Lesson 3
Coherent optical effects

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Outline

- Coherence
  - Temporal and spatial
- Interference
- Interferometers
  - Michelson
  - Mach-Zehnder
- Scattering➔Diffraction

- Coherence?
  - Spatial
  - Temporal

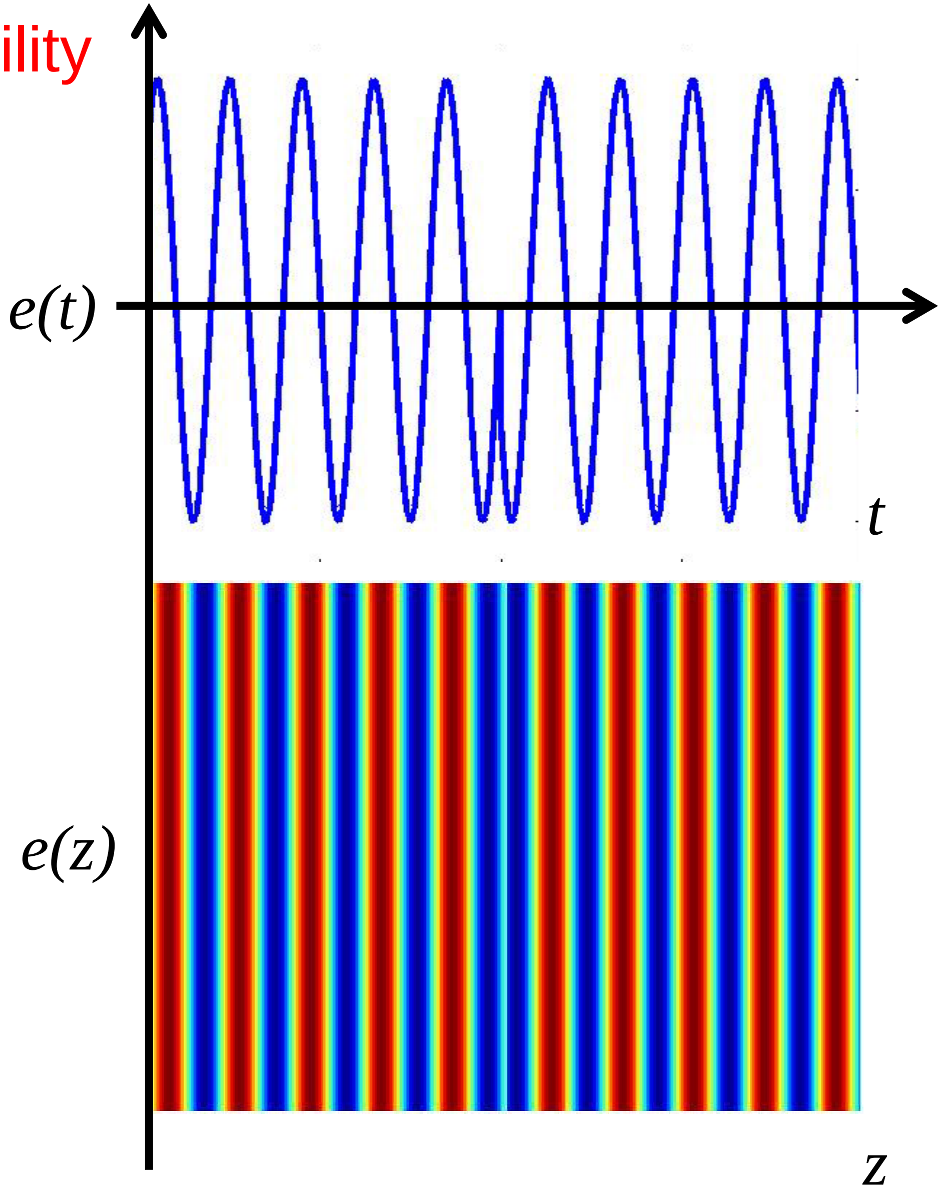
Coherence

- Unique for waves: phase stability
- Temporal coherence
  - Effect?

$$e(t) = A\cos[\omega t + \varphi(t)]$$

- Spatial coherence
  - Effect?

