Target: an optical communication system

Lesson 1
Light rays, ABCD matrix and thin lens

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Contents

- Light rays and two basic laws
- Ray tracing (ABCD matrix)
- Thin lens for imaging

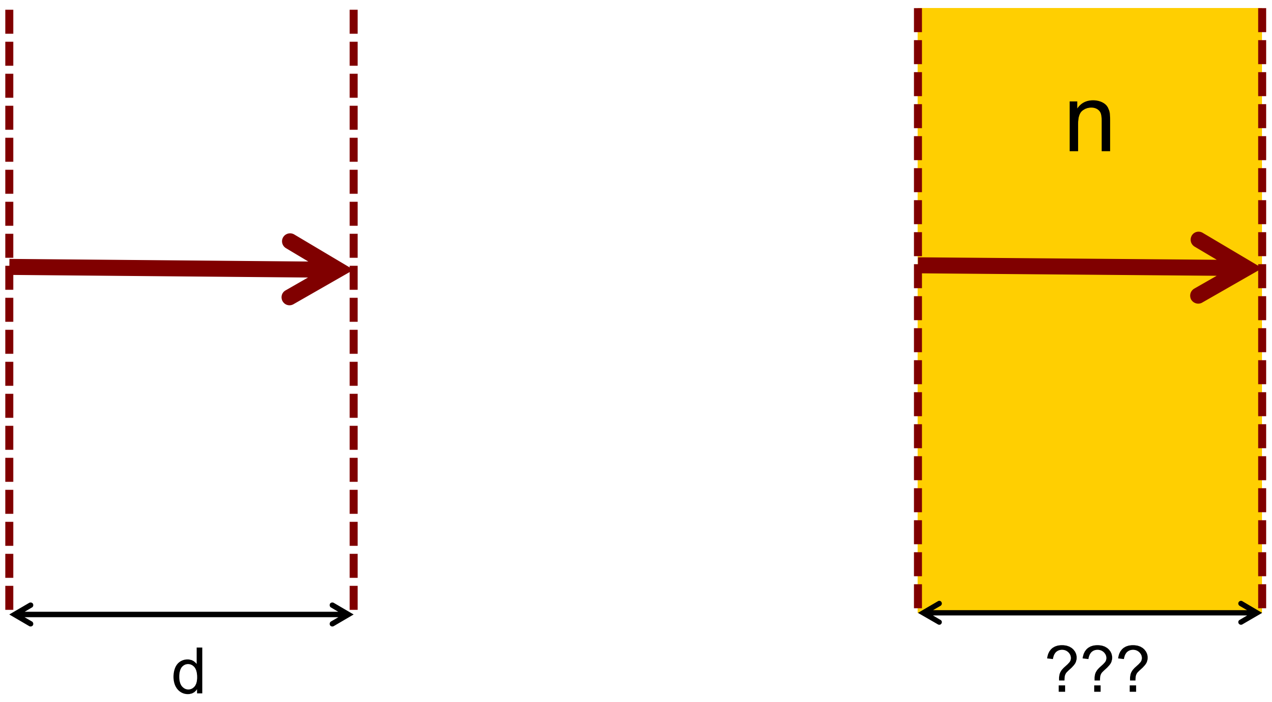
- Light rays
  - Law of reflection
  - Law of refraction

Light rays

- A simplified representation in describing light propagation
- Light travel as a straight line

Optical path length

- Within a medium, the effective length light travels



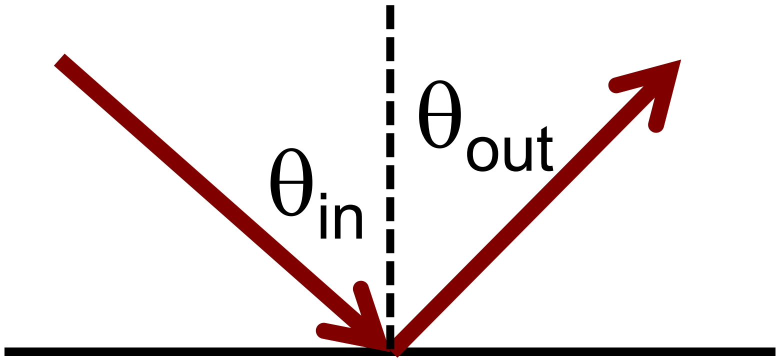
Refractive index

- Tell light how dense the medium is
- The ratio between speed of light in vacuum to in a medium
- Determines the speed of light
- Used to change the direction of light ray

$$\frac{c}{v} = n$$

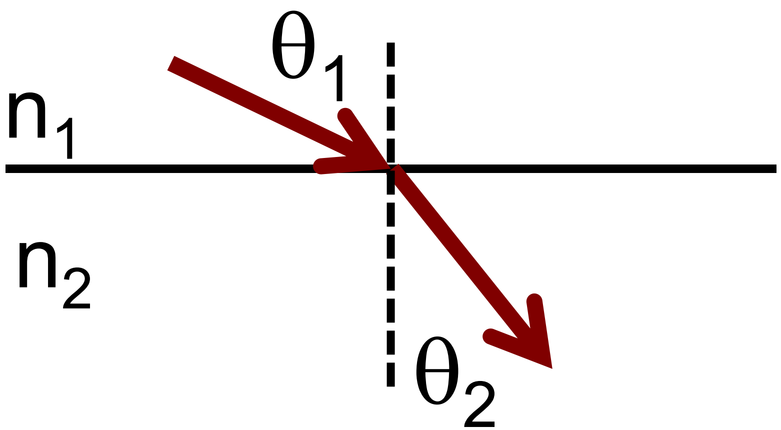
Two basic laws

- Law of reflection



$$\theta_{in} = \theta_{out}$$

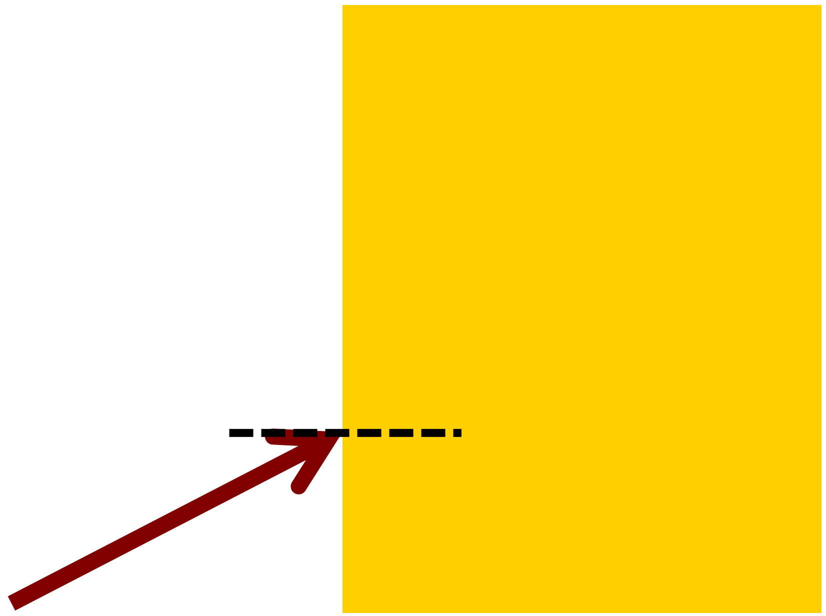
- Law of refraction (Snell's law)



