Lesson 2
Refraction and Reflection

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Outline

- Normal incidence of plane wave

- Oblique incidence of TE and TM plane waves
  - Law of reflection
  - Snell’s law

- Total internal reflection
  - Near-field enhancement

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清華大學

黃承彬老師

For detailed derivations, please refer to my MOOCs course:
http://mooc.nthu.edu.tw/sharecourse/course/view/courseInfo/53

Reflection

- Gives us the natural beauties

Refraction

Refractive index and the speed of light

The wavelength in a medium is shorter than in vacuum

$$\boxed{\lambda = \lambda_0 / n}$$

The speed of light is ω/k. Since k_0 becomes $k = nk_0$ in a medium,

$$v = \omega/(nk_0) = (\omega/k_0)/n \quad \Rightarrow \quad \boxed{v = c/n}$$

The refractive index, n, of a medium is defined as the ratio:

$$\boxed{n \equiv c/v}$$

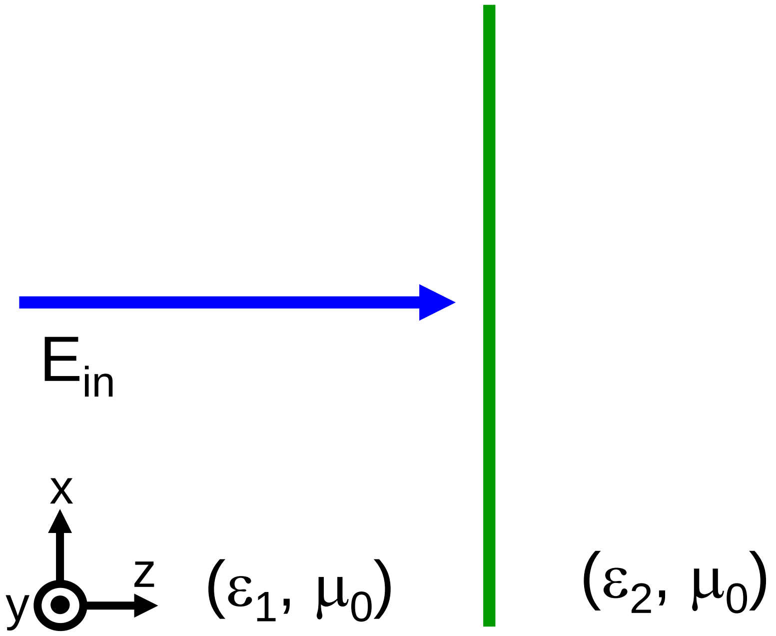
The refractive index is usually > 1. But it can sometimes be < 1.

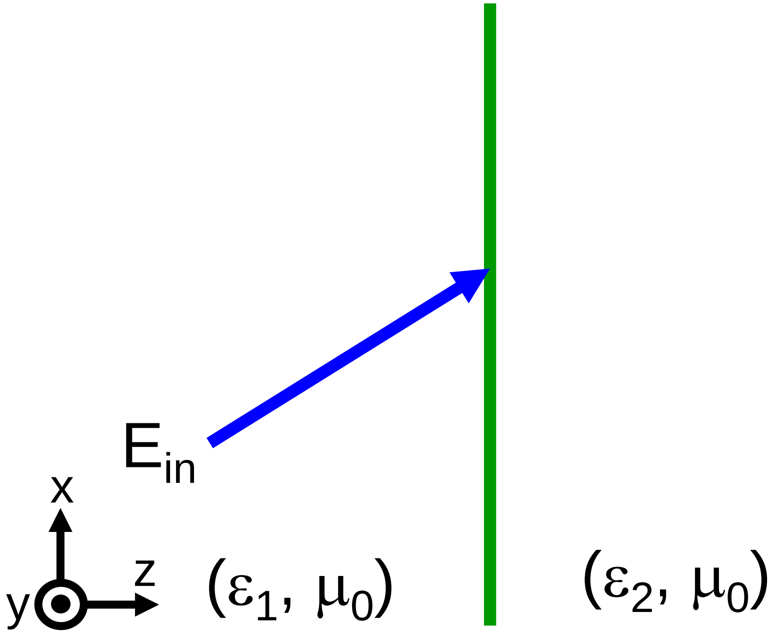
Derivations of Fresnel coefficients

- Normal incidence
- Oblique incidence
  - TE
  - TM

$$r_{TE} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad r_{TM} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$$t_{TE} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad t_{TE} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$



