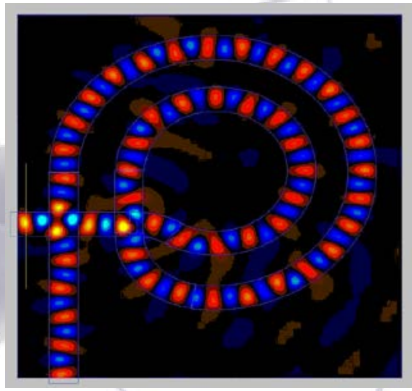

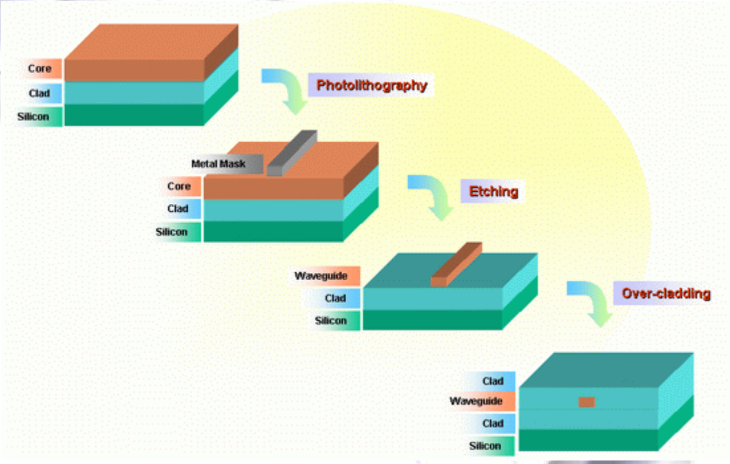

Lesson 8
Optical Slab Waveguides

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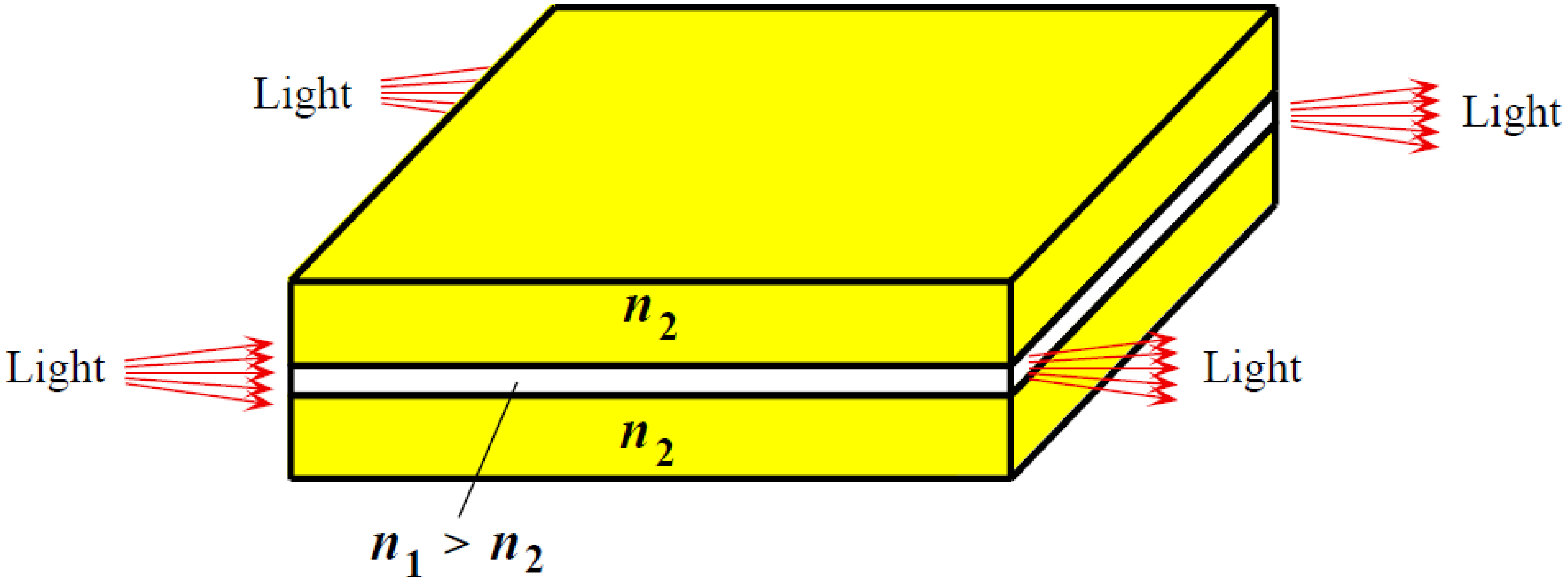
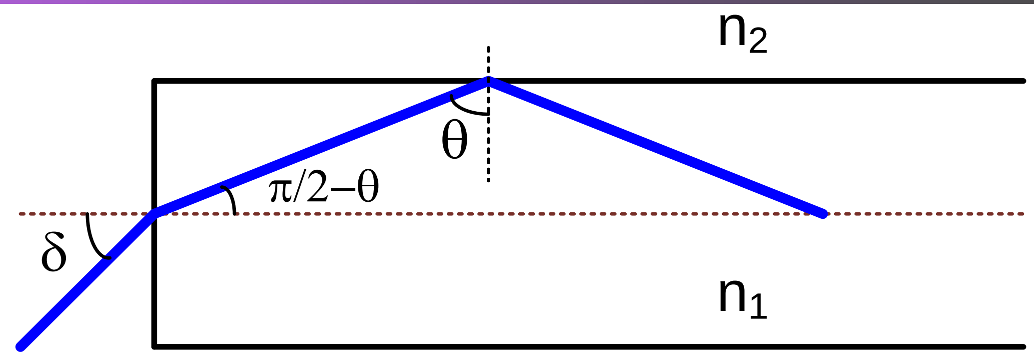
Waveguide structure

- Core (n_1), cladding (n_2)
- $n_1 > n_2$



Waveguide principle: TIR

- What you already learned
- Numerical aperture (NA)



$$\theta > \theta_c \quad , \; n_1 \sin\theta_c = n_2$$

$$1 \cdot \sin\delta = n_1 \sin\left(\frac{\pi}{2} - \theta\right) \le \sqrt{n_1^2 - n_2^2}$$