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Parallel plates

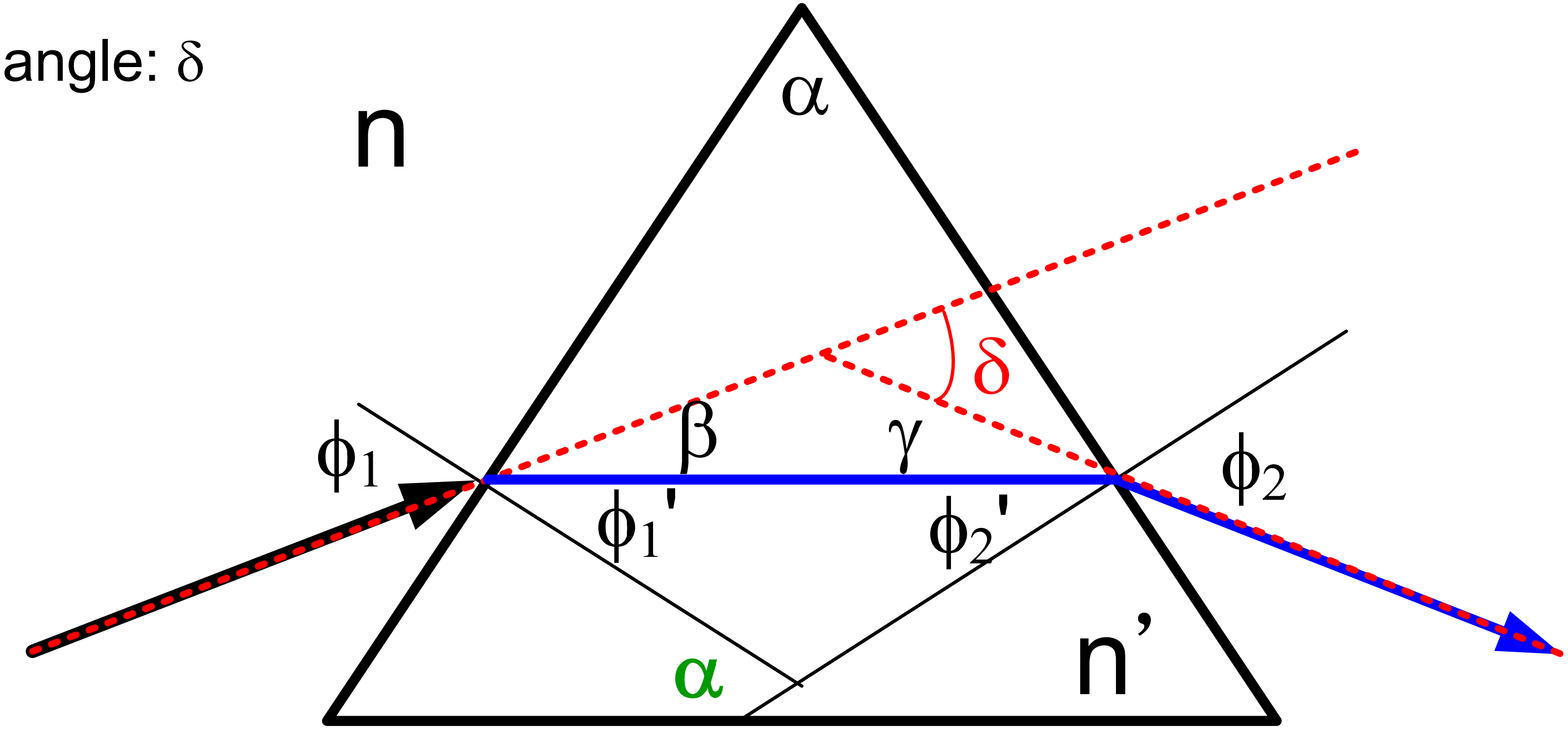
- Use the two laws
- How are the input and output rays related?
- What are potential applications?



Prism

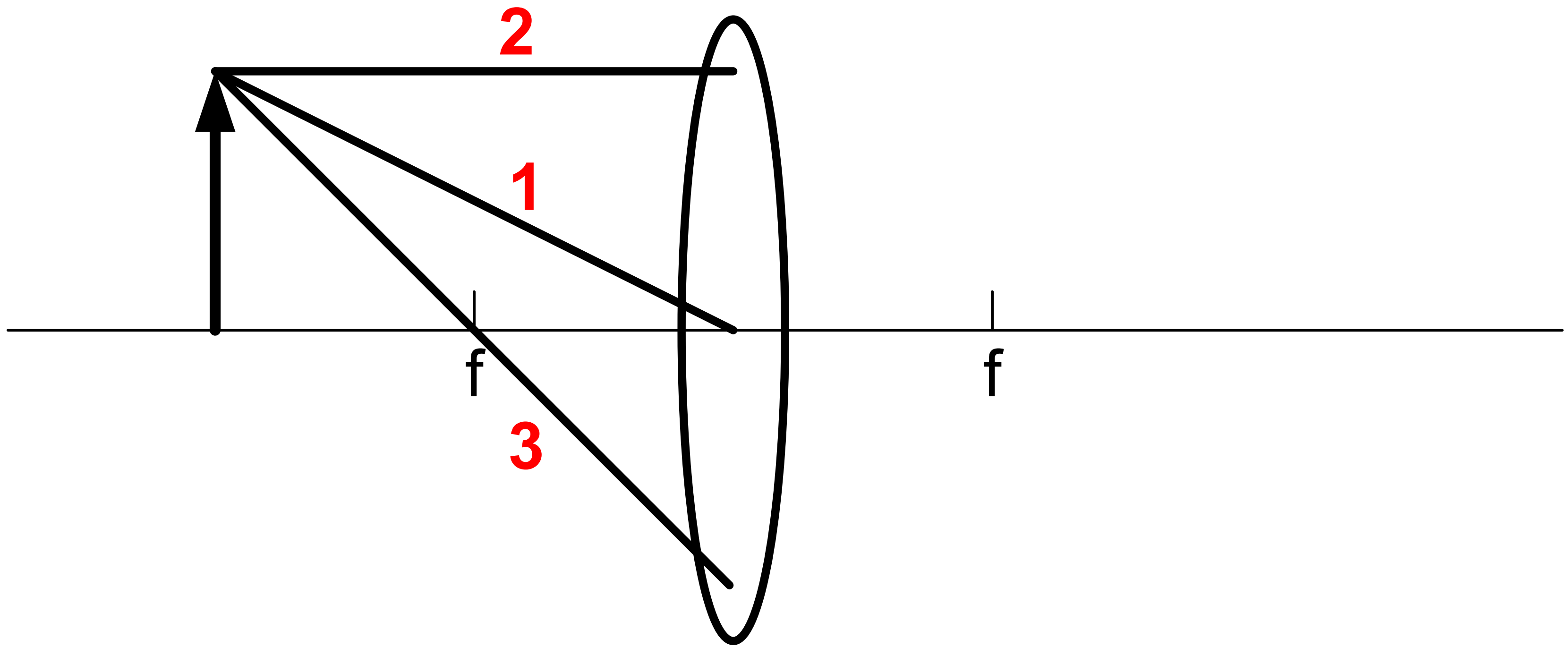
Deviation angle: δ

n > n' ?