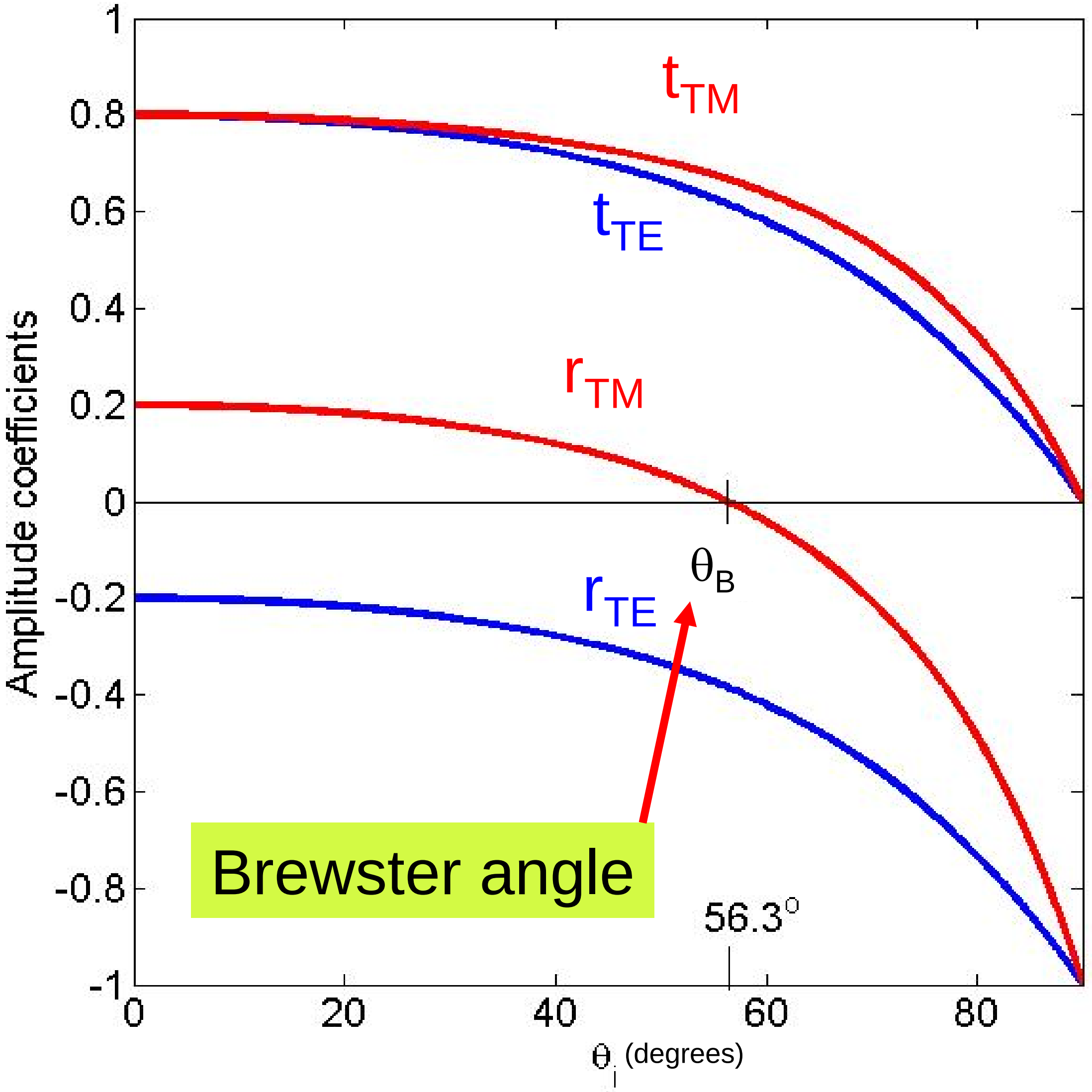


External reflection

- $n_1 < n_2$
- Air-glass interface

- Brewster angle (TM)
  - Special!