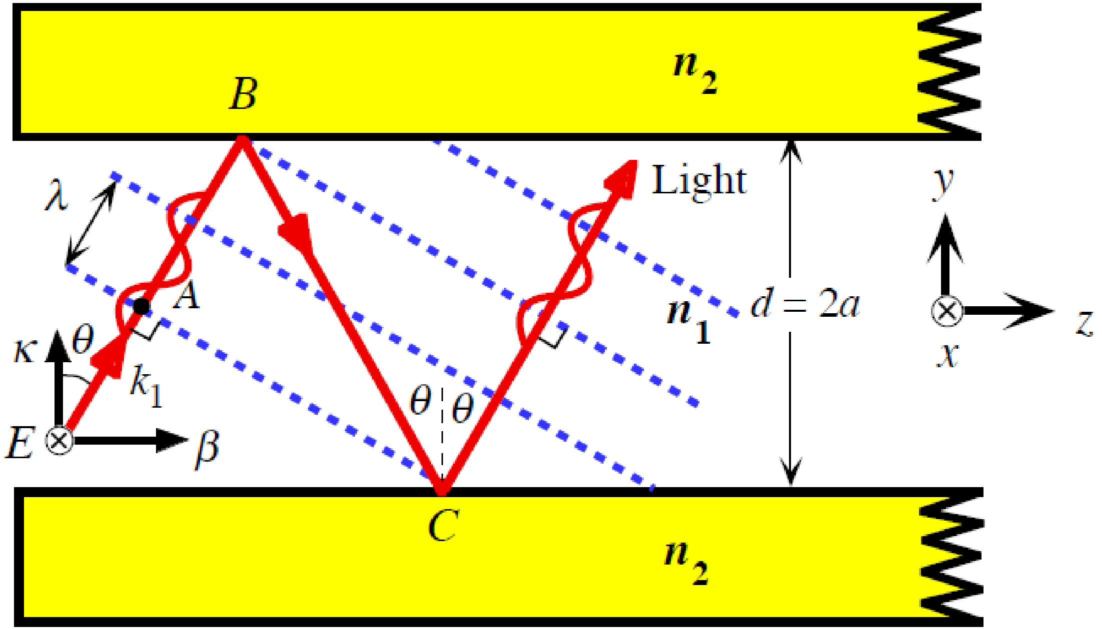
$$\delta \le \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right) \equiv \delta_{\max}$$

$$NA = \sin\delta_{\max} = n_1\sqrt{2\Delta}, \quad \Delta \equiv \frac{n_1^2 - n_2^2}{2n_1^2}$$

Weakly guiding waveguides: $n_1 \approx n_2 \; , \quad \Delta \approx \dfrac{n_1 - n_2}{n_1}$