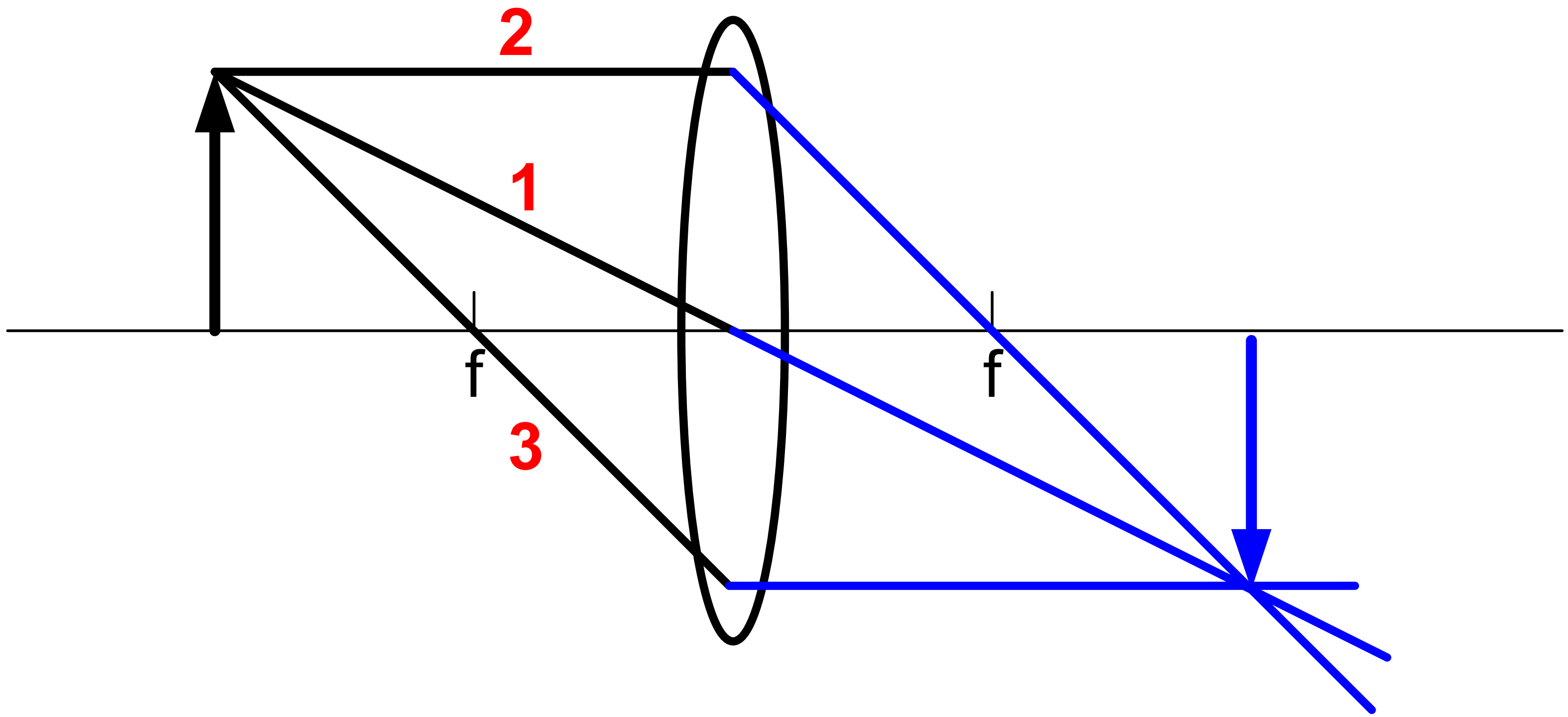
Thin lens

- Three major rays



Thin lens

- Three major rays

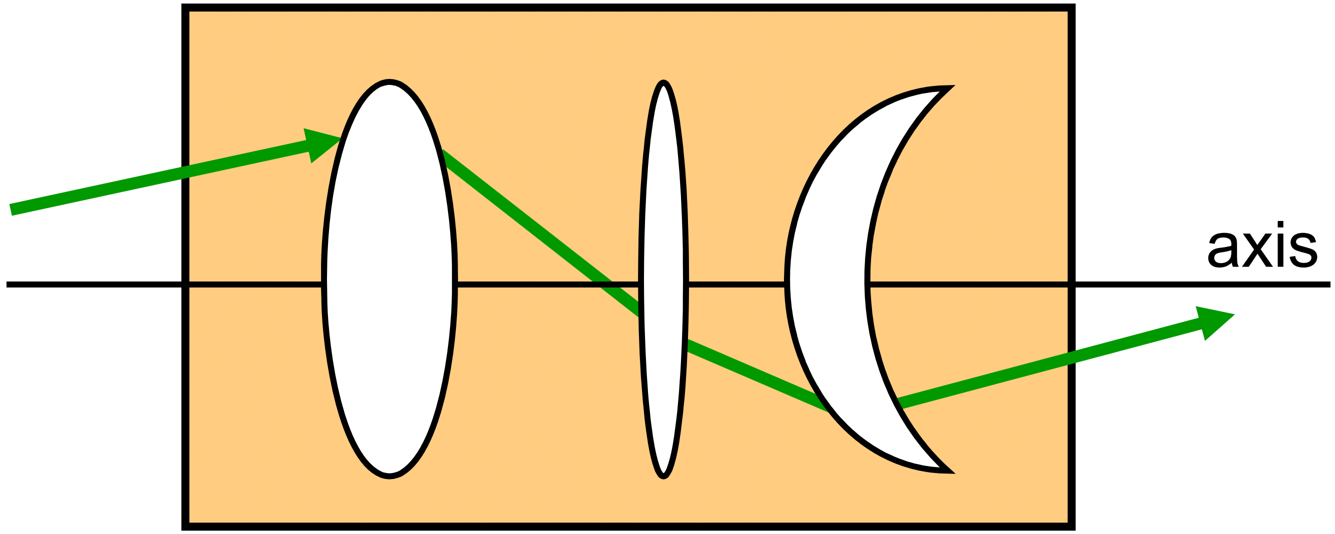


Thin lens sequence?

- What if you have two lenses?

- Ray tracing
  - Ray vector
  - ABCD matrix
  - The lens law

Ray optics



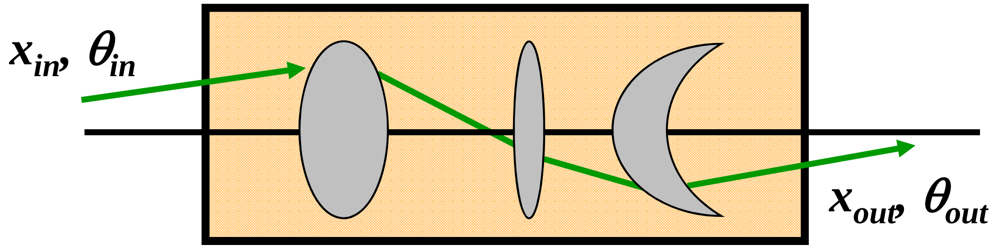
We'll define **light rays** as directions in space, corresponding, roughly, to k-vectors of light waves.

We won’t worry about the phase.

Each optical system will have an axis, and all light rays will be assumed to propagate at small angles to it. This is called the **Paraxial Approximation**.