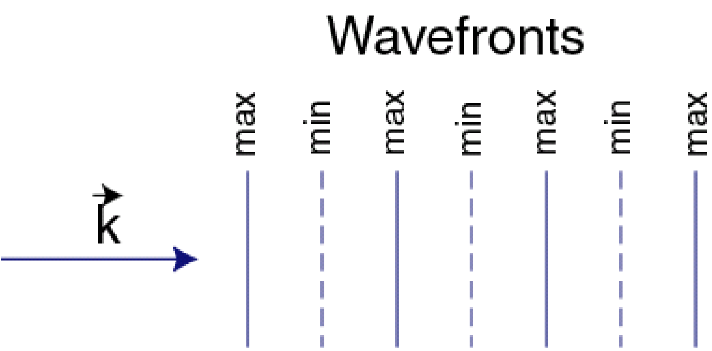
$$e(z) = A\cos[kz + \varphi(z)]$$

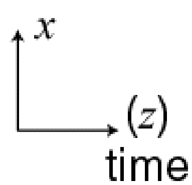


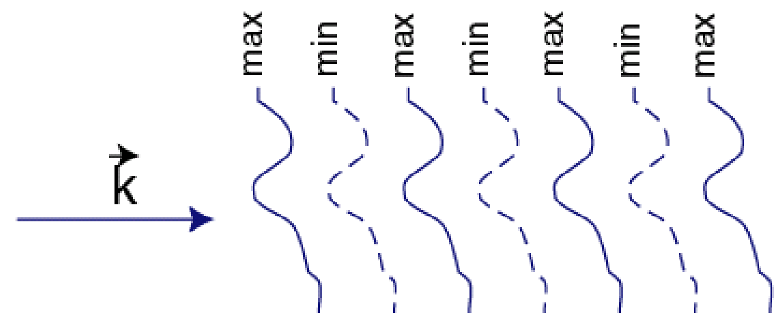
Mixture of two in two-dimensions

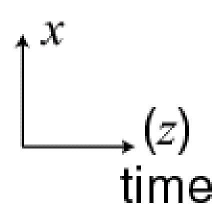
spatial and temporal coherent

temporal coherent; spatial incoherent



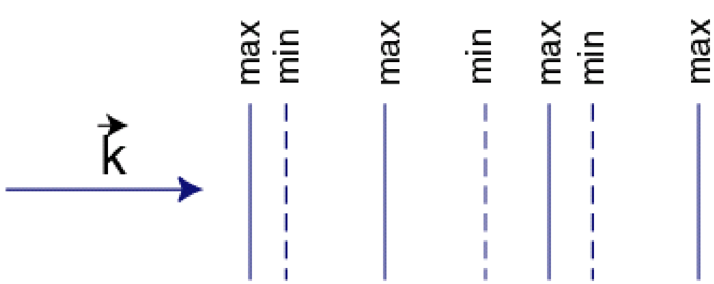


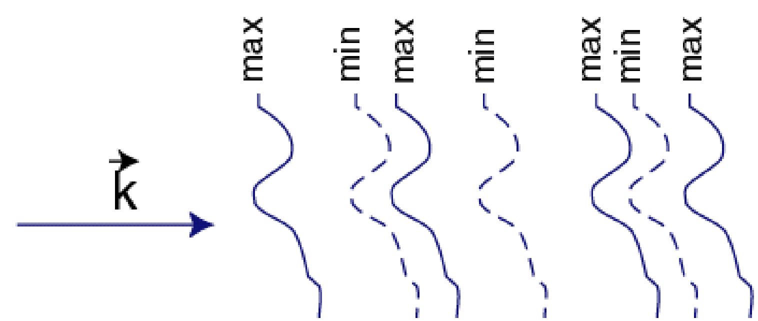




spatial coherent; temporal incoherent

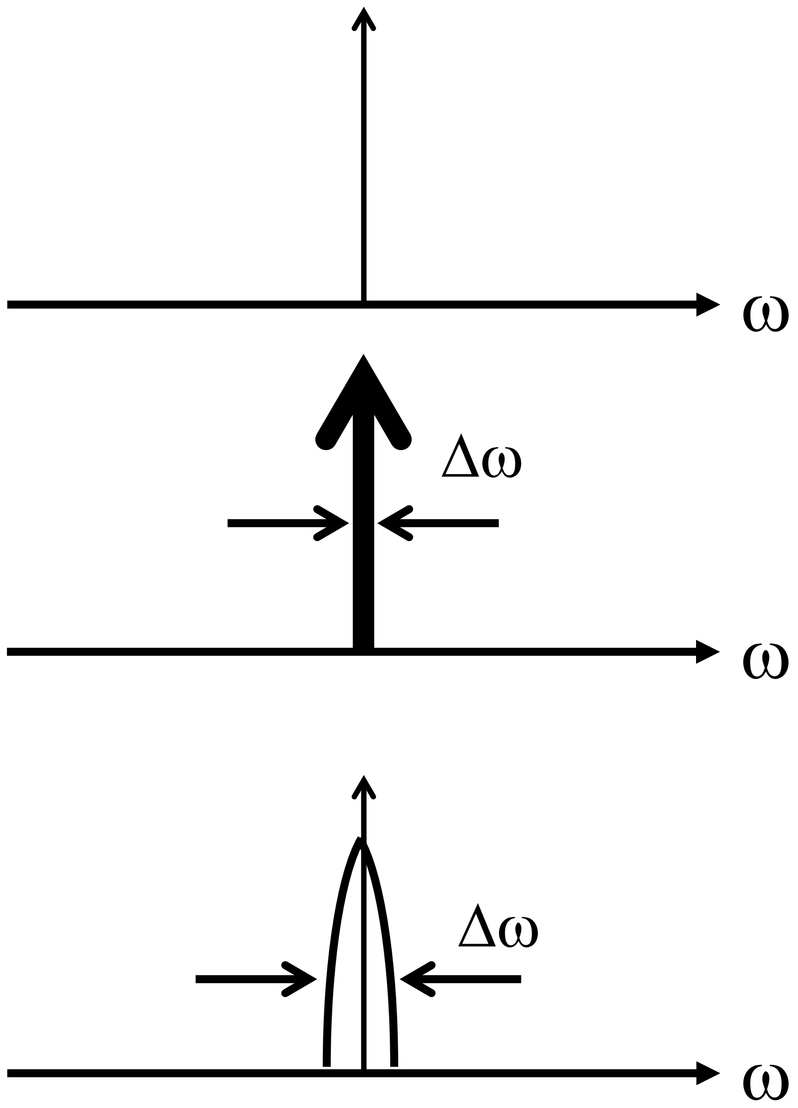
spatial and temporal incoherent





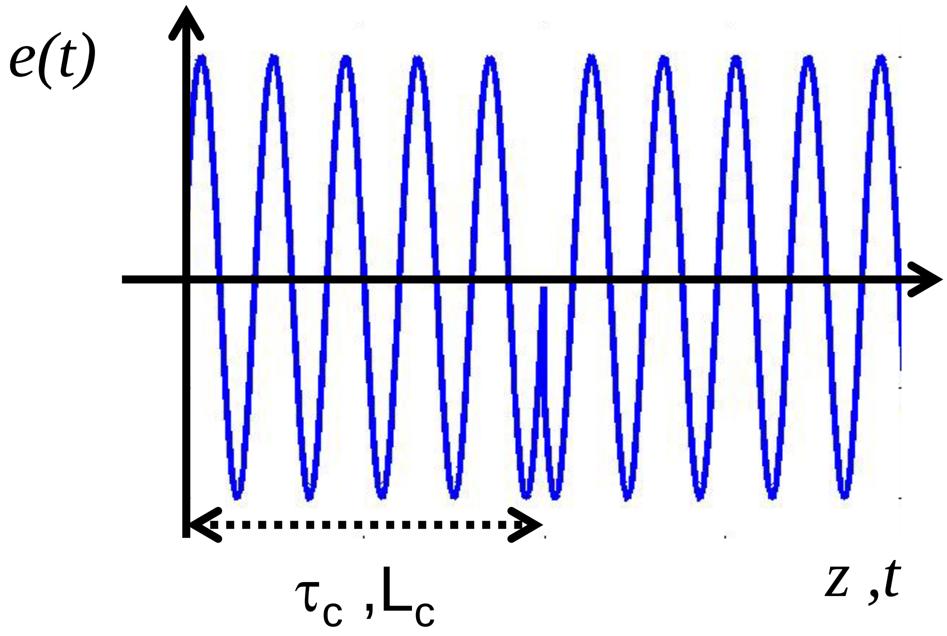
Coherence degradation

- Linewidth: impurity (Δω)
  - Perfectly coherent source
  - Highly coherent source
  - Poorly coherent source



The effect of linewidth

- Coherence time: τ_c



- Coherence length: L_c

$$L_c \simeq \tau_c \times c$$

Importance of coherence

- If you have two waves, for simplicity, we only investigate temporal characteristics

$$e(t) = A_1 \exp^{j\varphi_1(t)} + A_2 \exp^{j\varphi_2(t)}$$

$$I(t) \propto \frac{1}{2}\left\langle |e|^2 \right\rangle = \frac{1}{2}\left\langle A_1^2 + A_2^2 + 2A_1A_2\cos[\Delta\varphi(t)]\right\rangle$$

$$= \frac{1}{2}\left[A_1^2 + A_2^2 + 2A_1A_2\left\langle\cos[\Delta\varphi(t)]\right\rangle\right]$$

$\langle\ \rangle$: time average

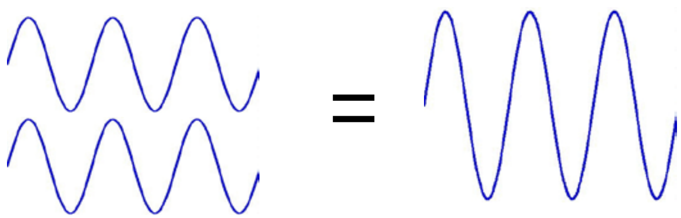
High degree of coherence is essential for stable optical signal!

- Interference
  - Constructive
  - Destructive
- Interferometer
  - Gravitational wave

Coherent interference

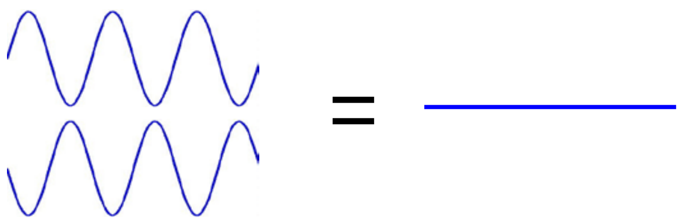
Again, a result of **coherence**: phase stability is crucial!!!

Waves that combine **in phase** add up to relatively high irradiance.



Constructive interference

Waves that combine **180˚ out of phase** cancel out and yield zero irradiance.



Destructive interference

How coherence affect added intensity?

- Perfect coherence
  - Stable output

$$I = \frac{1}{2}\left[A_1^2 + A_2^2 + 2A_1A_2\cos[\Delta\varphi]\right]$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos[\Delta\varphi]$$

- Partial coherence
  - Slowly changing against time
- Incoherent
  - Phase too random
  - Stable, but

$$I = I_1 + I_2$$

Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos[\Delta\varphi]$$

- Based on highly coherent optical sources

- Coherent addition: constructive interference (**$\Delta\phi$=0**)
  - 1+1 ➔ 4!!

$$I = I_1 + I_1 + 2\sqrt{I_1 I_1}$$

- Coherent subtraction: destructive interference (**$\Delta\phi$=π**)
  - 1+1 ➔ 0!!

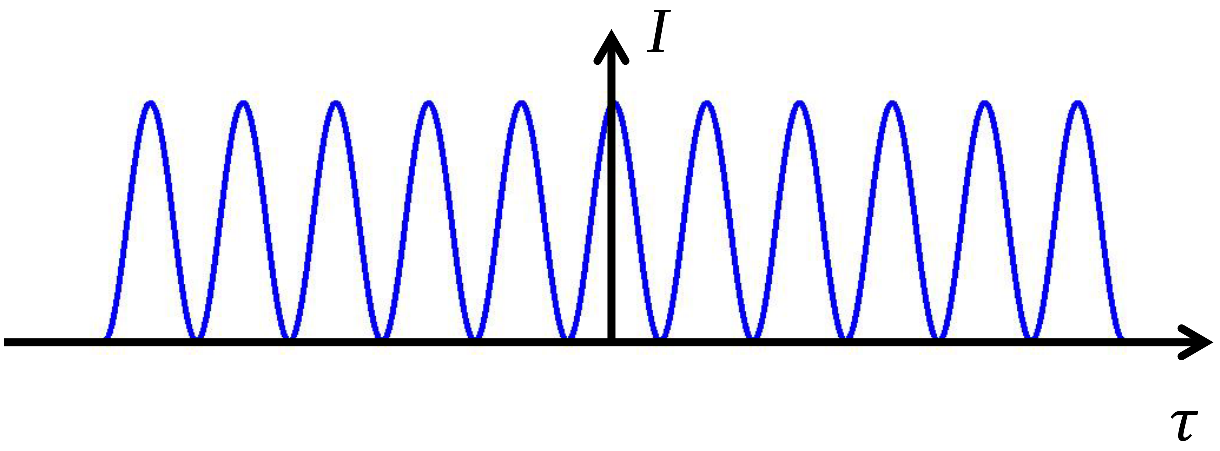
$$I = I_1 + I_1 - 2\sqrt{I_1 I_1}$$

Other absolute phase differences?