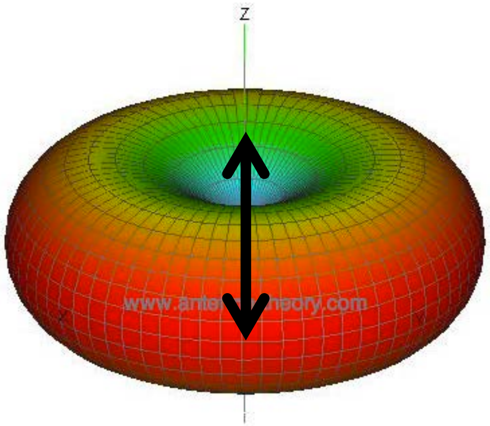
$$\theta_B + \theta_t = \frac{\pi}{2}$$



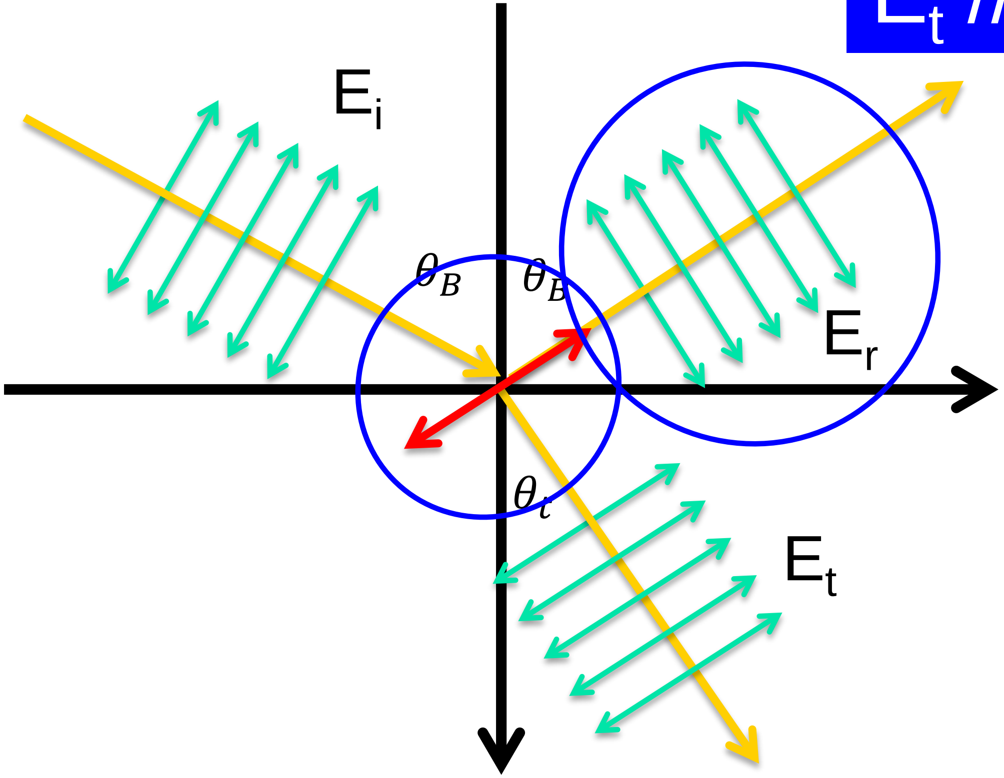
Brewster's angle: another perspective

- Why only TM mode?
- Recall that $\theta_B + \theta_t = \frac{\pi}{2}$

$$r_{TM} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_1 \cos\theta_t + n_2 \cos\theta_i} = 0$$

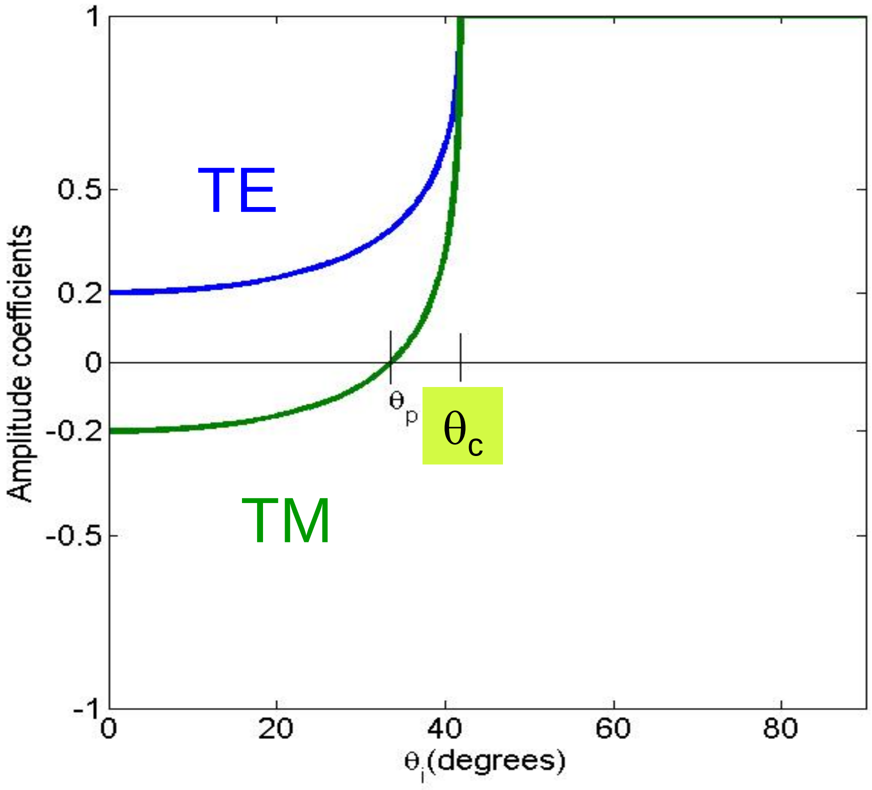


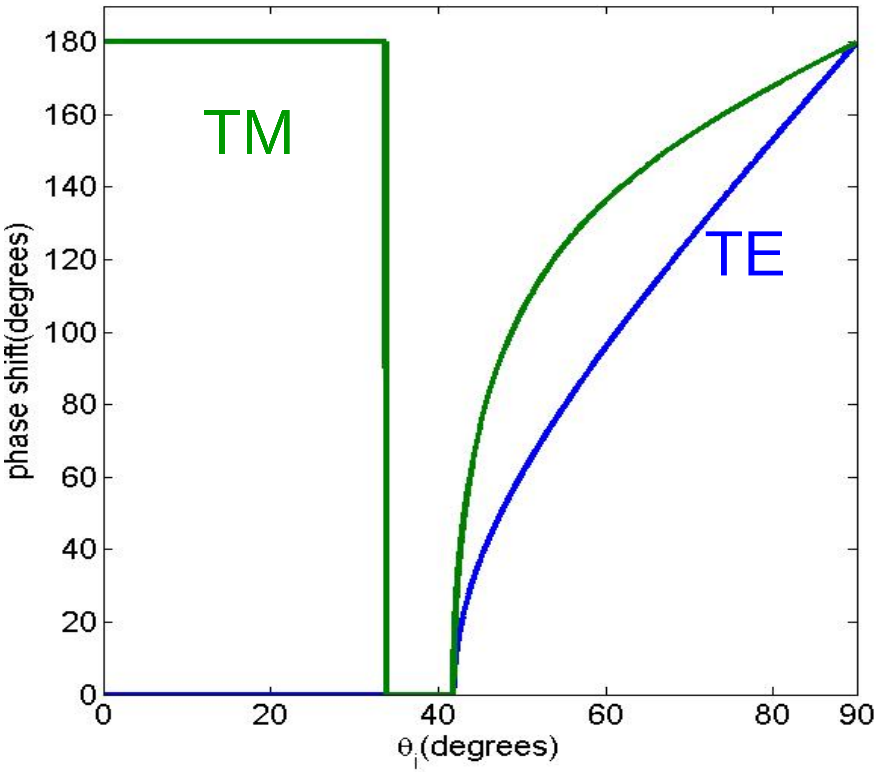
E_t // k_r !