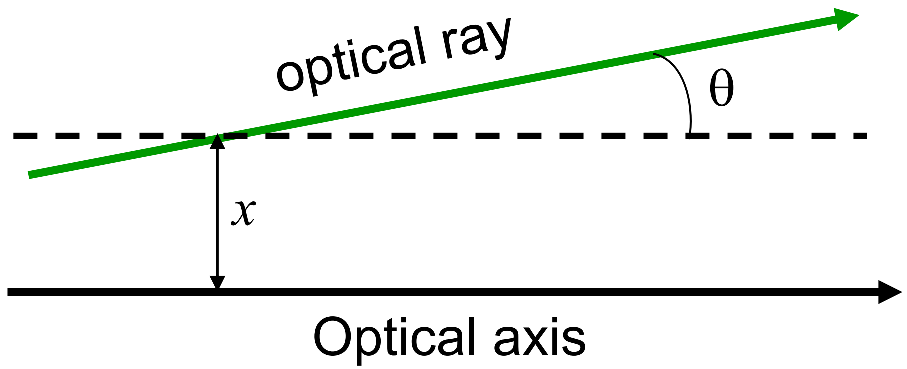
The ray vector

A light ray can be defined by two co-ordinates:



its position, *x*

its slope, θ



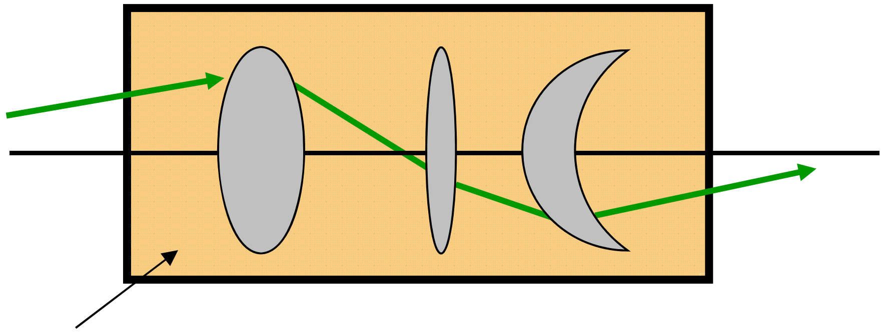
These parameters define a **ray vector**, which will change with distance and as the ray propagates through optics.

$$\begin{bmatrix} x \\ \theta \end{bmatrix}$$

Ray matrices

For many optical components, we can define 2 x 2 **ray matrices**.

An element's effect on a ray is found by multiplying its ray vector.

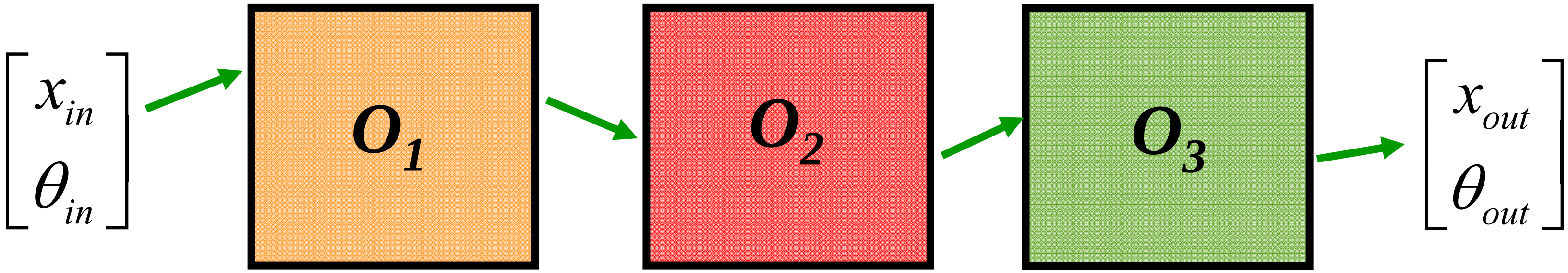


$$\begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} \rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow \begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix}$$

Ray matrices can describe simple and complex systems.

These matrices are often called **ABCD Matrices**.

Matrix for cascaded elements



$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = O_3 \left\{ O_2 \left(O_1 \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} \right) \right\} = O_3 \ O_2 \ O_1 \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

Notice that the order looks opposite to what it should be,
but it makes sense when you think about it.

The ABCD matrix

Since the displacements and angles are assumed to be small, we can think in terms of partial derivatives.

$$x_{out} = \frac{\partial x_{out}}{\partial x_{in}} x_{in} + \frac{\partial x_{out}}{\partial \theta_{in}} \theta_{in}$$

$$\theta_{out} = \frac{\partial \theta_{out}}{\partial x_{in}} x_{in} + \frac{\partial \theta_{out}}{\partial \theta_{in}} \theta_{in}$$

spatial magnification $\frac{\partial x_{out}}{\partial x_{in}} \rightarrow A$

$\frac{\partial x_{out}}{\partial \theta_{in}} \rightarrow B$

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

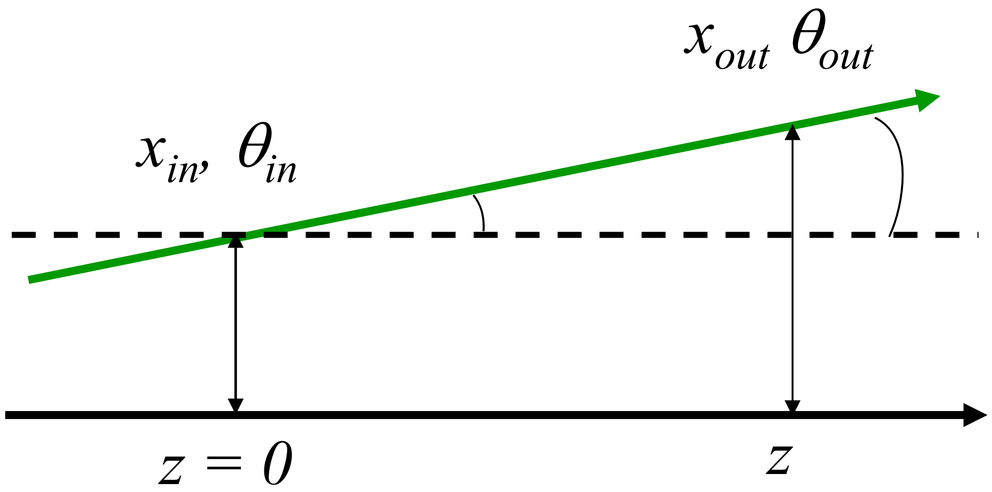
$\frac{\partial \theta_{out}}{\partial x_{in}} \rightarrow C$

$\frac{\partial \theta_{out}}{\partial \theta_{in}} \rightarrow D$ **angular magnification**

We can write these equations in matrix form.

Ray matrix for free space or a medium

If x_{in} and θ_{in} are the position and slope upon entering, let x_{out} and θ_{out} be the position and slope after propagating from $z = 0$ to z.



$$x_{out} = x_{in} + z\,\theta_{in}$$
$$\theta_{out} = \theta_{in}$$

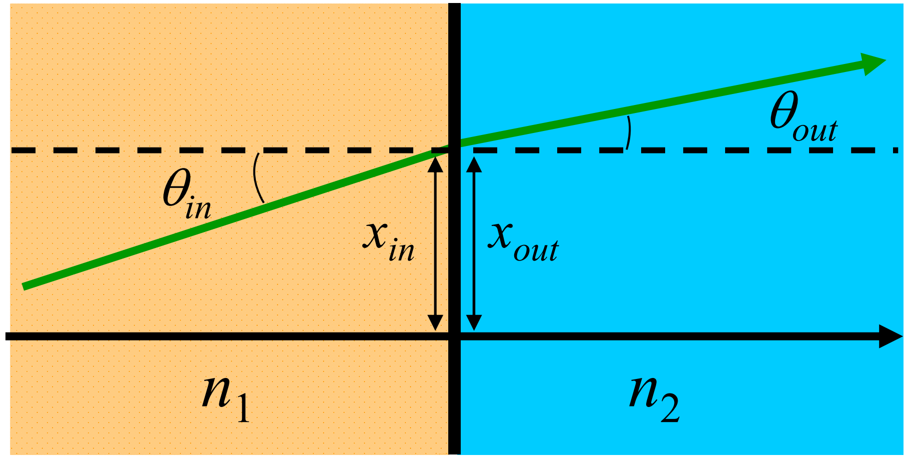
$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

$$O_{space} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}$$

Ray matrix for an interface

At the interface, clearly:

$$x_{out} = x_{in}.$$



Now calculate θ_{out}.