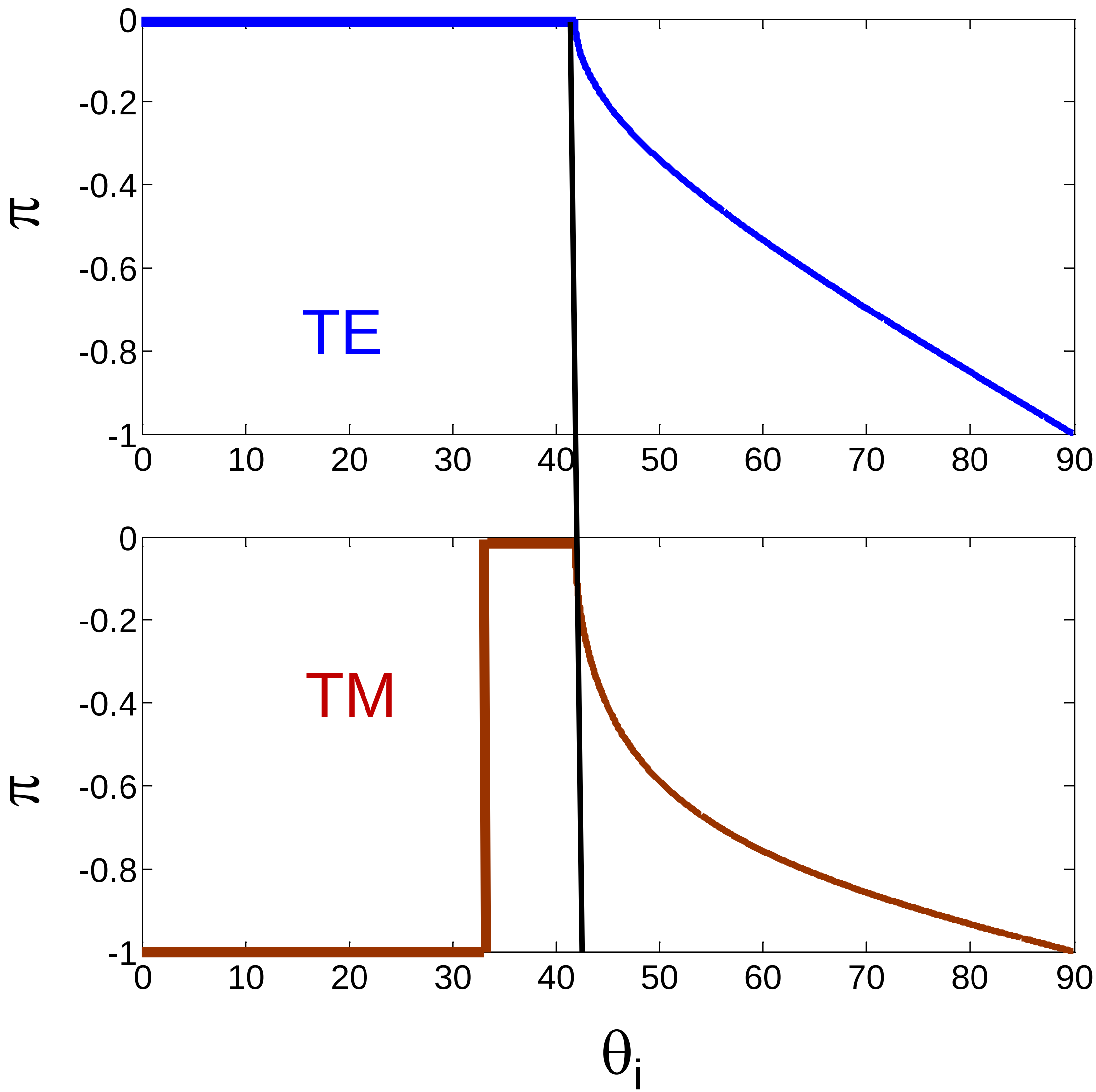
Formation of guided modes

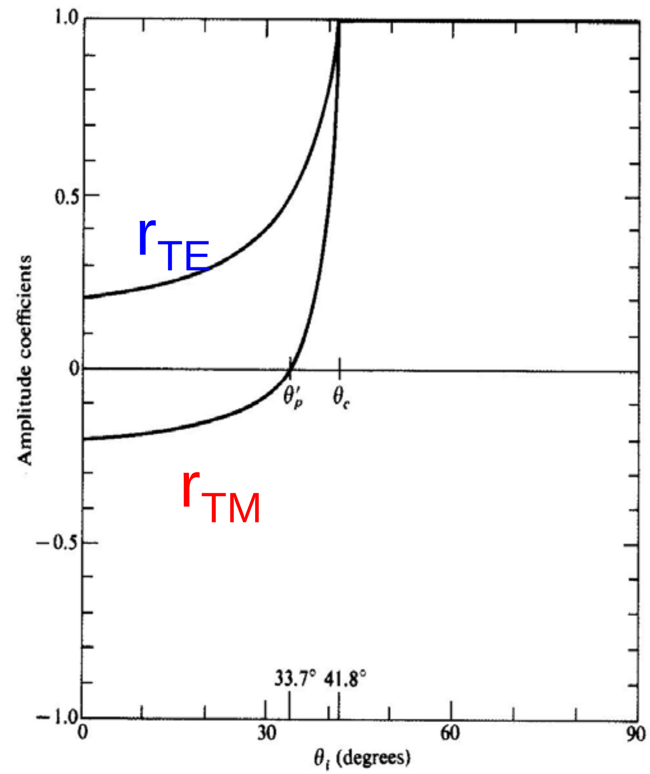
- Optical path length $\overline{AB}$, $\overline{BC}$ should give constructive interference
- But is that all?
  - NO! **TIR** gives additional phase ϕ!



Phase incurred during TIR

- $n_1 > n_2$





ϕ from Fresnel coefficients

- Refresh your memory when TIR:

$$\cos(\theta_t) = -j\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

- TE

$$r_{TE} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{n_1\cos\theta_i + j\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_1\cos\theta_i - j\sqrt{n_1^2\sin^2\theta_i - n_2^2}} = e^{-j\phi_{TE}}$$

$$\phi_{TE} = -2\tan^{-1}\left(\frac{\sqrt{n_1^2\sin^2\theta_i - n_2^2}}{n_1\cos\theta_i}\right) = -2\tan^{-1}\left(\sqrt{\frac{2\Delta}{\cos^2\theta_i} - 1}\right)$$

- TM

$$r_{TM} = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{jn_1\sqrt{(n_1/n_2)^2\sin^2\theta_i - 1} - n_2\cos\theta_i}{jn_1\sqrt{(n_1/n_2)^2\sin^2\theta_i - 1} + n_2\cos\theta_i} = e^{-j\phi_{TM}}$$

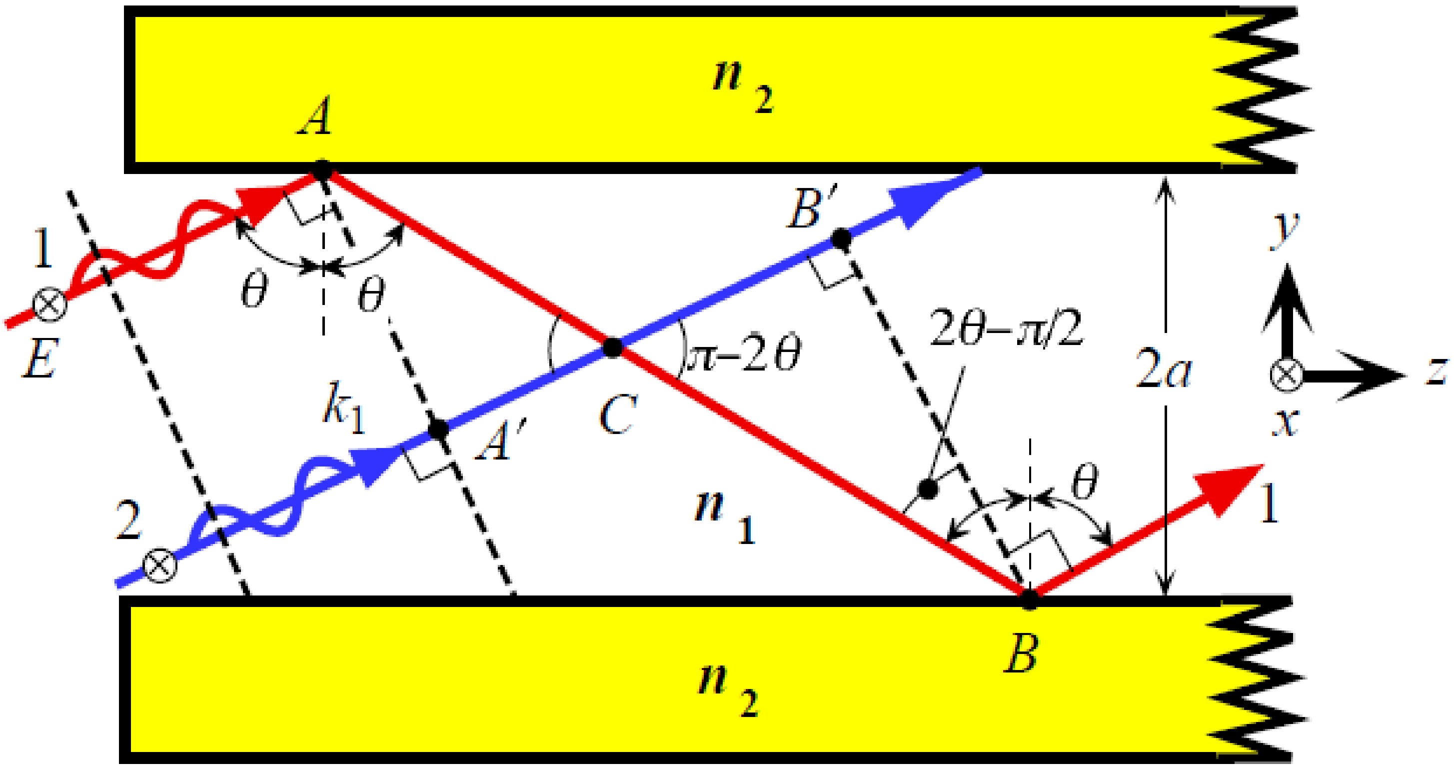
$$\phi_{TM} = -2\tan^{-1}\left(\frac{n_1\sqrt{(n_1/n_2)^2\sin^2\theta_i - 1}}{n_2\cos\theta_i}\right)$$