Internal reflection

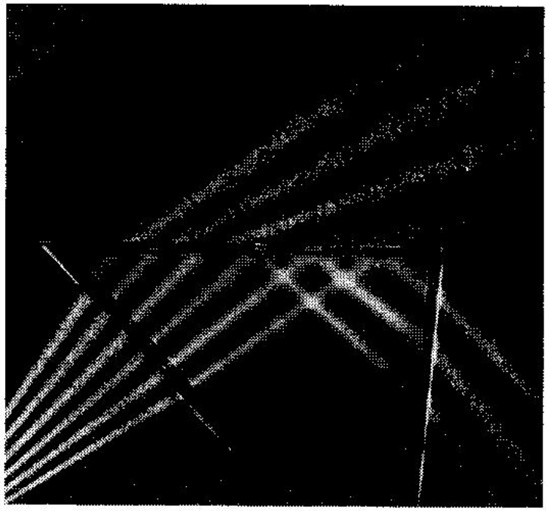
- $n_1 > n_2$
- Glass-air interface ($n_1/n_2=1.5$)

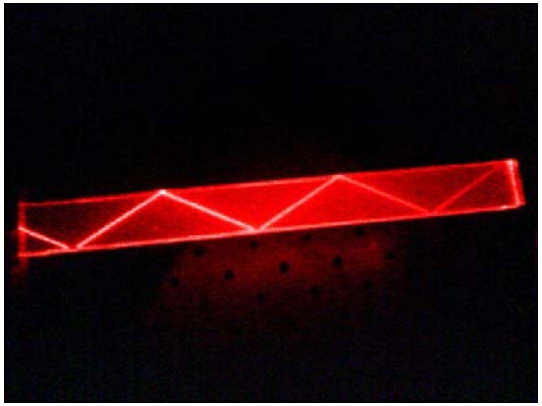




Total internal reflection

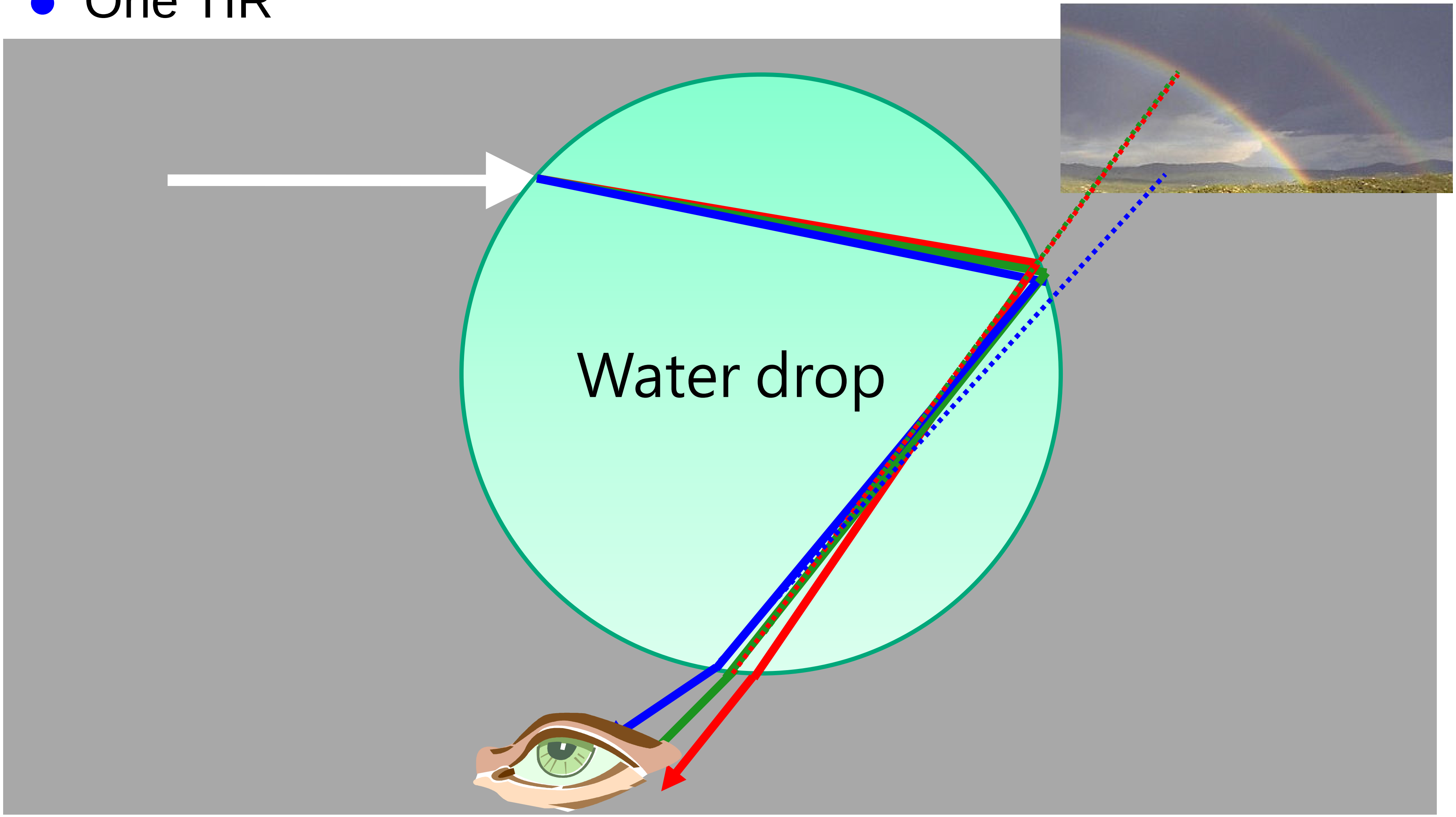
- Only if $n_1 > n_2$
- Incident angle > critical angle