Snell's Law says: $n_1 \sin(\theta_{in}) = n_2 \sin(\theta_{out})$

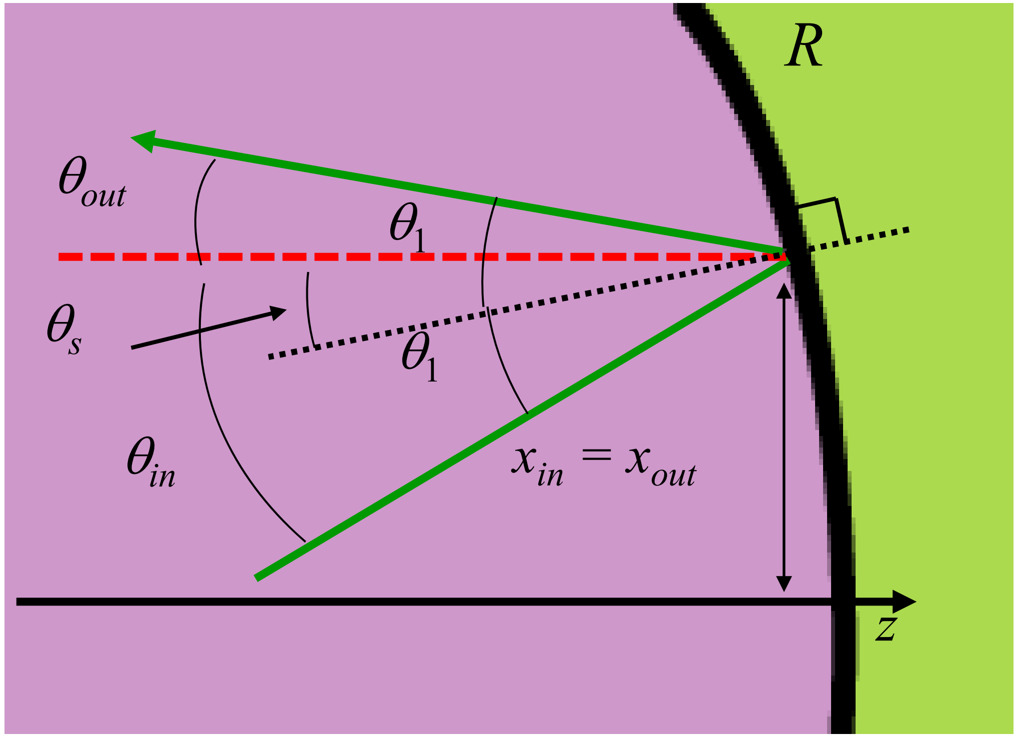
Paraxial Approximation: $n_1 \theta_{in} = n_2 \theta_{out}$

$$\Rightarrow \theta_{out} = [n_1 / n_2] \theta_{in}$$

$$O_{interface} = \begin{bmatrix} 1 & 0 \\ 0 & n_1 / n_2 \end{bmatrix}$$

Ray matrix for a curved mirror

Consider a mirror with radius of curvature, R, with its optic axis perpendicular to the mirror:



$$\theta_1 = \theta_{in} - \theta_s \qquad \theta_s \approx x_{in} / R$$

$$\theta_{out} = \theta_1 - \theta_s = (\theta_{in} - \theta_s) - \theta_s$$
$$\approx -2x_{in} / R + \theta_{in}$$

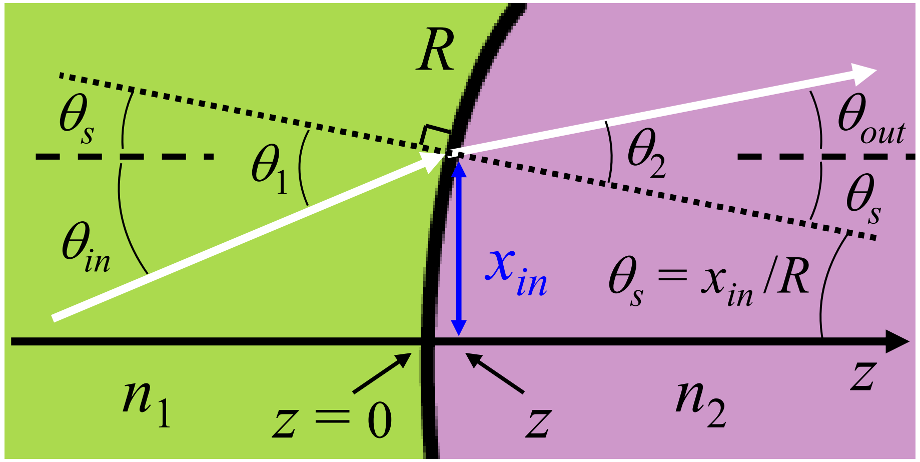
$$\Rightarrow \quad O_{mirror} = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$$

Like a lens, a curved mirror will focus a beam. Its focal length is $R/2$.

Note that a flat mirror has $R = \infty$ and hence an identity ray matrix.

Ray matrix for a curved interface

At the interface: $x_{out} = x_{in}$



$\theta_1 = \theta_{in} + \theta_s$ and $\theta_2 = \theta_{out} + \theta_s$

$\theta_1 = \theta_{in} + x_{in}/R$ and $\theta_2 = \theta_{out} + x_{in}/R$

Paraxial Approximation:
$x_{in} << R$

Snell's Law: $n_1 \theta_1 = n_2 \theta_2 \quad \Rightarrow n_1 (\theta_{in} + x_{in}/R) = n_2 (\theta_{out} + x_{in}/R)$

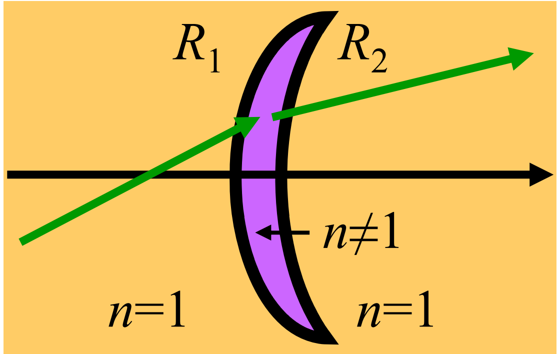
$$\Rightarrow \theta_{out} = (n_1/n_2)(\theta_{in} + x_{in}/R) - x_{in}/R$$

$$\Rightarrow \theta_{out} = (n_1/n_2 - 1)\, x_{in}/R + (n_1/n_2)\theta_{in}$$

$$O_{\substack{curved \\ interface}} = \begin{bmatrix} 1 & 0 \\ (n_1/n_2 - 1)/R & n_1/n_2 \end{bmatrix}$$

A thin lens is just two curved interfaces

We'll neglect the glass in between (it's a really thin lens!), and we'll take $n_1 = 1$.



$$O_{\substack{curved\\interface}} = \begin{bmatrix} 1 & 0 \\ (n_1/n_2 - 1)/R & n_1/n_2 \end{bmatrix}$$

$$O_{thin\ lens} = O_{\substack{curved\\interface\,2}} \; O_{\substack{curved\\interface\,1}} = \begin{bmatrix} 1 & 0 \\ (n-1)/R_2 & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (1/n-1)/R_1 & 1/n \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ (n-1)/R_2 + n(1/n-1)/R_1 & n(1/n) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (n-1)/R_2 + (1-n)/R_1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ (n-1)(1/R_2 - 1/R_1) & 1 \end{bmatrix}$$

This can be written: $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$

where: $1/f = (n-1)(1/R_1 - 1/R_2)$

The Lens-Maker's Formula

Meaning of negative sign?