Finding the modes

- Writing out the phase difference, eventually you see only the k_y component matters:

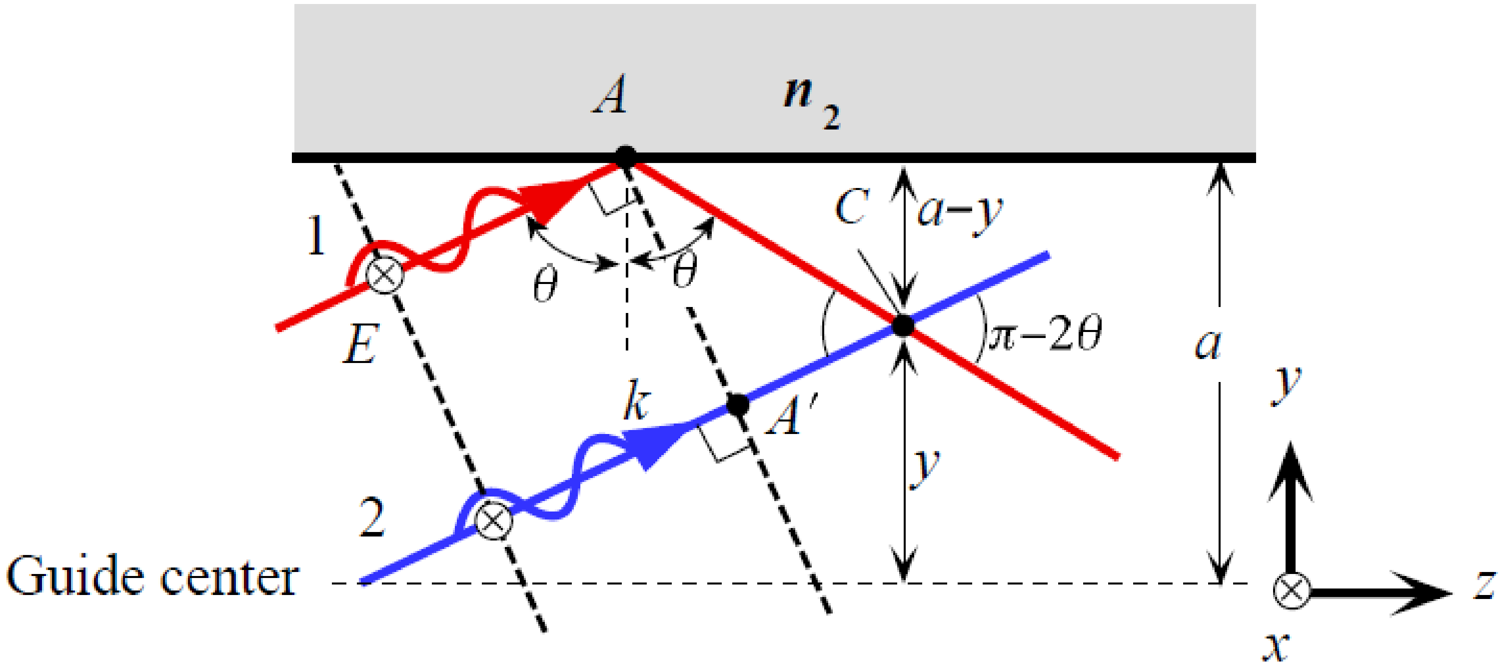
Consider multiple rays

- Interference ➔ transverse mode profile



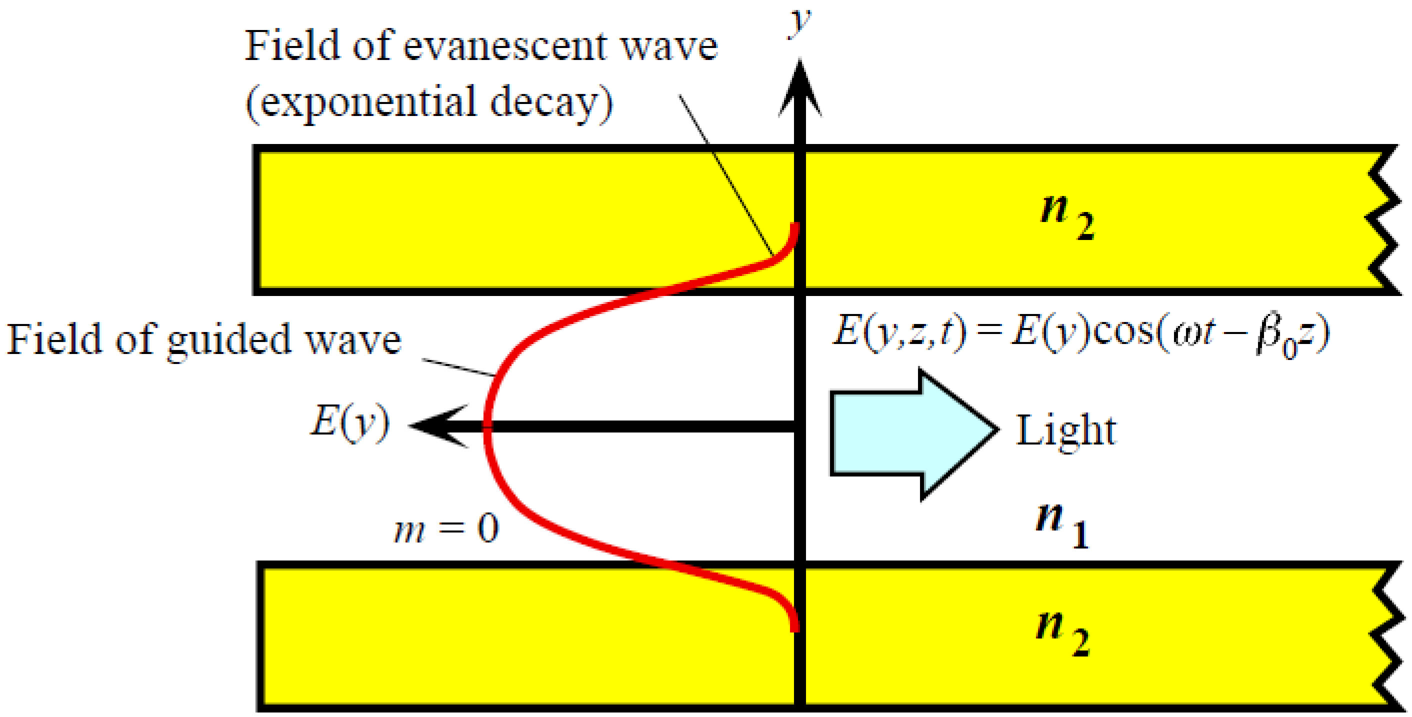
Standing wave

- In the y-direction only
- The transverse pattern propagates in z-direction



Waveguide mode profile