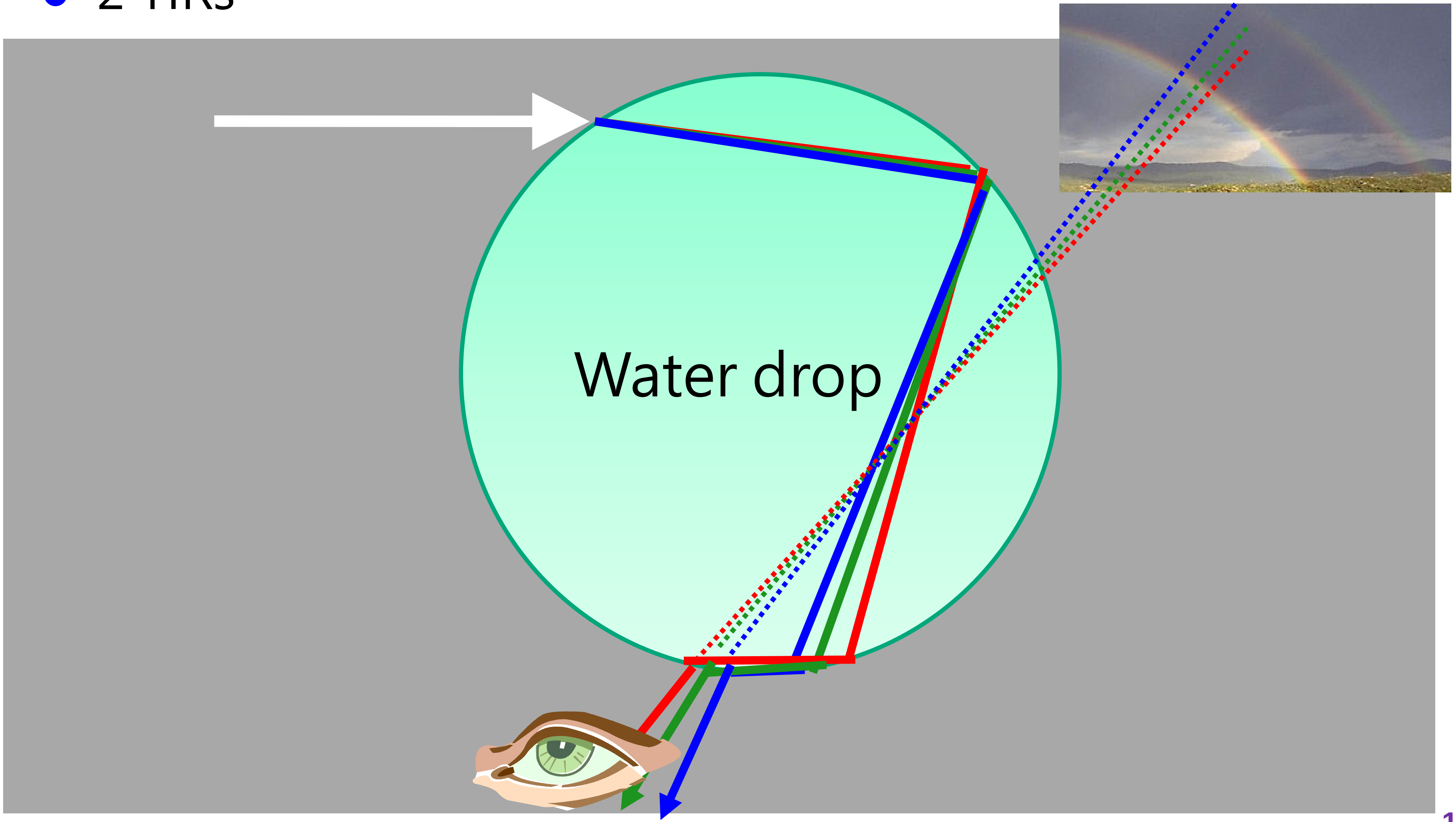
Rainbows

- Primary rainbow
- One TIR



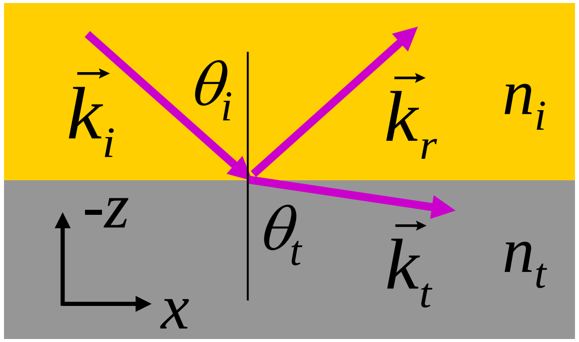
Rainbows

- Secondary rainbow
- 2 TIRs



The evanescent wave

The "transmitted wave" when total internal reflection occurs:



$$r_\perp = \frac{E_{0r}}{E_{0i}} = \frac{[n_i \cos(\theta_i) - n_t \cos(\theta_t)]}{[n_i \cos(\theta_i) + n_t \cos(\theta_t)]}$$

Since $\sin(\theta_t) > 1$, θ_t doesn't exist, so computing $r_\perp$ is impossible.