Sign conventions for curved interface

Assuming light propagates from left to right

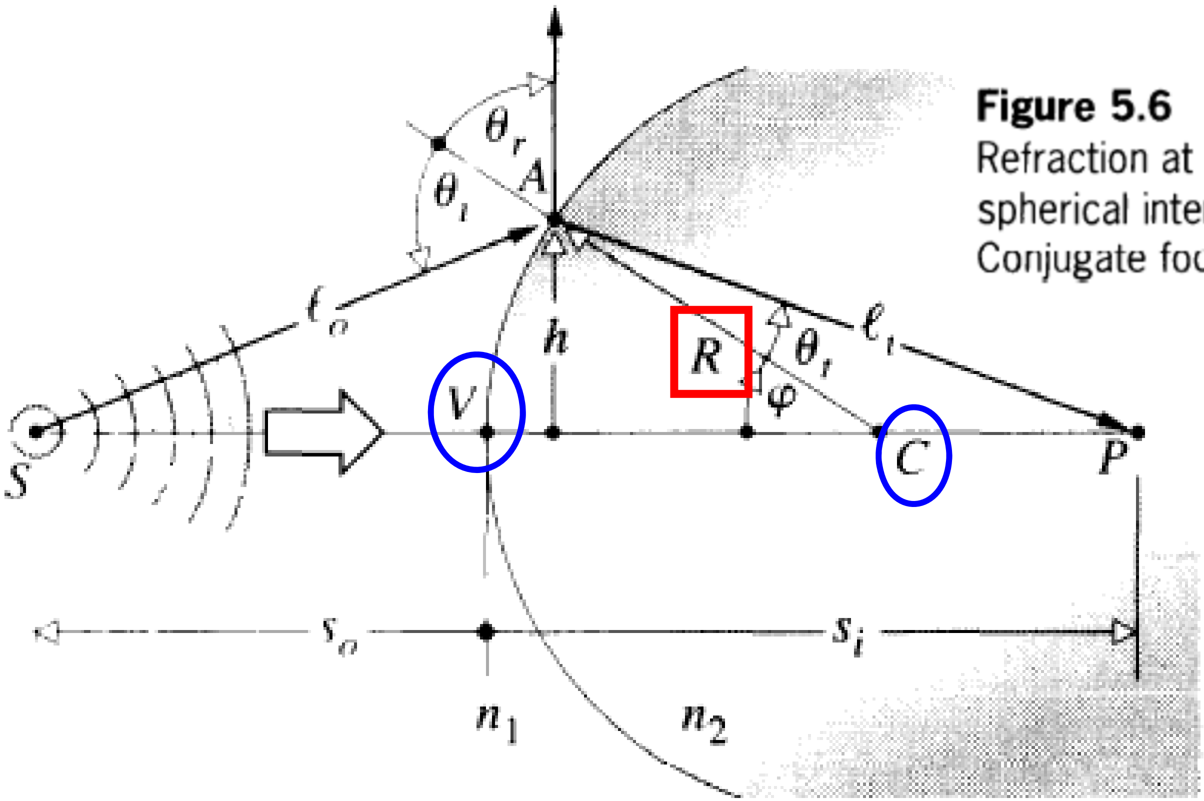
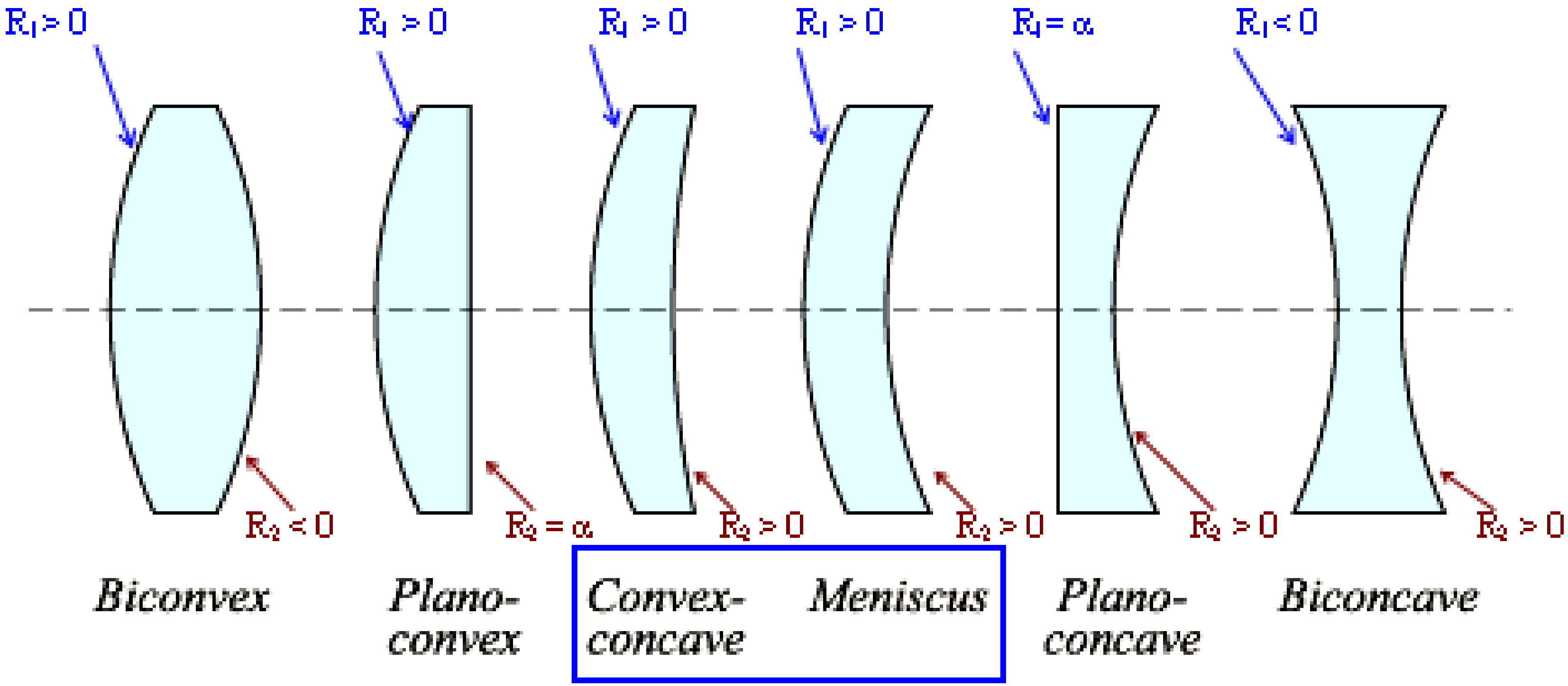


Figure 5.6
Refraction at a spherical interface. Conjugate foci.

R: + if C is right of V
- if C is left of V

Types of lenses



Which type of lens to use (and how to orient it) depends on the aberrations and application.