- Fundamental mode (m=0): almost axial



Normalized frequency *V*

- Determines how many modes in a waveguide

$$V \equiv \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda_0} NA$$

$$m \le \frac{(2V + \phi_m)}{\pi}$$, note ϕ_m <0

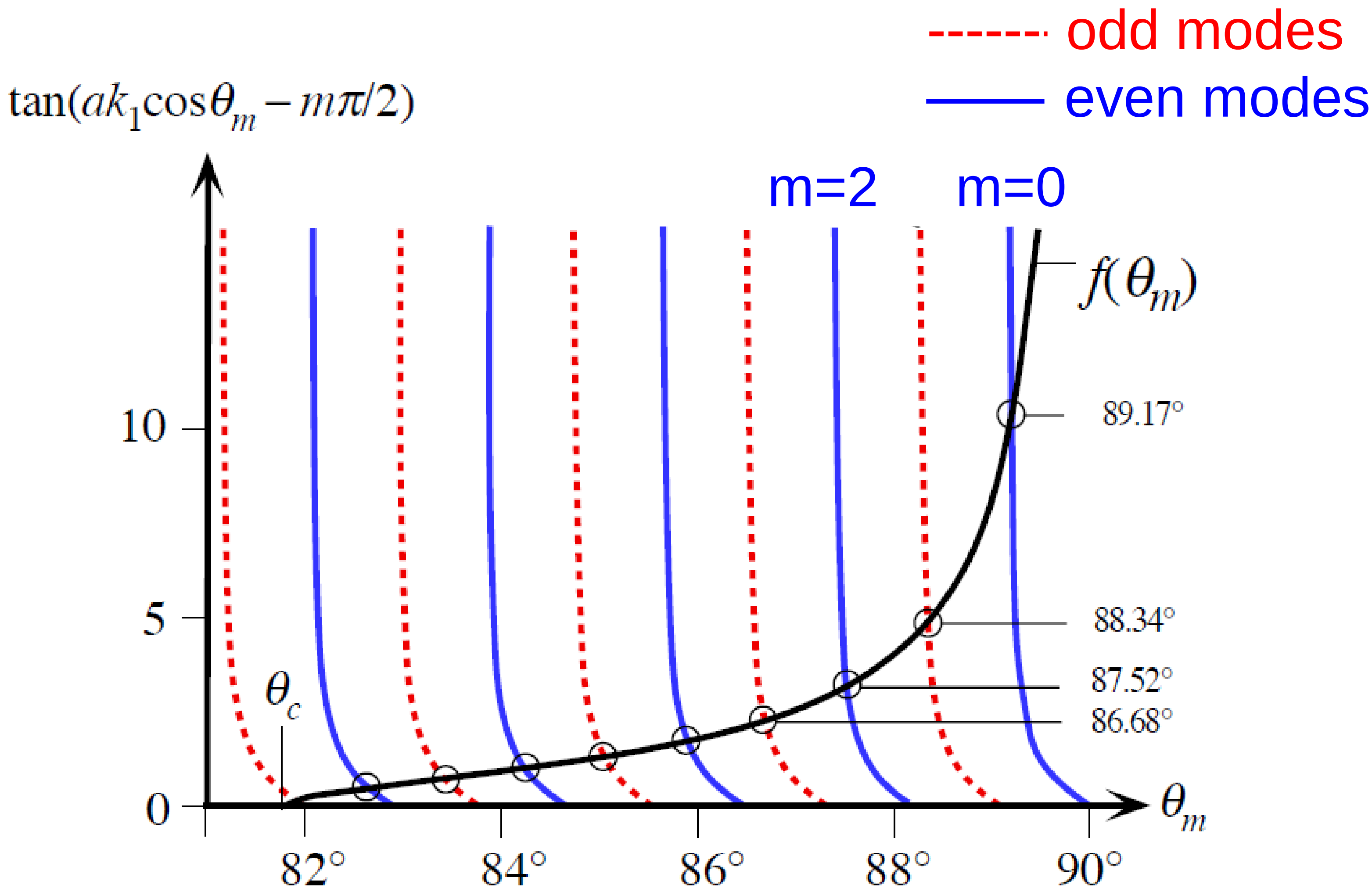
- Single-mode (only m=0 mode, ϕ_m ➔ -π) slab waveguide: $V < \frac{\pi}{2}$