Interferometers

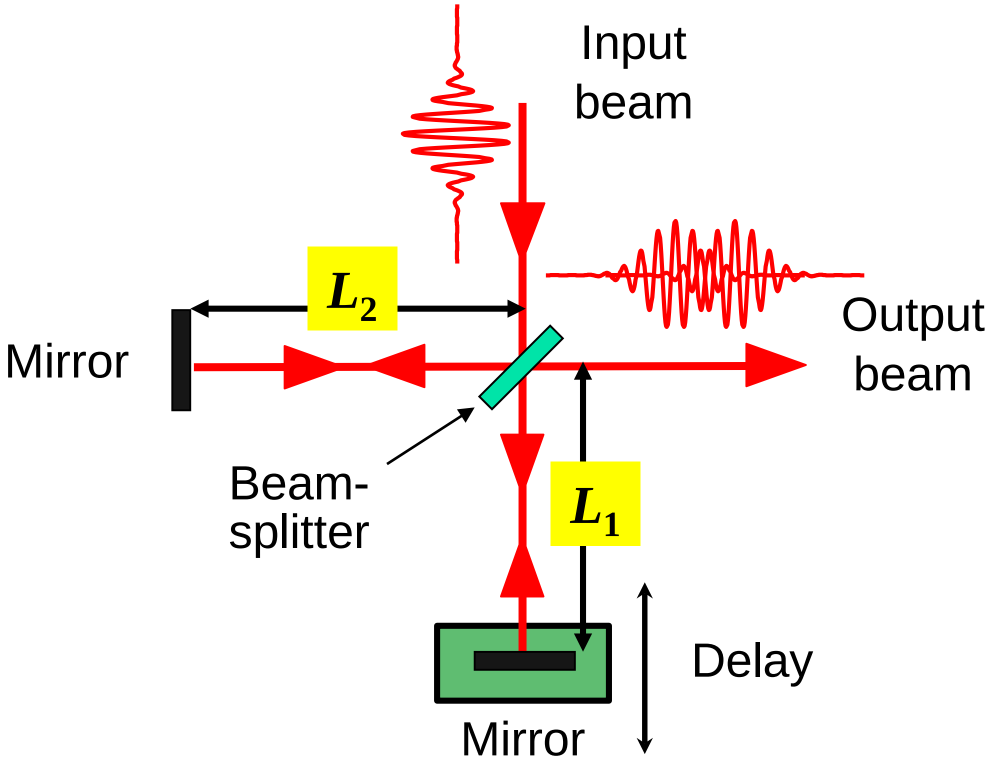
- Absolute phase difference change ➔ fringes
- Example: temporal delay τ for temporal phase control

$$I = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos[\omega\tau]$$

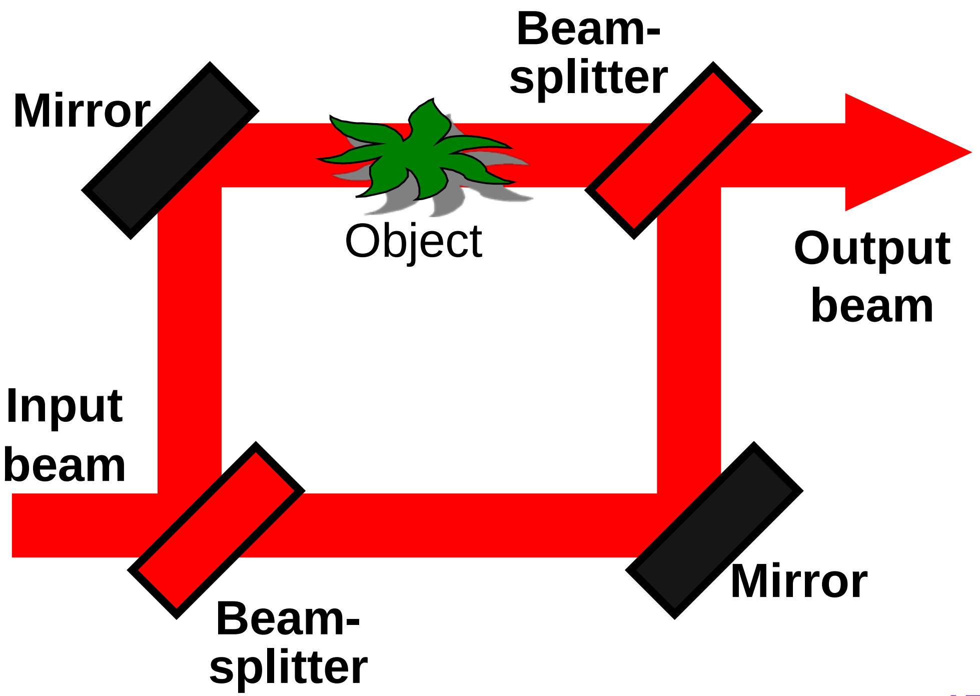


How to construct such setup?