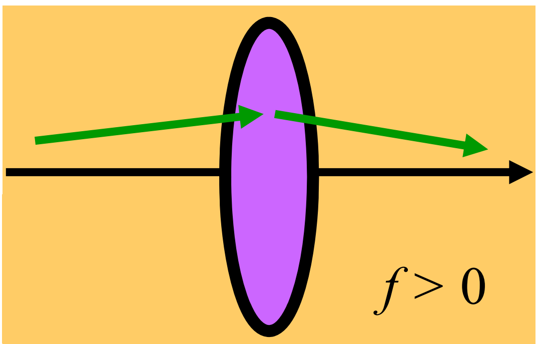
Ray matrix for a lens

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

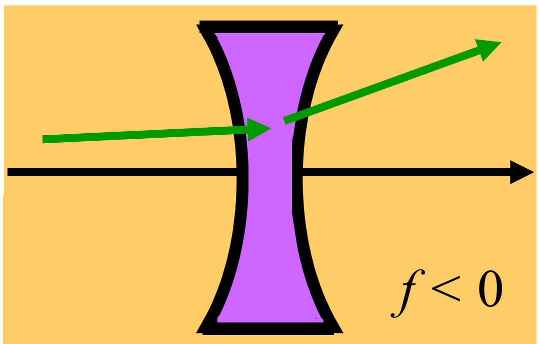
$$O_{lens} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

What happens if environment is not air?

The quantity, f, is the **focal length** of the lens. It's the single most important parameter of a lens. It can be positive or negative.



If $f > 0$, the lens deflects rays toward the axis.



If $f < 0$, the lens deflects rays away from the axis.

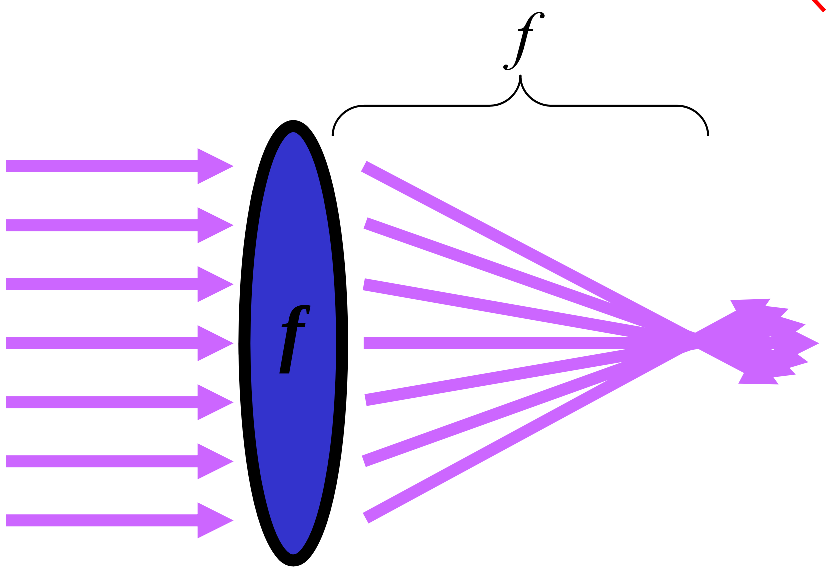
Effect of a lens: focusing A=0

A lens followed by propagation of one focal length:

For all rays $x_{out} = 0!$

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_{in}/f \end{bmatrix}$$

Assume all input rays have $\theta_{in} = 0$

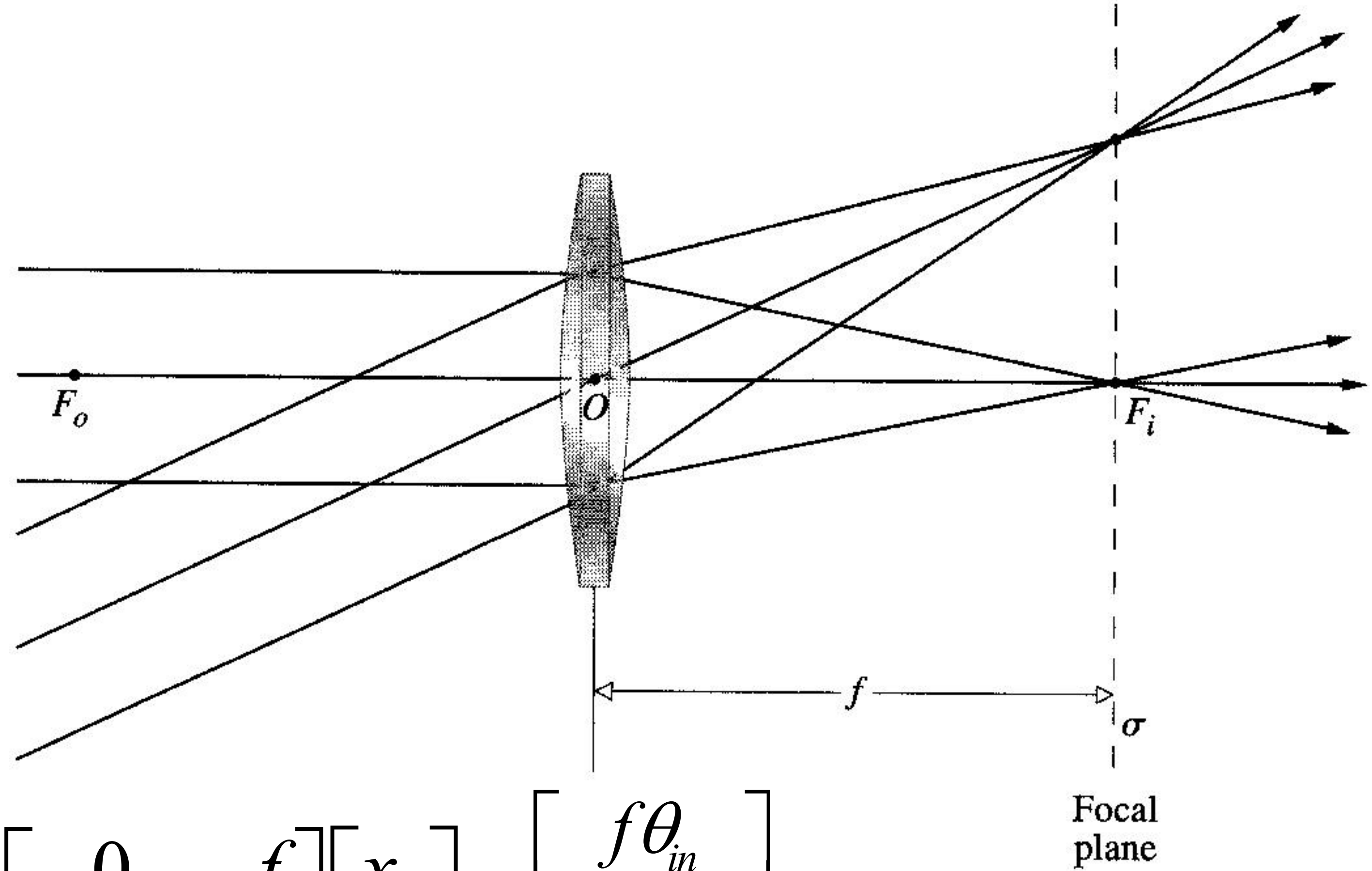


At the focal plane, all rays converge to the z axis $(x_{out} = 0)$ independent of input position.

Looking from right to left, rays diverging from a point are made parallel.