Solving the modes

- Analytical expression

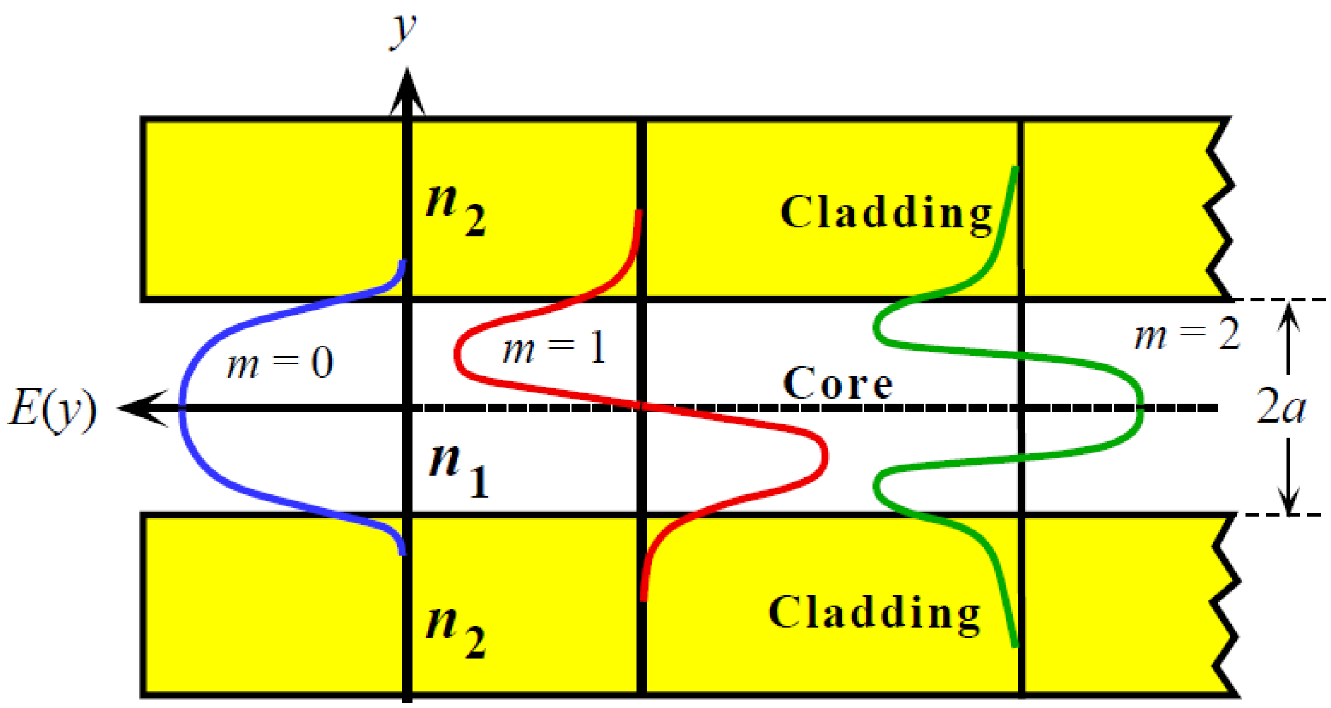
Graphical solution

- Higher-order modes have cut-off



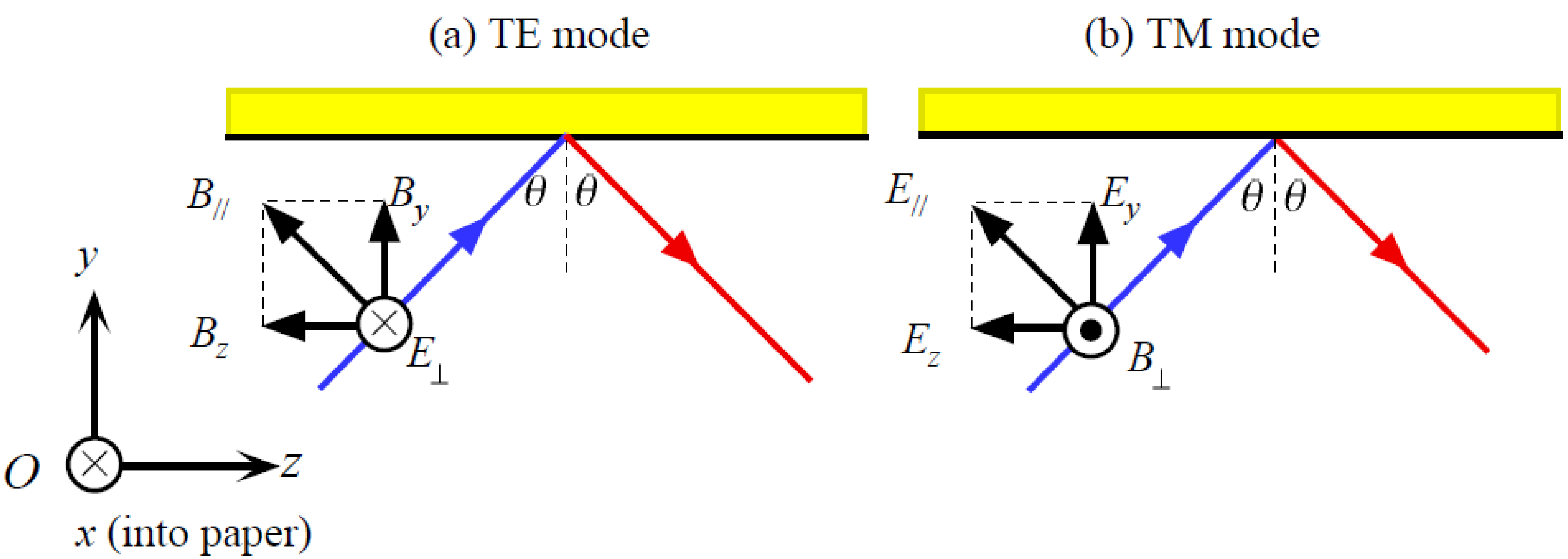
Waveguide mode profile

- Large V: higher-order modes exist
- Higher-order modes penetrates more into the cladding