Michelson interferometer

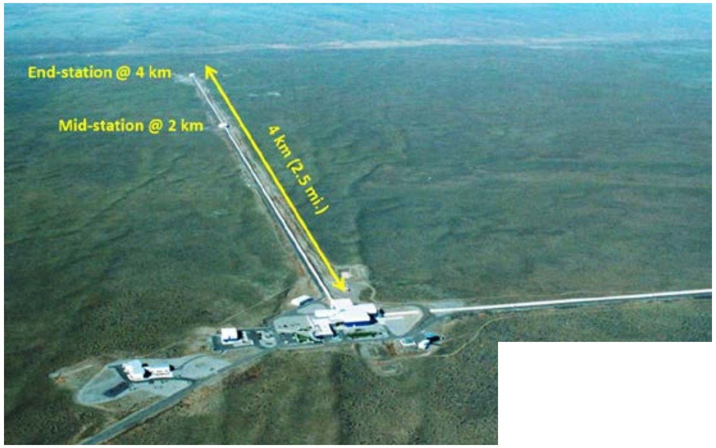


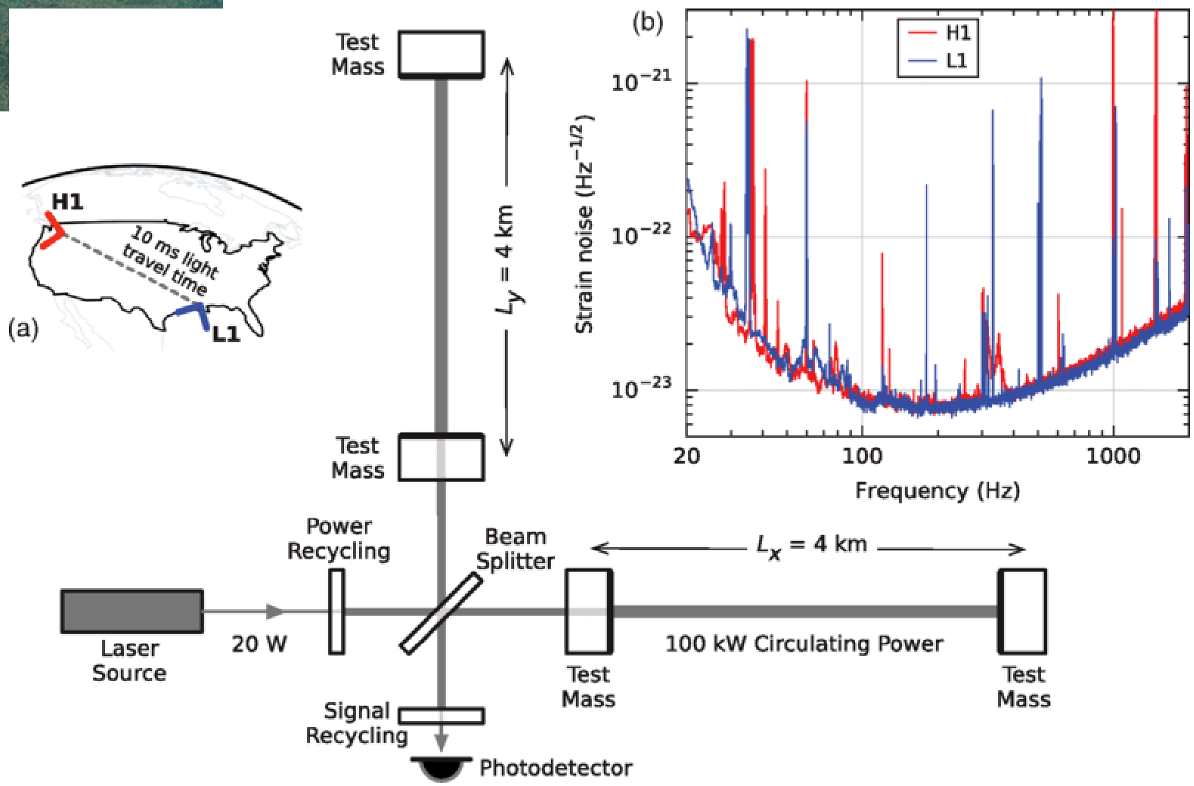
Mach-Zehnder interferometer



LIGO: detection of gravitational waves

- Based on interferometer



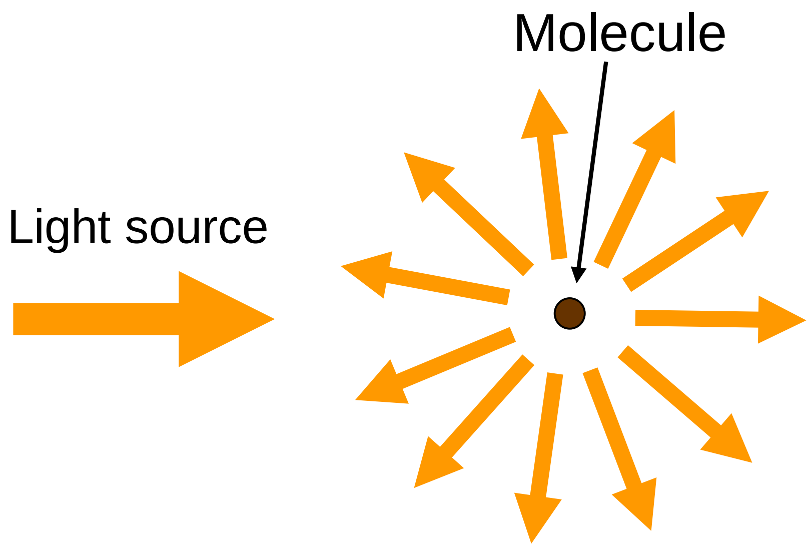


- Scattering
  - Spherical wavelets
  - Colors
  - Scattering + Interference → Law of reflection

Light scattering

When light encounters matter, matter re-emits light in all other directions.

This is called **scattering**



Light scattering is everywhere. All molecules scatter light. Surfaces scatter light. Scattering causes milk and clouds to be white. It is the basis of nearly all optical phenomena.

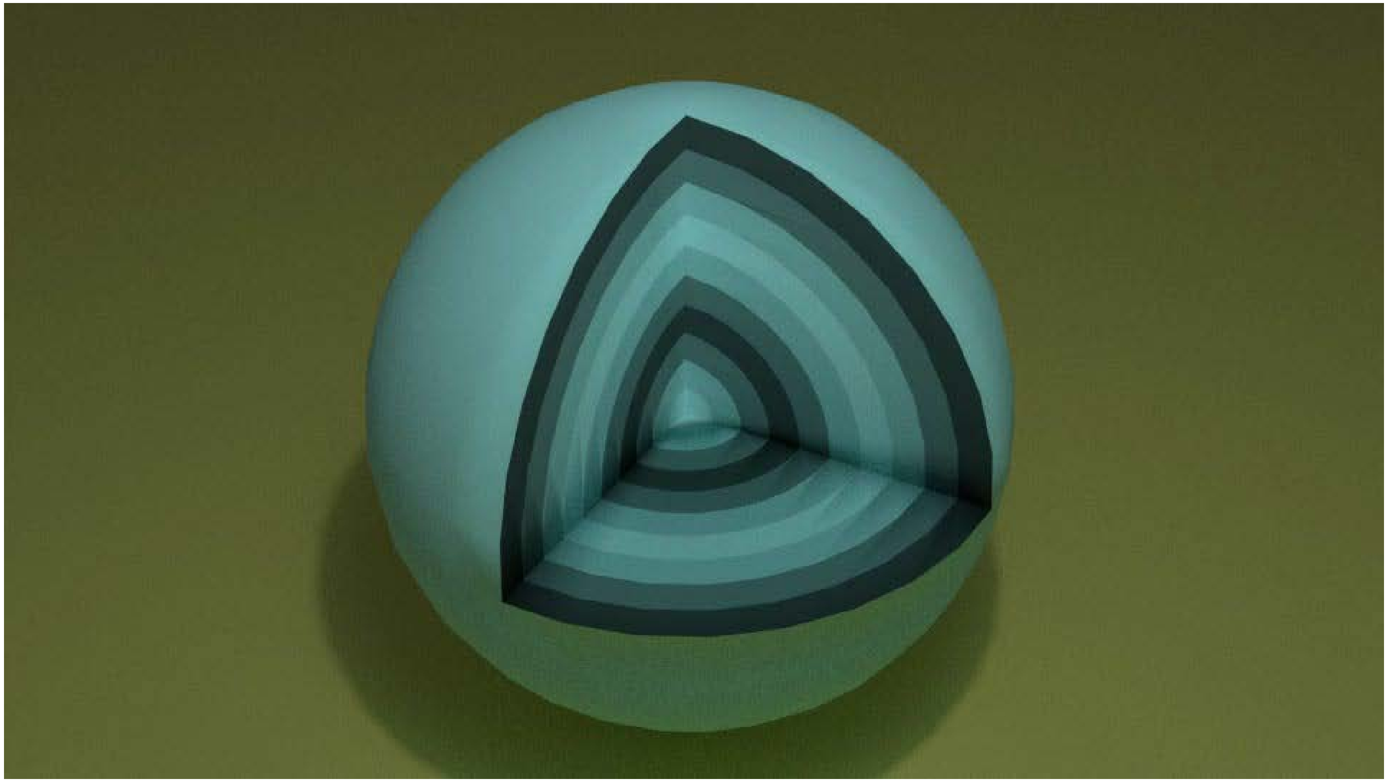
Let’s do an experiment!!

Spherical waves

A spherical wave is also a solution to Maxwell's equations and is a good model for the light scattered by a molecule.

$$E(\vec{r},t) = \frac{E_0}{r} \mathrm{Re}\{\exp[j(\omega t - kr)]\}$$

- where k is a scalar, and
- r is the radial magnitude.



Colors due to scattering

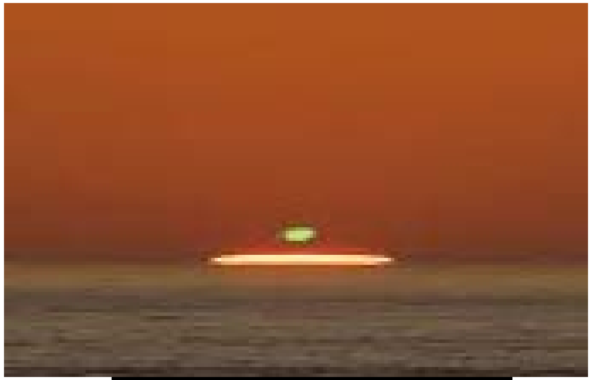
- Blue sky, yellow sunset
- **Rayleigh scattering**

$$S \propto f^{4}$$

Sunset can be Green!

Just as the sun sets, there is a green flash for a fraction of a second.

Pirates of the Caribbean: At World's End

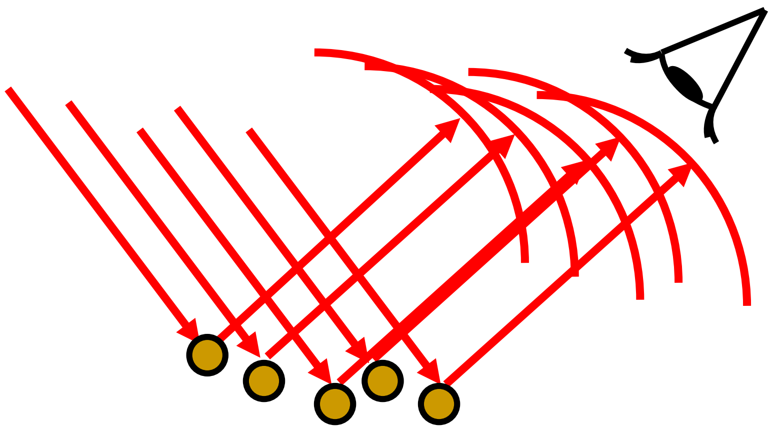


Refractive ray curvature, but the long path allows an otherwise unseen atmospheric absorption in the yellow to split the sun's spectrum into orange and green components.

Check one direction at a time, far away

This way we can approximate spherical waves as plane waves in that direction, vastly simplifying the math.

$$E(\vec{r},t)=\frac{E_0}{r}\mathrm{Re}\{\exp[j(\omega t-kr)]\}\quad \Rightarrow E_{r0}\,\mathrm{Re}\{\exp[j(\omega t-kz)]\}$$



Far away, spherical wave-fronts are almost flat…

Usually, coherent constructive interference will occur in one direction, and destructive interference will occur in all others.