Let's check the reflectivity, R, anyway. Use Snell's Law to eliminate θ_t :

$$\cos(\theta_t) = \sqrt{1 - \sin^2(\theta_t)} = \sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2(\theta_i)} = \sqrt{\text{Neg. Number}}$$

Substituting this expression into the above one for $r_\perp$ and

redefining R yields: $R \equiv r_\perp r_\perp^* = \left(\frac{a - bi}{a + bi}\right)\left(\frac{a + bi}{a - bi}\right) = 1$

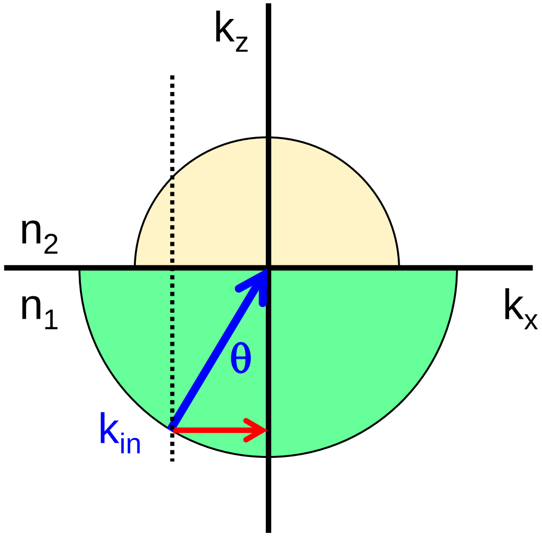
So all power is reflected; the evanescent wave contains no power.

Some interfaces are unique!

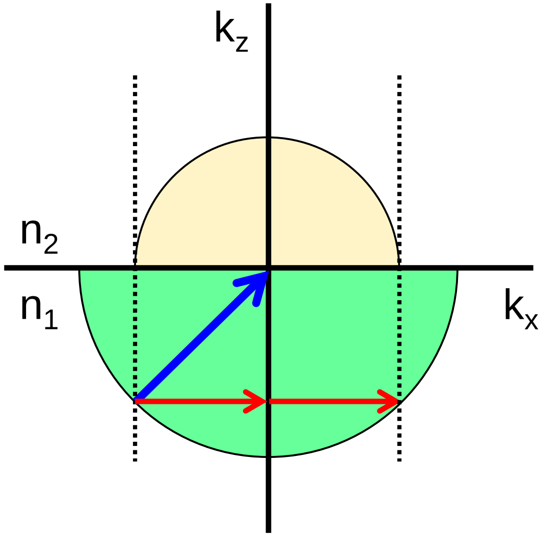
- The start medium is having a higher index
- Formation of evanescent wave
  - Imaginary wave-vector component in z-direction

$$k_2^2 = n_2^2\left(\frac{\omega}{c}\right)^2 = k_x^2 + k_z^2$$

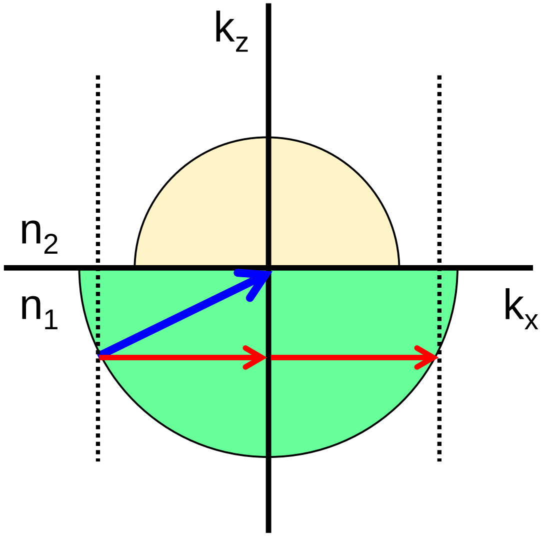
Interface 2



Interface 2: critical angle



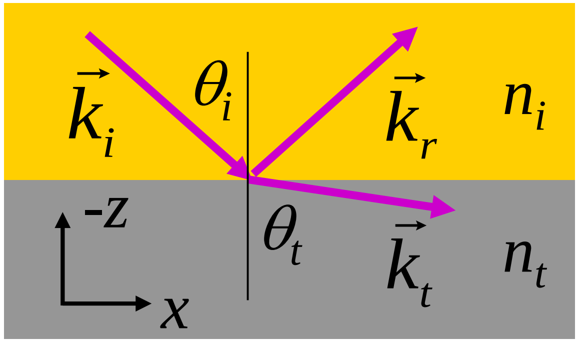
Interface 2: evanescent wave



The evanescent-wave k-vector

The evanescent wave k-vector must have x and y components:

$$k_{tz} = k_t \cos(\theta_t)$$



Using Snell's Law,

$\sin(\theta_t) = (n_i / n_t) \sin(\theta_i)$, so k_{tz} is meaningful.

And again:

$$\cos(\theta_t) = [1 - \sin^2(\theta_t)]^{1/2} = [1 - (n_i / n_t)^2 \sin^2(\theta_i)]^{1/2}$$
$$= \pm j\gamma$$

$$\boxed{E_t(x,z,t) = E_0 \exp[-k\gamma z] \exp j[\omega t - k\,(n_i / n_t) \sin(\theta_\mathrm{i})\, x]}$$

The evanescent wave decays exponentially in the transverse direction.