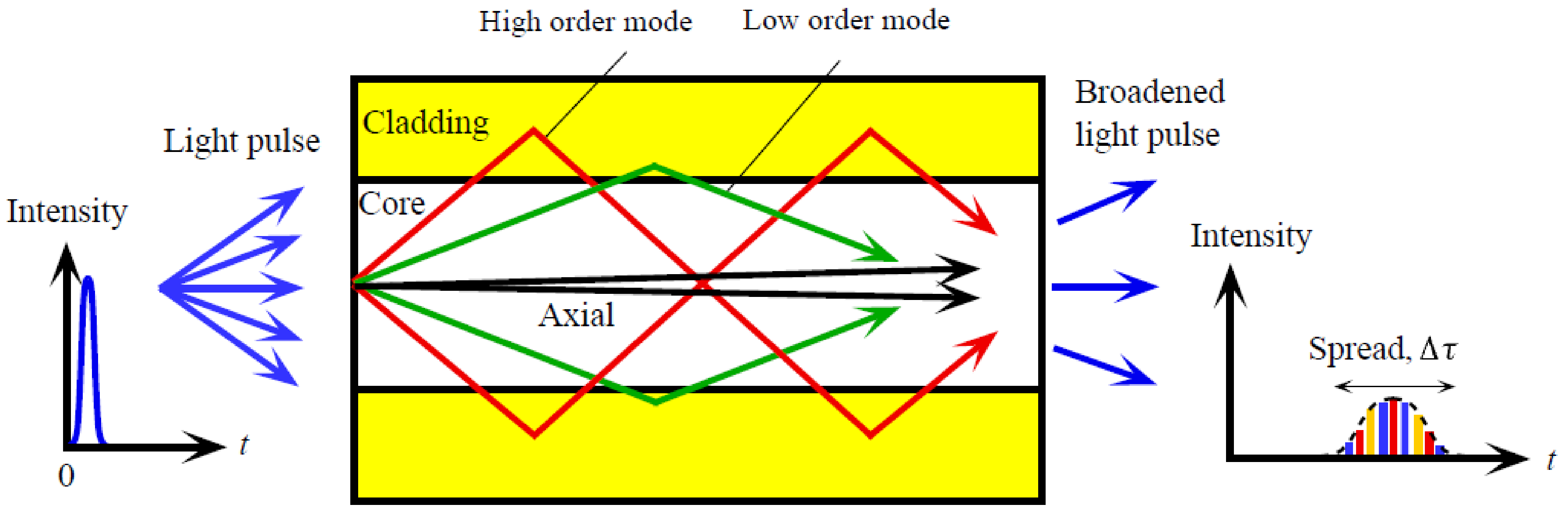
TE and TM modes

- Finding the TM modes: all you need to do is change the $\phi_{m,TE}$ in the above derivations to $\phi_{m,TM}$
- Of course, the resulting same order (TE, TM)-modes will have different θ_m
- Note: B_z for TE and E_z for TM!!
  - Only possible in waveguides



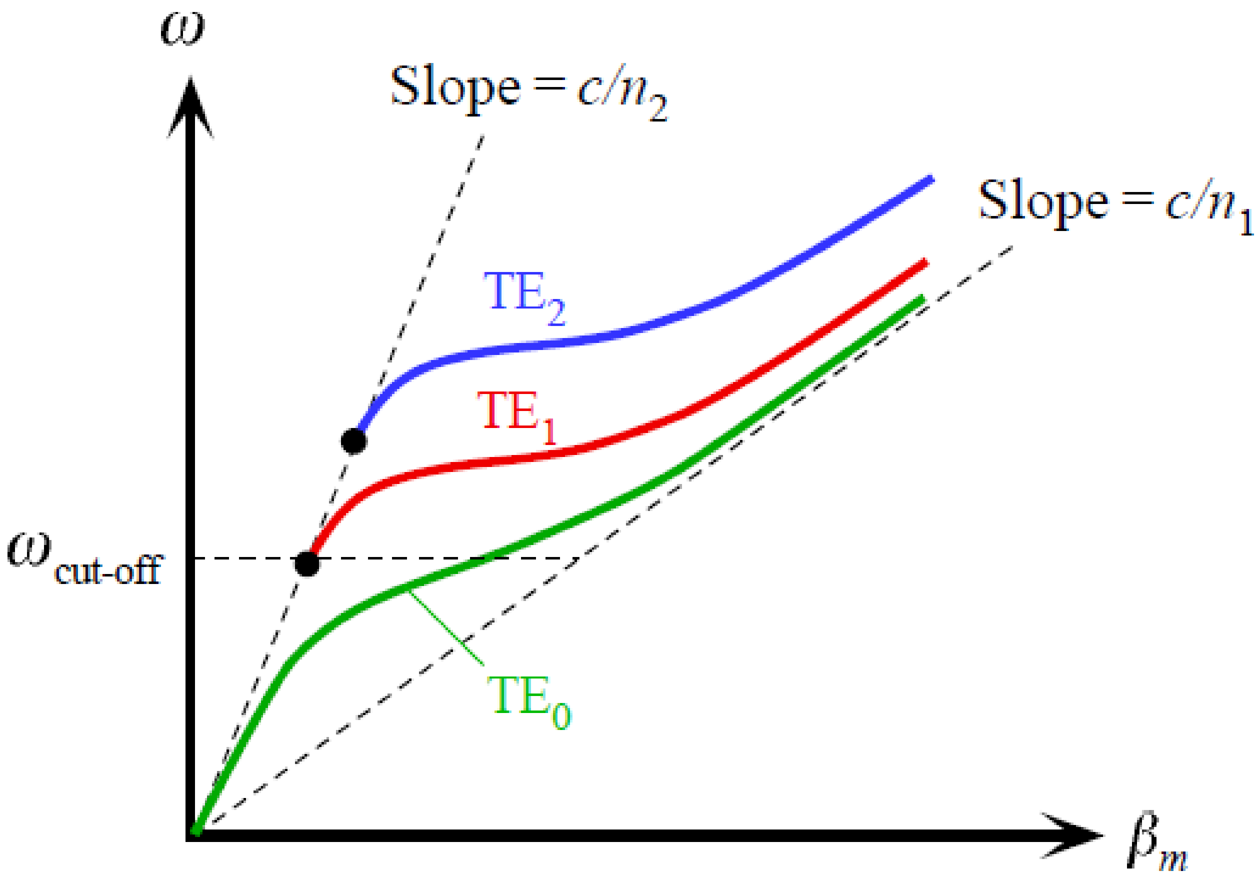
Intermodal dispersion

- Different transverse modes travel at different velocities
- But do higher-order modes travel slower?
    - We need to understand **dispersion diagram/relation**