If incoherent interference occurs, it is usually omni-directional.

The mathematics of scattering

The math of light scattering is analogous to that of interference.

If the phases are not random (coherent process), we add the fields:

$$\boxed{\bullet\ E_{total} = E_1 + E_2 + \ldots + E_n}$$

$$I_{total} = \underbrace{I_1 + I_2 + \ldots + I_N}_{} + \underbrace{c\varepsilon\,\mathrm{Re}\left\{E_1E_2^* + E_1E_3^* + \ldots + E_{N-1}E_N^*\right\}}_{}$$

$I_1, I_2, \ldots I_n$ are the irradiances of the various beamlets. They're all positive real numbers and add.

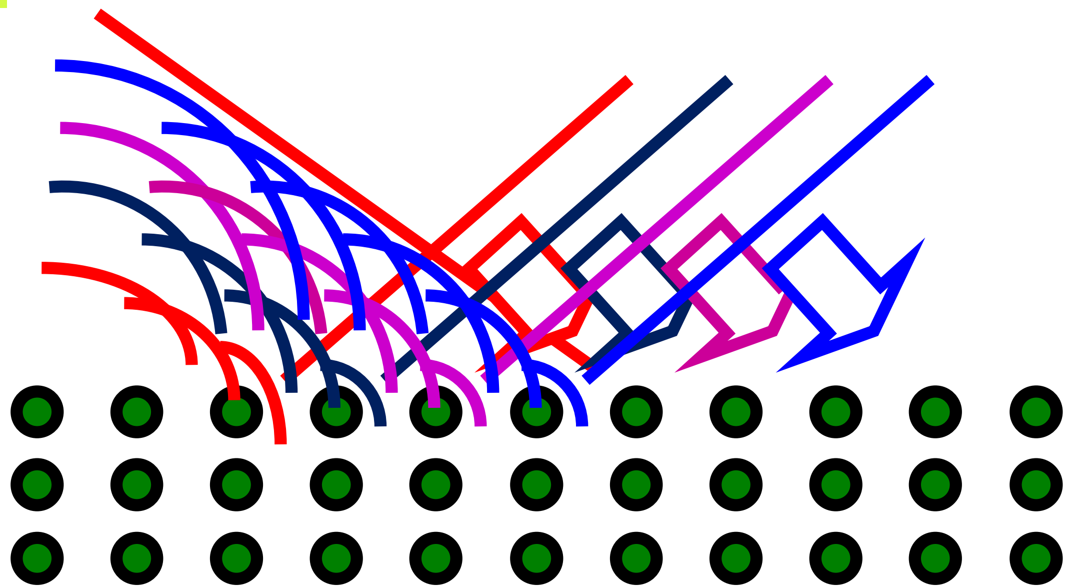
$E_i\, E_j^*$ are cross terms, which have the phase factors: $\exp[j(\theta_i - \theta_j)]$. When the θ's are not random, they don't cancel out!

Scattered spherical waves → plane waves

A plane wave impinging on a surface (that is, lots of very small closely spaced scatterers!) will produce a reflected plane wave because all the spherical wavelets interfere constructively along a flat surface.

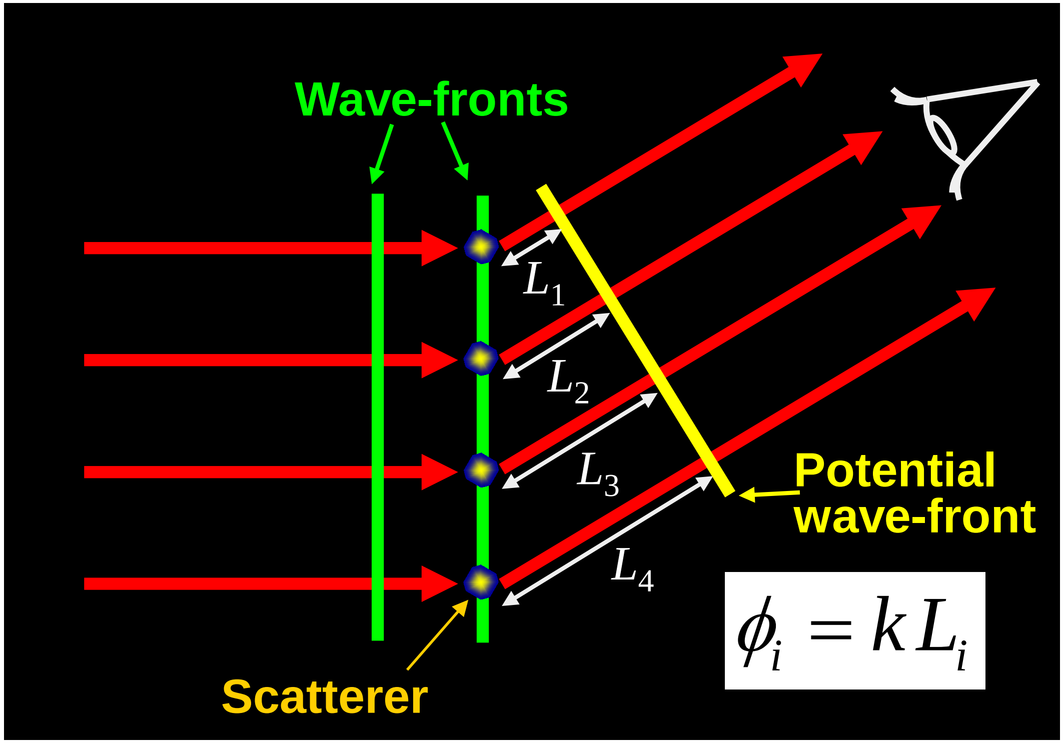
What Huygens taught us

Christian Huygens (1629-1695)



Phase delays

Because the phase is constant along a wave-front, we compute the phase delay from one wave-front to another potential wave-front.



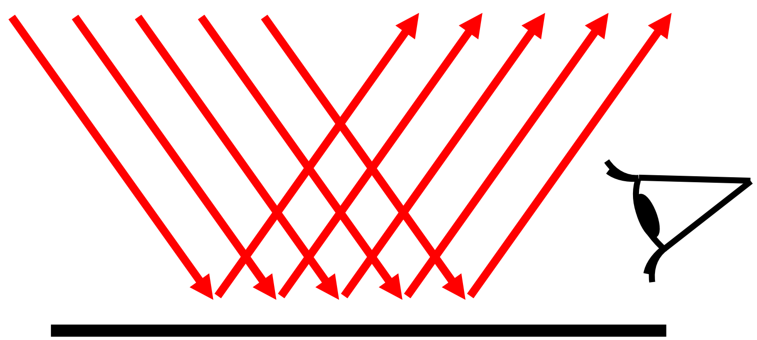
Phase delays all same (modulo 2π): constructive and coherent.

Phases vary uniformly: destructive and coherent.

Phases random: incoherent.

Direction of coherent scattering

A smooth surface scatters light coherently and constructively only in the direction whose **angle of reflection equals the angle of incidence**.

