.$

$\left. \times \left(\dfrac{2 \times 10^{14}}{5 \times 10^{15}} \right) \left(\dfrac{1}{5 \times 10^{15} + 2 \times 10^{14}} \right) \right]^{1/2}$

or

$x_p = 3.16 \times 10^{-4}$ cm $= 3.16 \, \mu$m

(Minimum base width)

Depletion width into the collector

$x_n = \left[\dfrac{2 \in_s V_{BC}}{e} \left(\dfrac{N_B}{N_C} \right) \left(\dfrac{1}{N_B + N_C} \right) \right]^{1/2}$

$= \left[\dfrac{2(11.7)(8.85 \times 10^{-14})(1000)}{(1.6 \times 10^{-19})} \right.$

$\left. \times \left(\dfrac{5 \times 10^{15}}{2 \times 10^{14}} \right) \left(\dfrac{1}{5 \times 10^{15} + 2 \times 10^{14}} \right) \right]^{1/2}$

or

$x_n = 78.9 \times 10^{-4}$ cm $= 78.9 \, \mu$m

(Minimum collector width)

_____

15.10

(a) $BV_{CEO} = \dfrac{BV_{CBO}}{\sqrt[n]{\beta}}$

(i) $BV_{CEO} = \dfrac{300}{\sqrt[3]{10}} = 139$ V

(ii) $BV_{CEO} = \dfrac{300}{\sqrt[3]{50}} = 81.4$ V

(b)

(i) $BV_{CEO} = \dfrac{125}{\sqrt[3]{10}} = 58.0$ V

(ii) $BV_{CEO} = \dfrac{125}{\sqrt[3]{50}} = 33.9$ V

_____

15.11

(a) We have

$\beta_{eff} = \beta_A \beta_B + \beta_A + \beta_B$

so

$180 = 25\beta_B + 25 + \beta_B$

or

$155 = 26\beta_B$

which yields

$\beta_B = 5.96$

(b) We have

$\beta_B i_{EA} = i_{CB}$

or

$\beta_B \left(\dfrac{1 + \beta_A}{\beta_A} \right) \cdot i_{CA} = i_{CB}$

so

$(5.96) \left(\dfrac{1 + 25}{25} \right) \cdot i_{CA} = 20$

which yields

$i_{CA} = 3.23$ A

_____

15.12

(b) $P_T = \left(\dfrac{1}{2} \cdot I_{C,\max} \right)(V_{CEQ})$

$30 = \left(\dfrac{1}{2} \cdot I_{C,\max} \right)(60)$

$\Rightarrow I_{C,\max} = 1.0$ A

$R_L = \dfrac{V_{CEQ}}{I_{CQ}} = \dfrac{60}{0.5} = 120 \, \Omega$

$V_{CE,\max} = 120$ V

_____

(c) $P_T = \left(\frac{1}{2} \cdot I_{C,\max}\right)\left(V_{CEQ}\right)$

$30 = \left(\frac{2}{2}\right)V_{CEQ}$

$\Rightarrow V_{CEQ} = 30\,\text{V}$

$R_L = \frac{V_{CEQ}}{I_{CQ}} = \frac{30}{1} = 30\,\Omega$

$V_{CE,\max} = 2V_{CEQ} = 2(30) = 60\,\text{V}$

(d) Same as part (b)