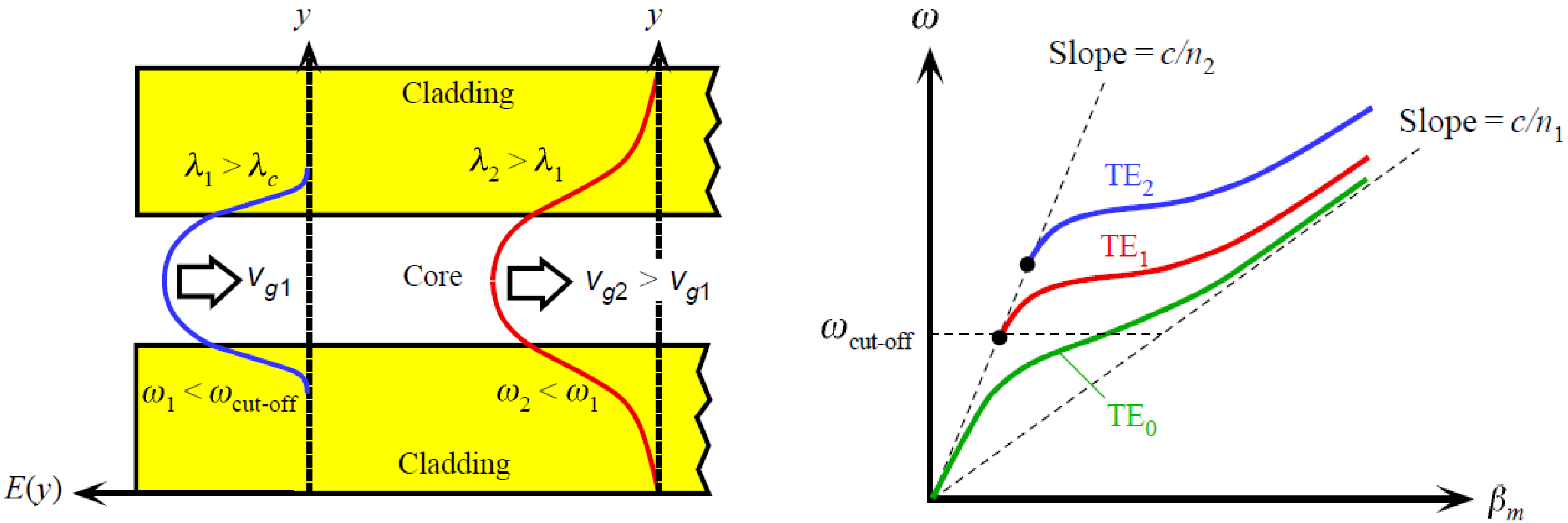
Dispersion diagram

- For TE_m modes
- The slope gives the **group velocity v_g**



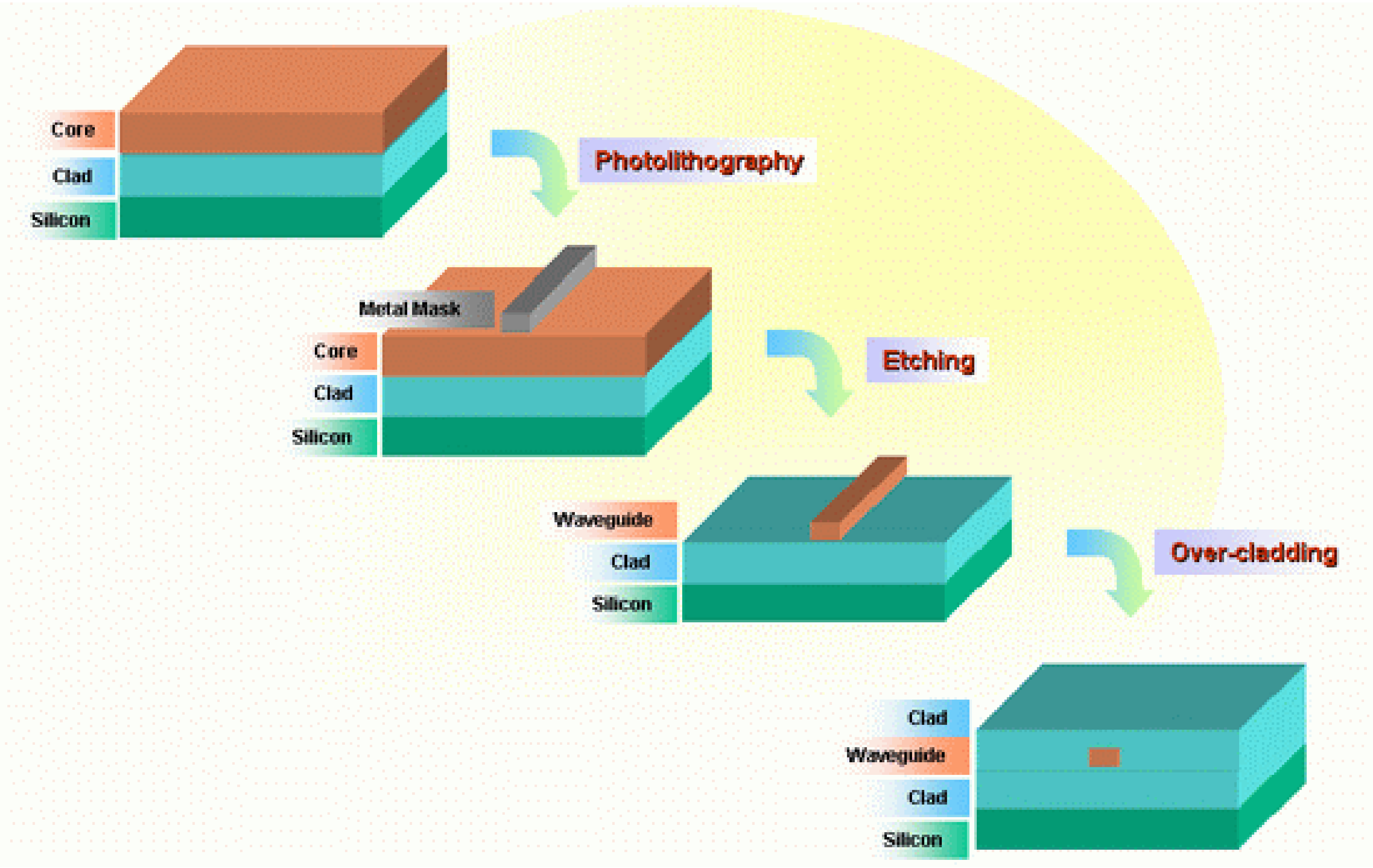
Intramodal dispersion

- Same transverse mode: wavelength dependence
- TE_0 mode: longer wavelength travels faster
  - Matches our understanding to typical n(λ)



Planar lightwave circuit (PLC)

- Fabrication processes



Real PLC

- Arrayed waveguide grating