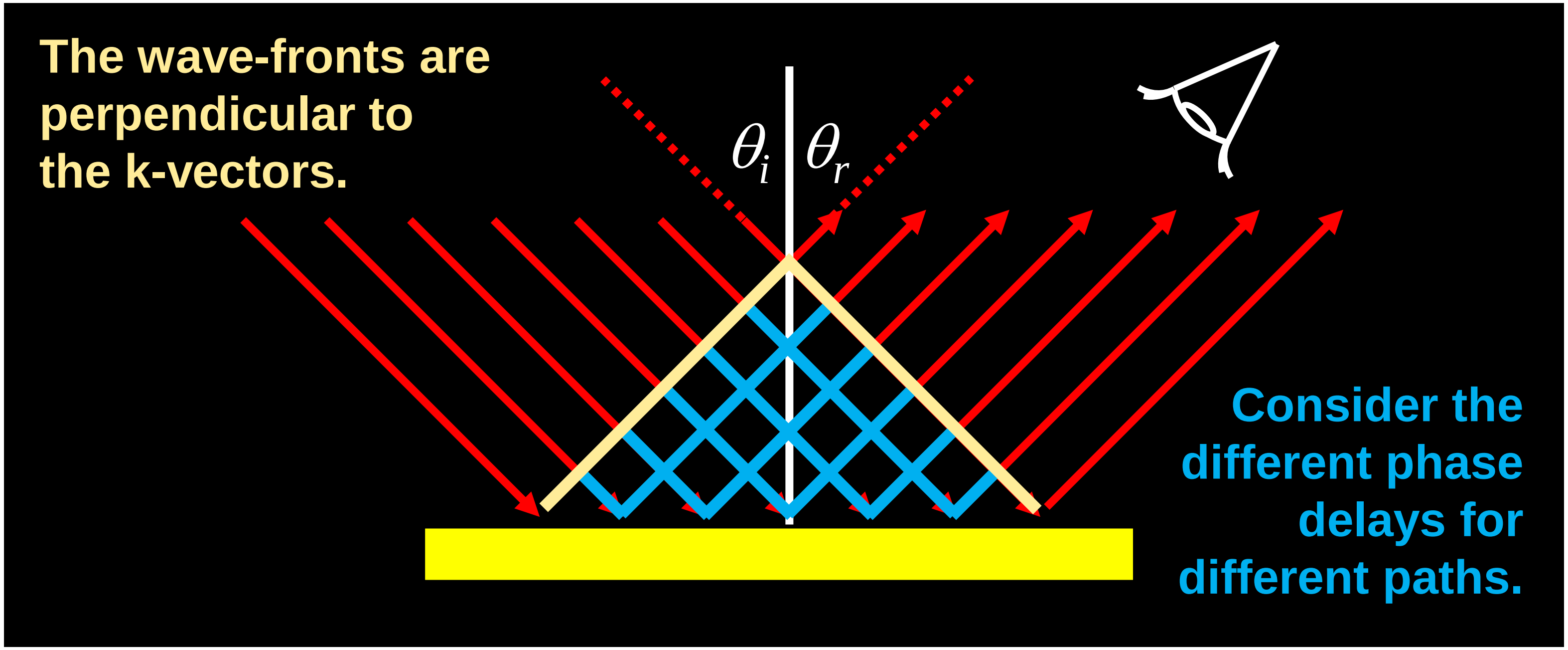


Looking from any other direction, you'll see no light at all due to coherent destructive interference.

Let's do an experiment!!

Coherent constructive scattering

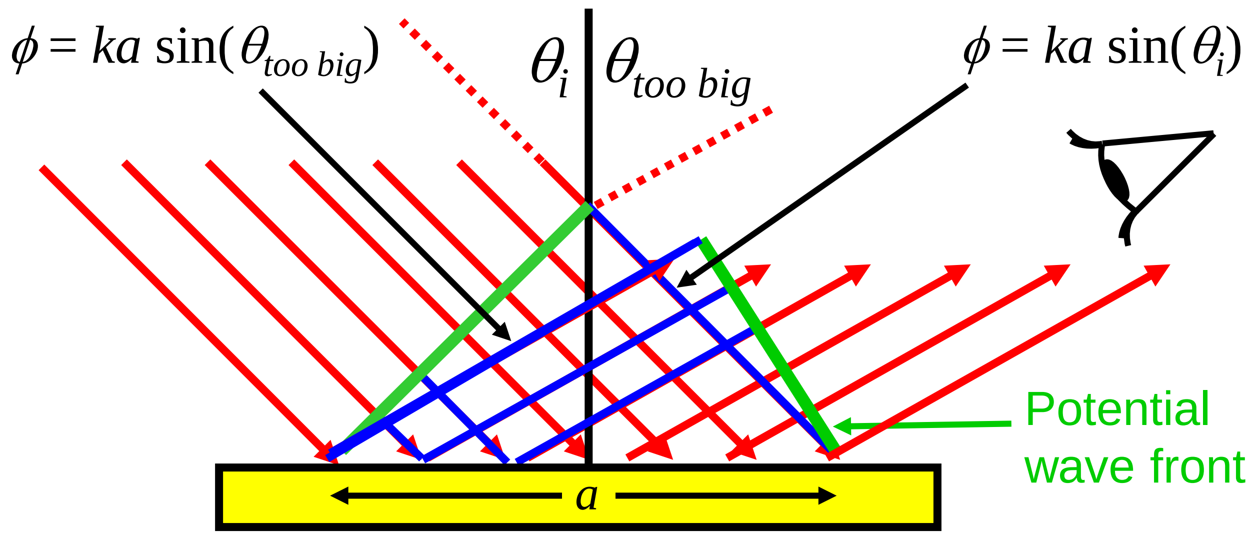
A beam can only remain a plane wave if there's a direction for which coherent constructive interference occurs.



Coherent constructive interference occurs for a reflected beam if the angle of incidence = the angle of reflection: $\theta_i = \theta_r$.

Coherent destructive scattering: different angles

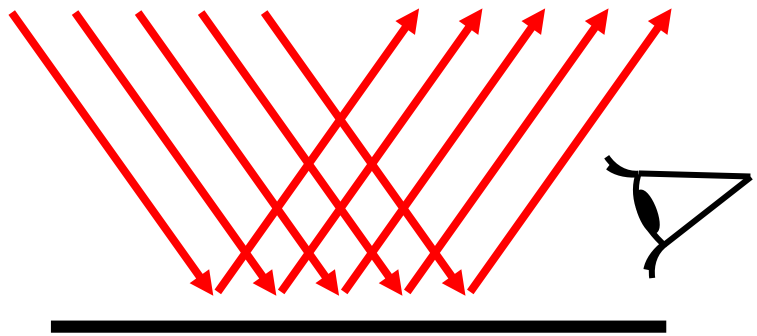
Imagine that the reflection angle is too big, the symmetry is now gone, and the phases (path lengths) are now all different



Coherent destructive interference occurs for a reflected beam direction if the angle of incidence ≠ the angle of reflection: $\theta_i \neq \theta_r$.

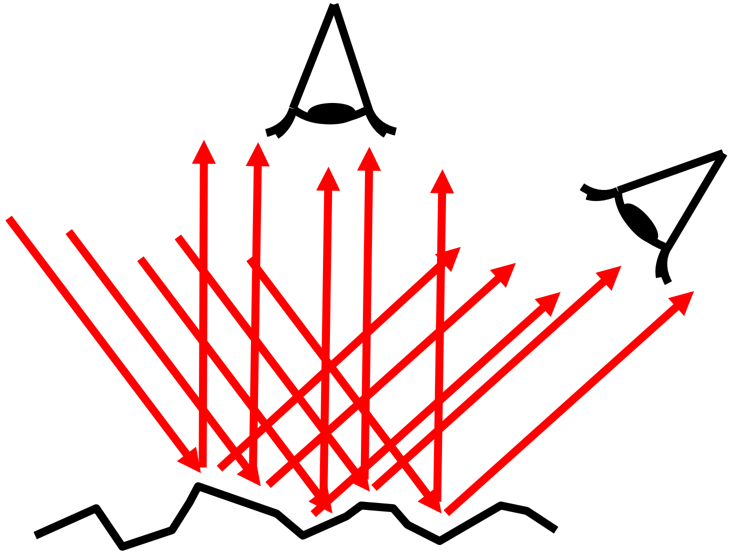
Direction of coherent scattering

A **smooth surface** scatters light coherently and constructively only in the direction whose angle of reflection equals the angle of incidence.



Looking from any other direction, you'll see no light at all due to coherent destructive interference.

Incoherent scattering: rough surface



No matter which direction we look at it, each scattered wave from a rough surface has a different phase. So scattering is incoherent, and we'll see weak light in all directions.

This is why rough surfaces look different from smooth surfaces and mirrors.