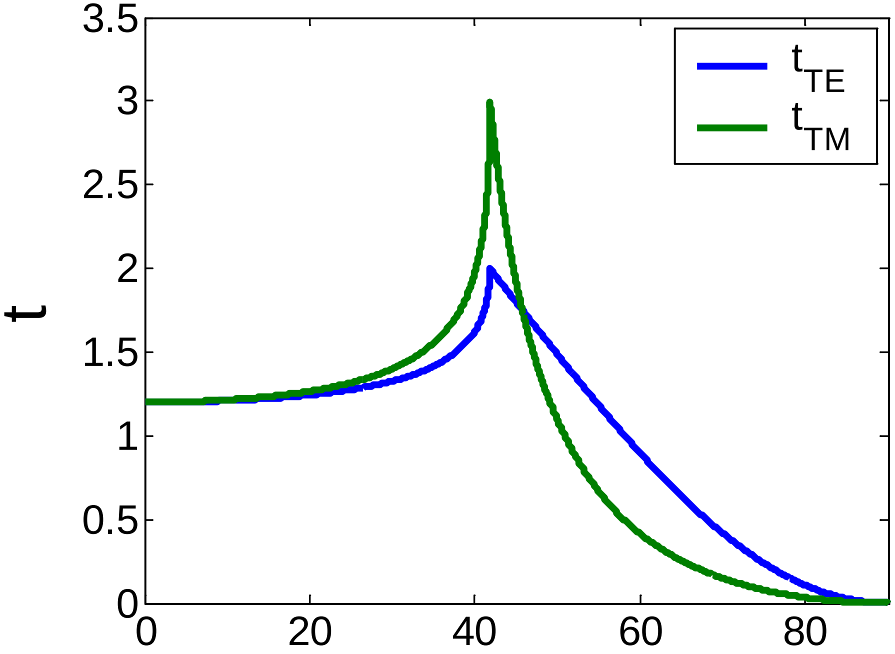
TIR: field enhancement at surface!

- At critical angle ($\theta_t=\pi/2$):
- TM mode is special!!
  - Sanity?

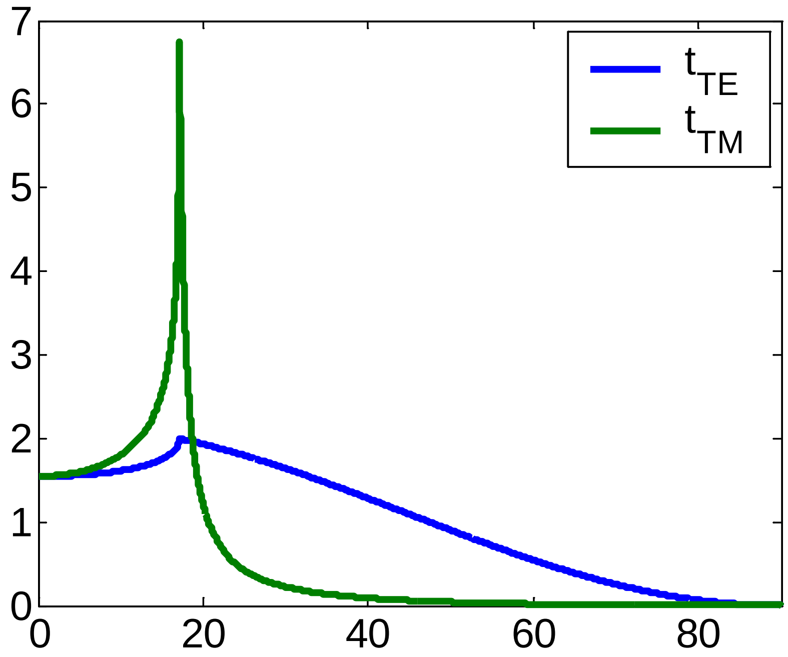
$$t_{TE}=\frac{2n_1\cos\theta_i}{n_1\cos\theta_i+n_2\cos\theta_t}\qquad t_{TM}=\frac{2n_1\cos\theta_i}{n_1\cos\theta_t+n_2\cos\theta_i}$$