_____

15.13

(a) $P_T = V_{CEQ}I_{CQ} = \left(\frac{V_{CC}}{2}\right) \cdot I_{CQ}$

$10 = 6I_{CQ} \Rightarrow I_{CQ} = 1.667\,\text{A}$

$R_L = \frac{V_{CEQ}}{I_{CQ}} = \frac{6}{1.667} = 3.60\,\Omega$

(b) $I_{C,\max} = 2I_{CQ} = 2(1.667) = 3.333\,\text{A}$

_____

15.14

If $V_{CC} = 25$ V, then

$I_C(\max) = \frac{V_{CC}}{R_L} = \frac{25}{100} = 0.25\ \text{A} < I_{C,rated}$

The power

$P = I_C V_{CE} = I_C(V_{CC} - I_C R_L)$

Now, to find the maximum power point

$\frac{dP}{dI_C} = 0 = V_{CC} - 2I_C R_L = 25 - I_C(2)(100)$

which yields

$I_C = 0.125\,\text{A}$

So

$P(\max) = (0.125)[25 - (0.125)(100)]$

or

$P(\max) = 1.56\,\text{W} < P_T$

So maximum V_{CC} is $V_{CC} = 25$ V

_____

15.15

Now $R_{on} = \frac{V_{DS}}{I_D}$

Power dissipated in the transistor

$P = I_D V_{DS} = \frac{V_{DS}^2}{R_{on}}$

We have

$I_D = \frac{200 - V_{DS}}{100}$

so we can write

$P = \left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{R_{on}}$

For $T = 25°\text{C}$, $R_{on} = 2\,\Omega$.

Then

$\left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{2}$

which yields

$V_{DS} = 3.92$ V

The power is

$P = \left(\frac{200 - 3.92}{100}\right)(3.92) = 7.69$ W

We then have

T (C)	R_{on} (Ω)	V_{DS} (V)	P (W)
25	2.0	3.92	7.69
50	2.33	4.56	8.91
75	2.67	5.19	10.1
100	3.0	5.83	11.3

_____

15.16

(a) We have, for three devices in parallel,

$\frac{V}{1.8} + \frac{V}{2} + \frac{V}{2.2} = 5 \Rightarrow V(1.51) = 5$

or

$V = 3.311\,\text{V}$

Then, $I = \frac{V}{R}$, so that

$I_1 = 1.839\,\text{A}$

$I_2 = 1.656\,\text{A}$

$I_3 = 1.505\,\text{A}$

Now, $P = IV$, so

$P_1 = 6.09\,\text{W}$

$P_2 = 5.48\,\text{W}$

$P_3 = 4.98\,\text{W}$

(b) Now

$V\left(\frac{1}{1.8} + \frac{1}{3.6} + \frac{1}{2.2}\right) = 5 \Rightarrow V = 3.882$ V

Then

$I_1 = 2.157\,\text{A},\quad P_1 = 8.37\,\text{W}$

$I_2 = 1.078\,\text{A},\quad P_2 = 4.19\,\text{W}$

$I_3 = 1.765\,\text{A},\quad P_3 = 6.85\,\text{W}$

_____

15.17

(a) Let the n-drift region doping concentration be $N_d = 10^{14}\,\text{cm}^{-3}$.

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{14})(10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.516\,\text{V}$$

For the base region,

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+200)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{14}}{10^{15}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_p = 4.86\times10^{-4}\,\text{cm} = 4.86\,\mu\text{m}$$
$$= \text{channel length}$$

$$x_n = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+200)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_n = 4.86\times10^{-3}\,\text{cm} = 48.6\,\mu\text{m}$$
$$= \text{drift region width}$$

(b) Assume $N_d = 10^{14}\,\text{cm}^{-3}$

$$V_{bi} = 0.516\,\text{V}$$

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+80)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{14}}{10^{15}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_p = 3.08\times10^{-4}\,\text{cm} = 3.08\,\mu\text{m}$$
$$= \text{channel length}$$

$$x_n = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+80)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_n = 3.08\times10^{-3}\,\text{cm} = 30.8\,\mu\text{m}$$
$$= \text{drift region width}$$

_____

15.18

(b) In the saturation region,

$$I_D = K_n(V_{GS}-V_T)^2 = (0.20)(V_{GS}-2)^2$$
$$V_{DS} = V_{DD} - I_D R_L = 60 - I_D(10)$$

For $V_{GS} = 4\,\text{V}$, $I_D = 0.8\,\text{A}$, $V_{DS} = 52$ V

$$P = I_D V_{DS} = (0.8)(52) = 41.6\,\text{W}$$

For $V_{GS} = 6\,\text{V}$, $I_D = 3.2\,\text{A}$, $V_{DS} = 28$ V

$$P = (3.2)(28) = 89.6\,\text{W}$$

For $V_{GS} = 8\,\text{V}$, transistor biased in the nonsaturation region.