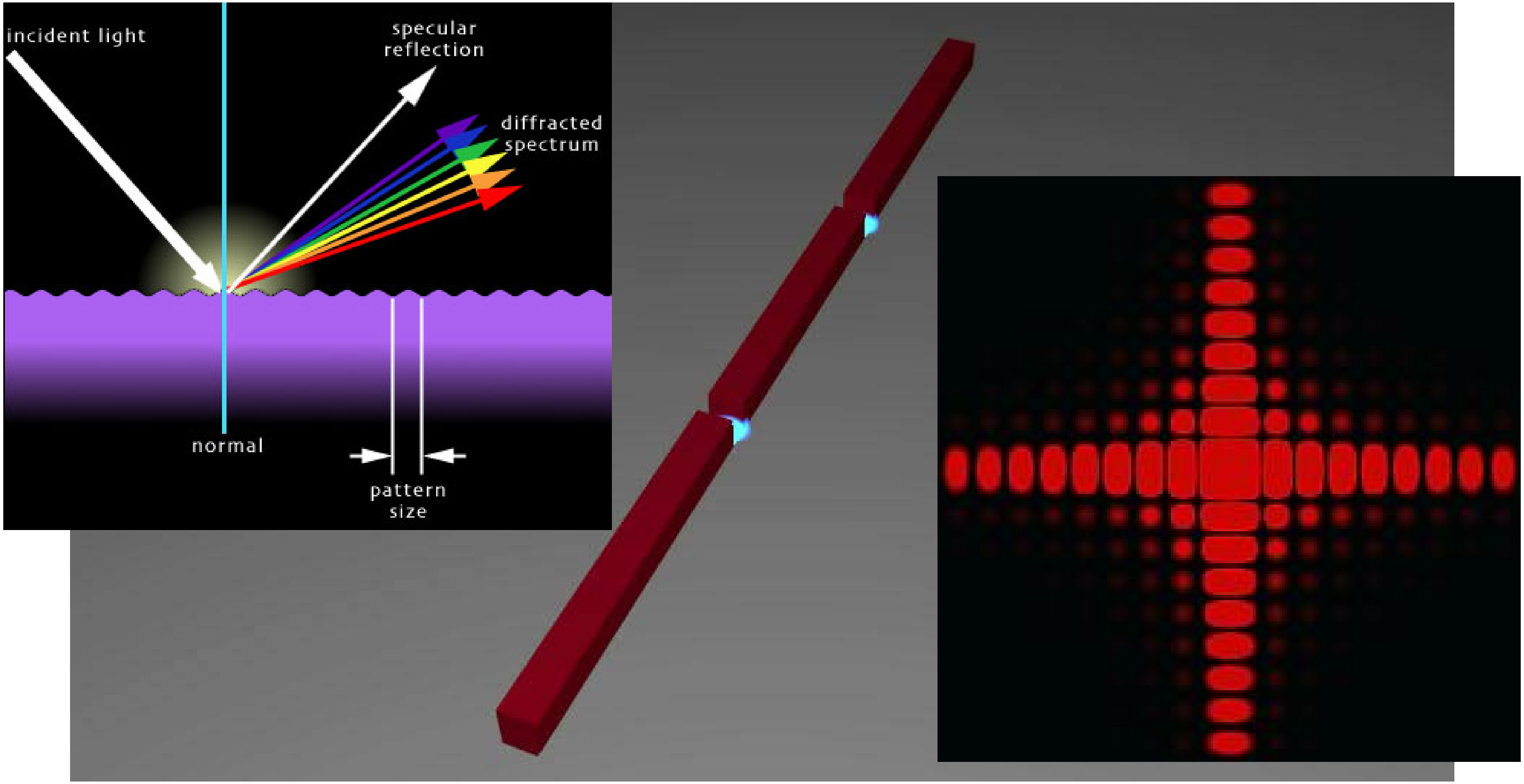
Let's do an experiment!!

- Diffraction
  - Optical grating

Diffraction?

- A coherent process

Scattering **(periodic)** → Interference → Diffraction

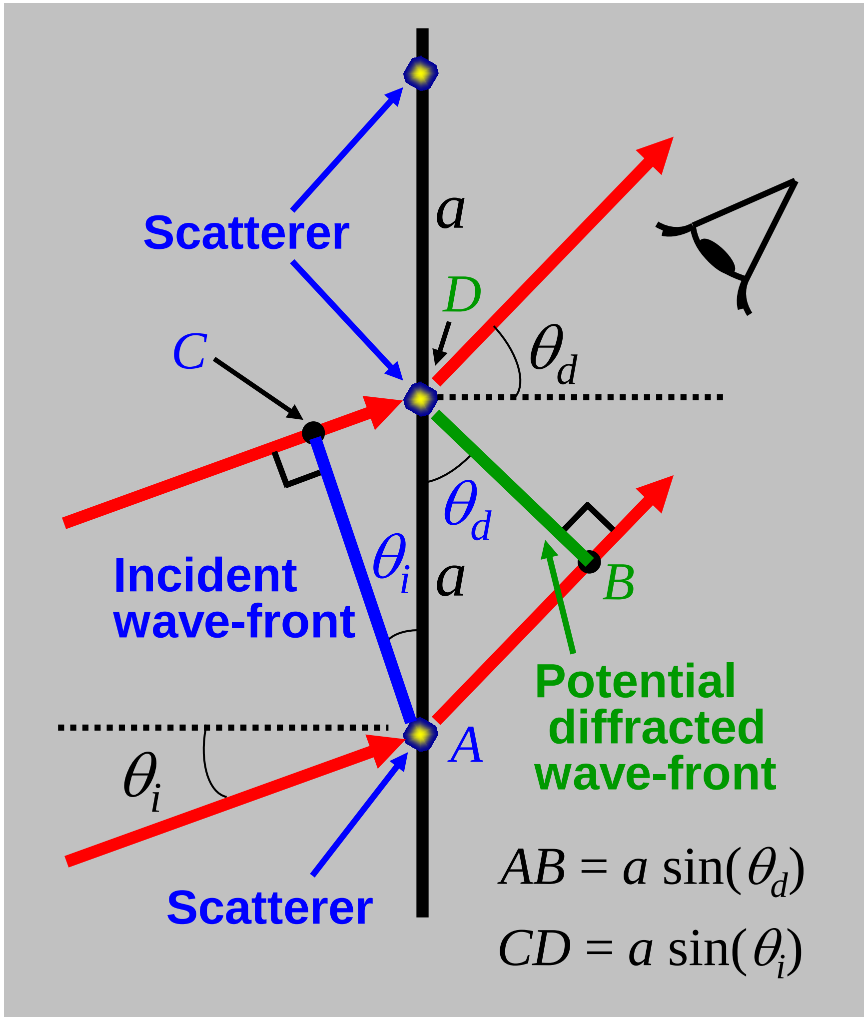


Diffraction gratings

Scattering explain what happens when light impinges on a periodic array of grooves. Constructive interference occurs if the delay between adjacent beamlets is an integral number, *m*, of wavelengths.

Path difference: $AB - CD = m\lambda$

$$\boxed{a\left[\sin(\theta_d) - \sin(\theta_i)\right] = m\lambda}$$



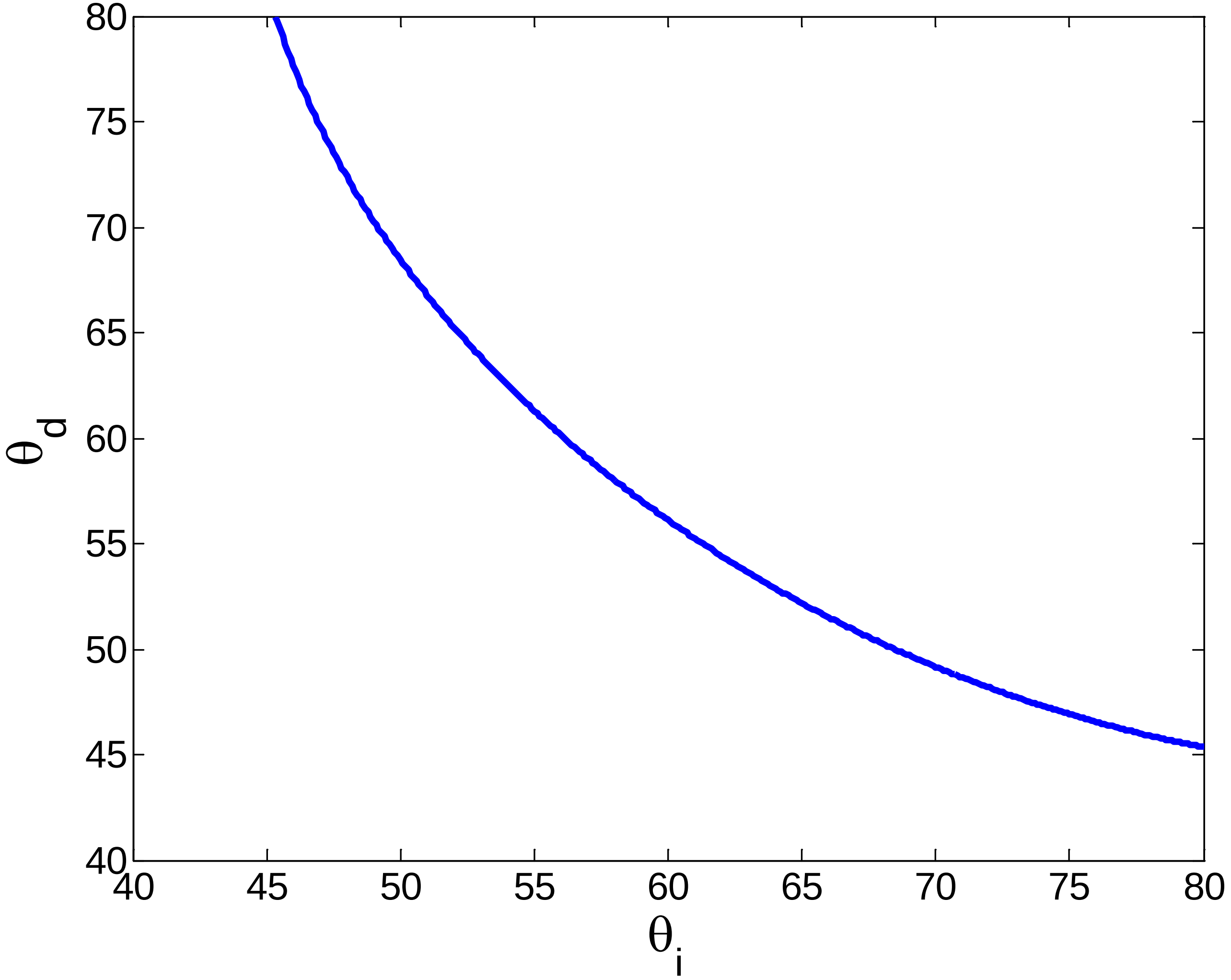
where m is any integer.