$$t_{TE}=2;\qquad \boxed{t_{TM}=2(\frac{n_1}{n_2})}$$

$n_1/n_2=1.5$



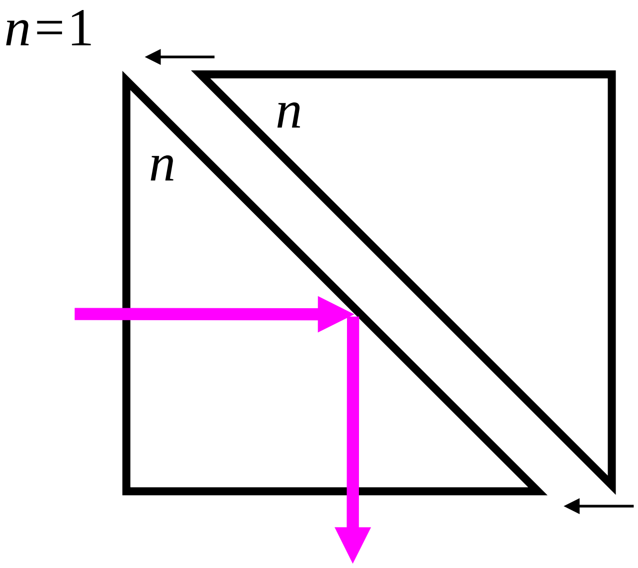
$n_1/n_2=3.4$



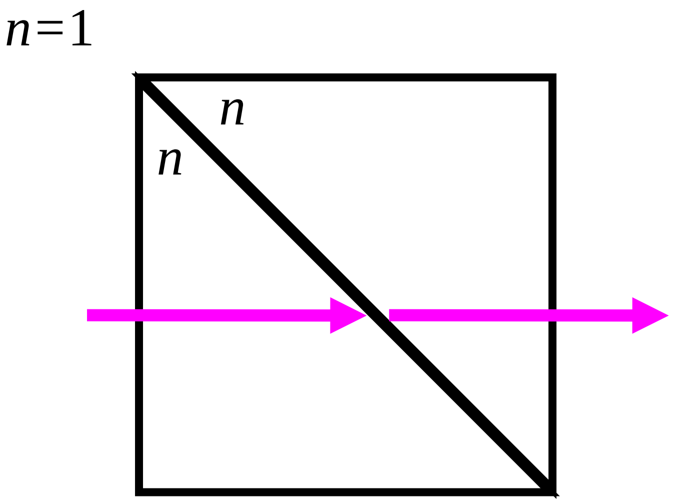
Frustrated total internal reflection

By placing another surface in contact with a totally internally reflecting one, total internal reflection can be **frustrated**.

Total internal reflection

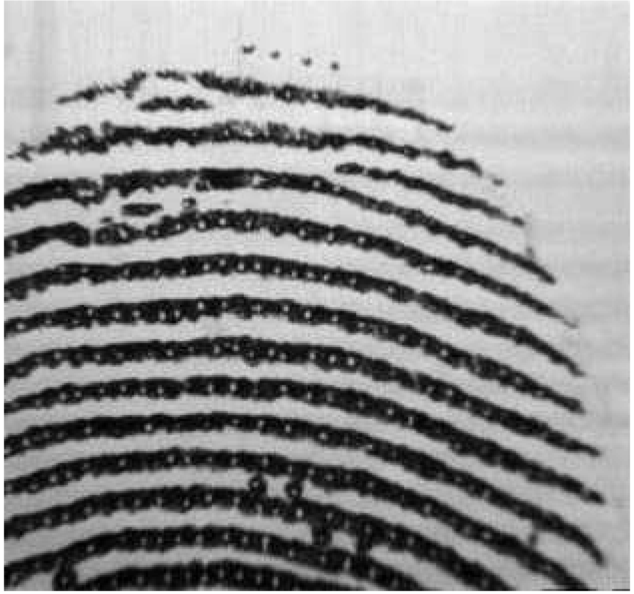


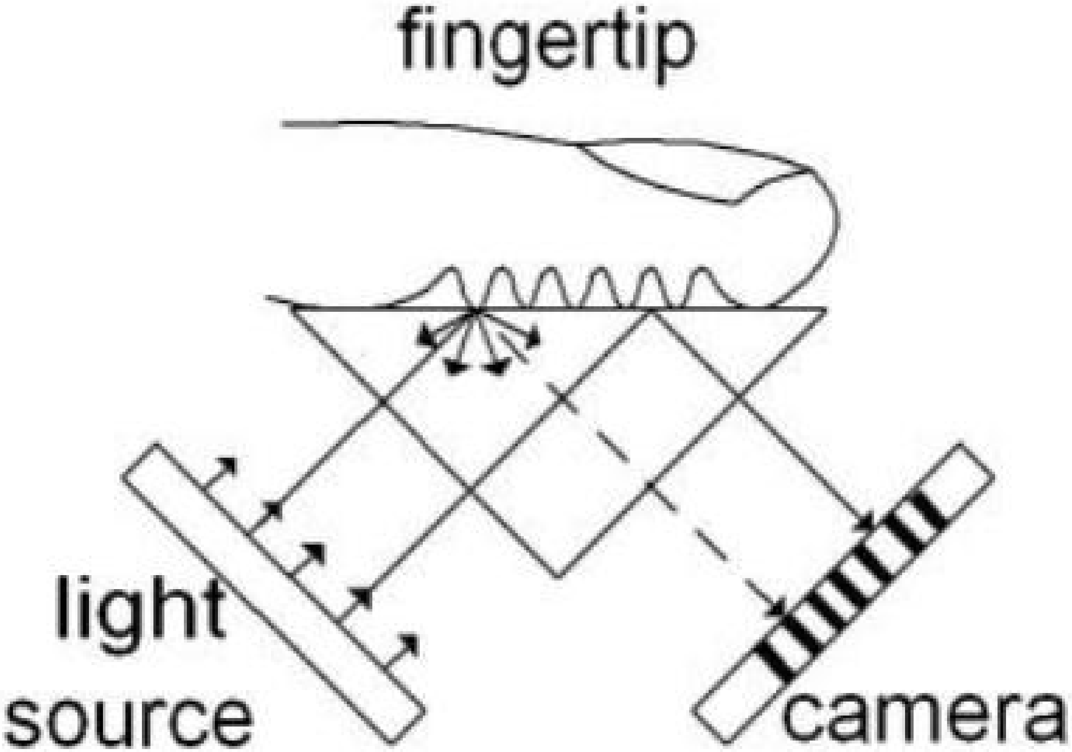
Frustrated total internal reflection



Engineering aspect!

FTIR, the evanescent wave, and fingerprinting





Summary

- Normal and oblique incidence of plane waves
- TM mode is special
  - Brewster's angle
- Total internal reflection
  - TM mode is special