$$\frac{60-V_{DS}}{10} = (0.20)\left[2(8-2)V_{DS} - V_{DS}^2\right]$$

We obtain $2.0V_{DS}^2 - 25V_{DS} + 60 = 0$

$$\Rightarrow V_{DS} = 3.24\,\text{V}, \ I_D = 5.676\,\text{A}$$
$$\Rightarrow P = (3.24)(5.676) = 18.39\,\text{W}$$

For $V_{GS} = 6\,\text{V}$, $P > P_T$ so transistor may be damaged.

_____

15.19

(a) $$P = \left(\frac{1}{2}\cdot V_{DD}\right)\left(\frac{1}{2}\cdot I_{D,\max}\right)$$

$$45 = \left(\frac{60}{2}\right)\left(\frac{I_D}{2}\right) \Rightarrow I_{D,\max} = 3\,\text{A}$$

$$R_L = \frac{V_{DD}}{I_{D,\max}} = \frac{60}{3} = 20\,\Omega$$

(b) $$P = \left(\frac{V_{DD}}{2}\right)\cdot\left(\frac{I_{D,\max}}{2}\right)$$

$$R_L = \frac{V_{DD}}{I_{D,\max}} = 10 \Rightarrow I_{D,\max} = \frac{V_{DD}}{10}$$

Then

$$P = \left(\frac{V_{DD}}{2}\right)\cdot\left(\frac{V_{DD}}{20}\right)$$

Or

$$45 = \frac{V_{DD}^2}{40}$$

$$\Rightarrow V_{DD} = 42.4\,\text{V}$$

_____

_____

15.20

We have $\alpha_1 + \alpha_2 = 1$. Now

$$\alpha_1 = \frac{\beta_1}{1+\beta_1} \quad \text{and} \quad \alpha_2 = \frac{\beta_2}{1+\beta_2}$$

so

$$\alpha_1 + \alpha_2 = \frac{\beta_1}{1+\beta_1} + \frac{\beta_2}{1+\beta_2} = 1$$

which can be written as

$$1 = \frac{\beta_1(1+\beta_2) + \beta_2(1+\beta_1)}{(1+\beta_1)(1+\beta_2)}$$

or

$$(1+\beta_1)(1+\beta_2) = \beta_1(1+\beta_2) + \beta_2(1+\beta_1)$$

Expanding, we find

$$1 + \beta_1 + \beta_2 + \beta_1\beta_2$$
$$= \beta_1 + \beta_1\beta_2 + \beta_2 + \beta_1\beta_2$$

which yields

$$\beta_1\beta_2 = 1$$

_____

15.21

The reverse-biased p-well to substrate junction corresponds to the J_2 junction in an SCR. The photocurrent generated in this junction will be similar to the avalanche generated current in an SCR, which can trigger the device.

_____

15.22

Case 1: Terminal 1(+), terminal 2(-), and I_G negative: this triggering was discussed in the text.

Case 2: Terminal 1(+), terminal 2(-), and I_G positive: the gate current enters the P2 region directly so that J3 becomes forward biased. Electrons are injected from N2 and diffuse into N1, lowering the potential of N1. The junction J2 becomes more forward biased, and the increased current triggers the SCR so that P2N1P1N4 turns on.

Case 3: Terminal 1(-), terminal 2(+), and I_G positive: the gate current enters the P2 region directly so that the J3 junction becomes more forward biased. More electrons are injected from N2 into N1 so that J1 also becomes more forward biased. The increased current triggers the P1N1P2N2 device into its conducting state.

Case 4: Terminal 1(-), terminal 2(+), and I_G negative: in this case, the J4 junction becomes forward biased. Electrons are injected from N3 and diffuse into N1. The potential of N1 is lowered which increases the forward biased potential of J1. This increased current then triggers the P1N1P2N2 device into its conducting state.

_____