A grating has solutions or zero, one, or many values of m, or **orders**.

Remember that m and θ_m can be negative, too.

Incident angle vs. diffraction angle

$$a\left[\sin(\theta_d)-\sin(\theta_i)\right]=\lambda$$



Diffraction-grating dispersion

Because diffraction gratings are used to separate colors, it's helpful to know the variation of the diffracted angle vs. wavelength.

$$a\left[\sin(\theta_d) - \sin(\theta_i)\right] = m\lambda$$

[θ_i is constant]

Differentiating the grating equation, with respect to wavelength:

$$a\cos(\theta_d)\frac{d\theta_d}{d\lambda} = m$$

Rearranging:

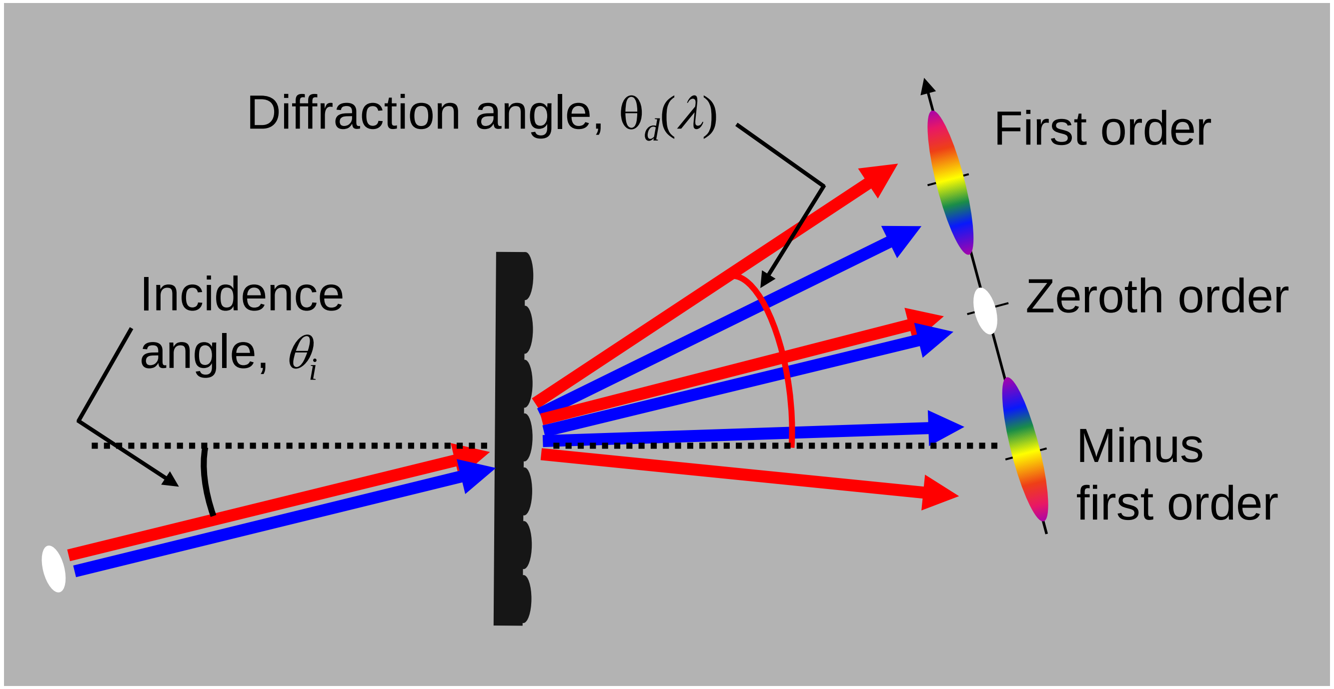
$$\boxed{\frac{d\theta_d}{d\lambda} = \frac{m}{a\cos(\theta_d)}}$$

Gratings typically have an order of magnitude more dispersion than prisms.

Thus, to separate different colors maximally, make a small, work in high order (make m large), and use a diffraction angle near 90 degrees.

Diffraction orders

Because the diffraction angle depends on λ, different wavelengths are separated in the nonzero orders.

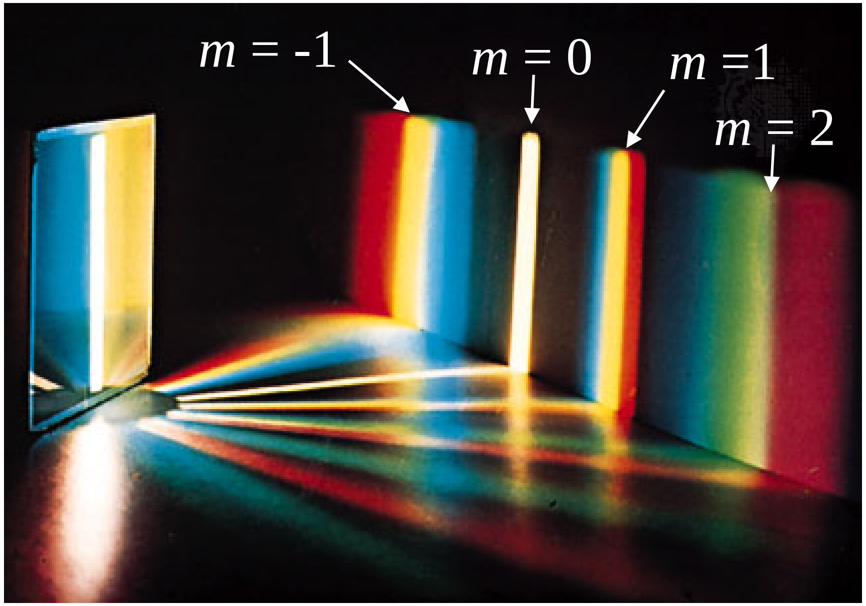


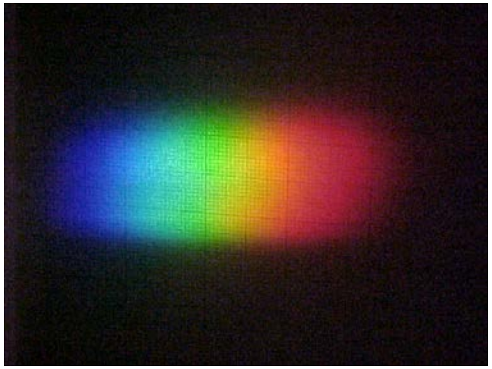
No wavelength dependence occurs in zero order.

The longer the wavelength, the larger its deflection in each nonzero order.

Real diffraction gratings

Diffracted white light



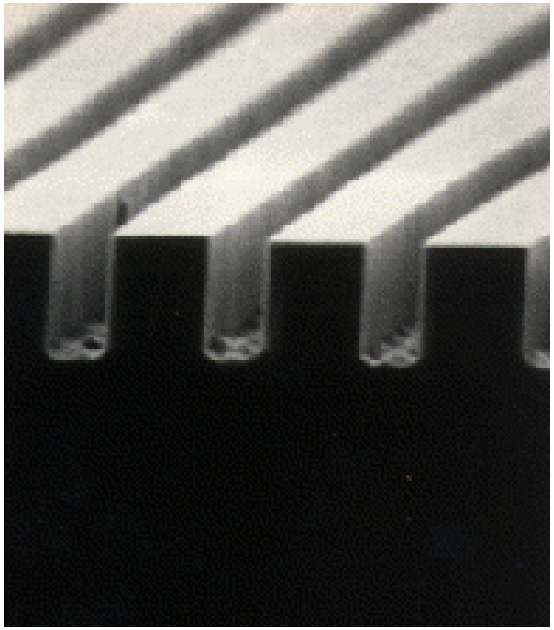


Diffraction gratings

The dots on a CD are equally spaced (although some are missing, of course), so it acts like a diffraction grating.

Any periodic surface is a diffraction grating

Gratings can work in reflection (r) or transmission (t).



Transmission gratings can be amplitude (α) or phase (n) gratings.

World's largest diffraction grating

Lawrence Livermore National Lab