Effect of a lens

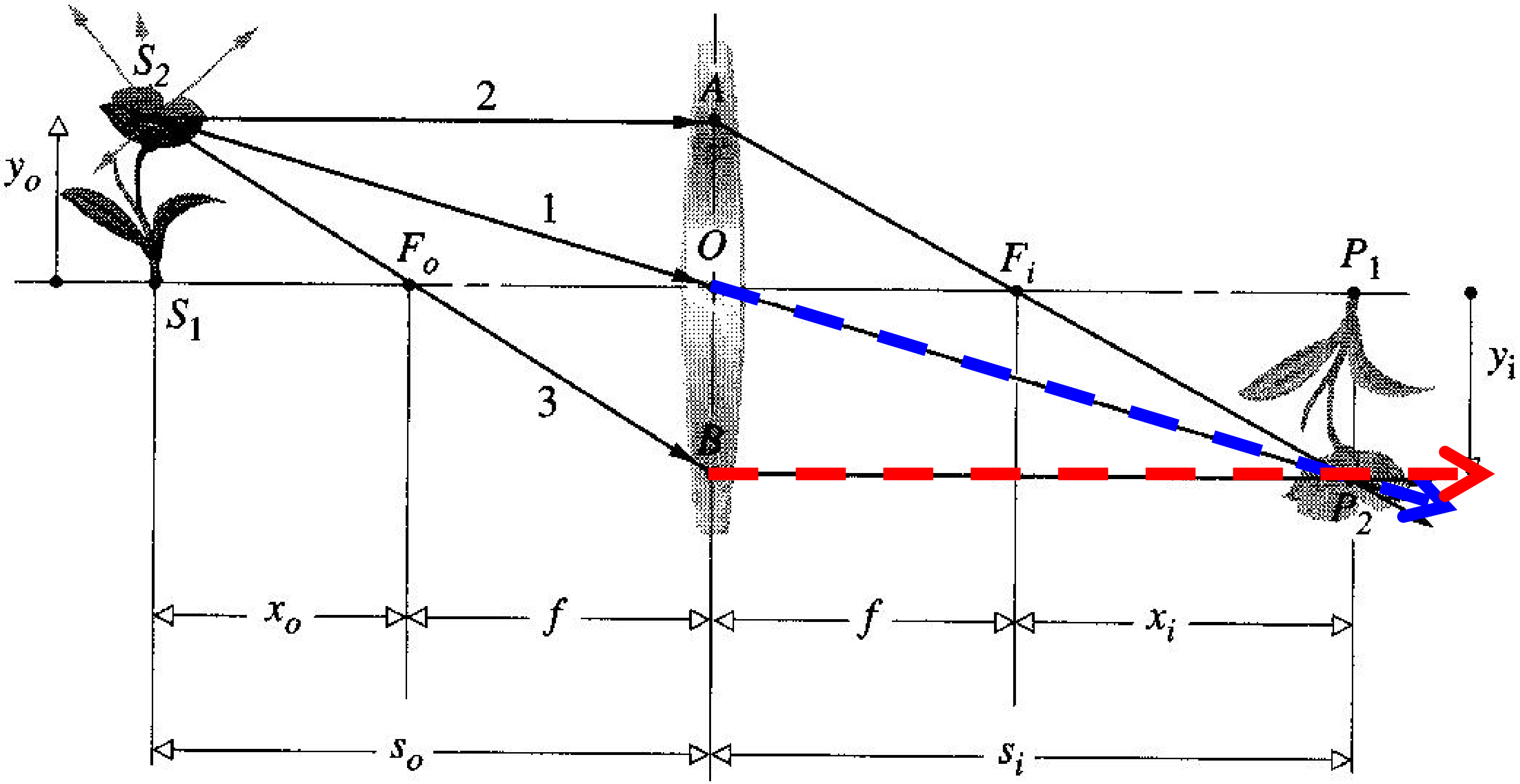
- Parallel rays at a different angle focus at a different x_{out}.



$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} f\theta_{in} \\ \theta_{in} - \dfrac{x_{in}}{f} \end{bmatrix}$$

Locating the image: matrix perspective

- Let's verify the three major rays



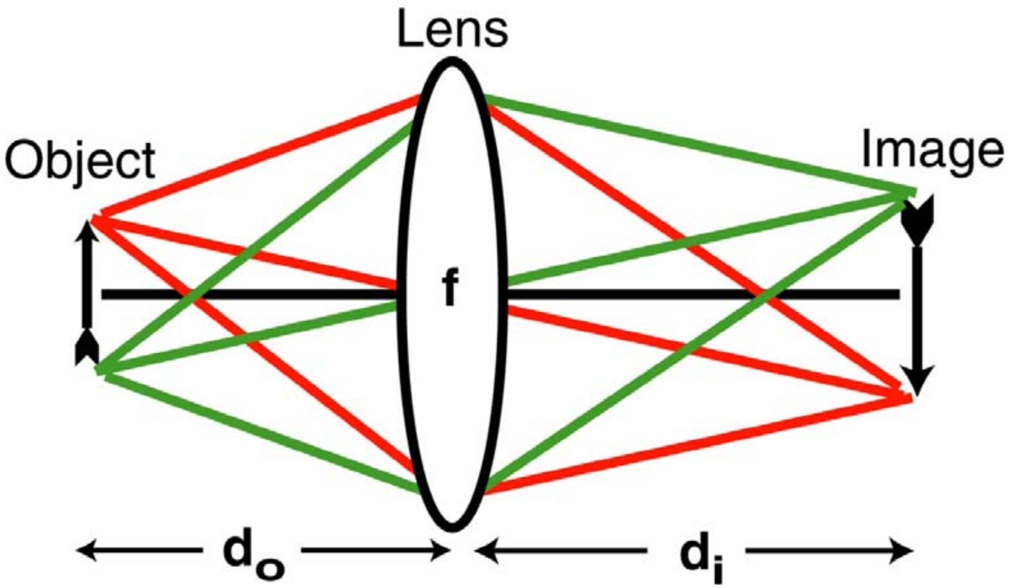
A system images an object when $B = 0$

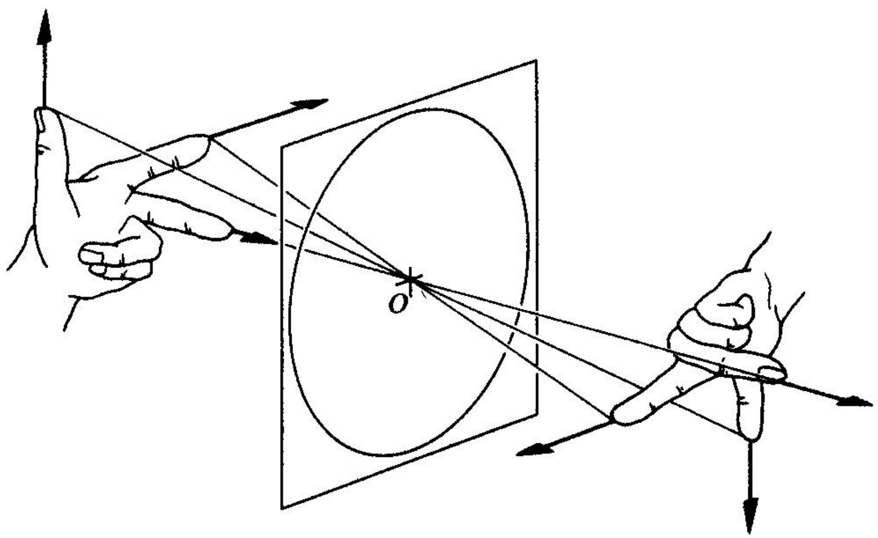
When $B = 0$, all rays from a point x_{in} arrive at a point x_{out}, independent of angle.

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} A x_{in} \\ C x_{in} + D\,\theta_{in} \end{bmatrix}$$

$$x_{out} = A\, x_{in}$$

$B = 0$, A is the magnification

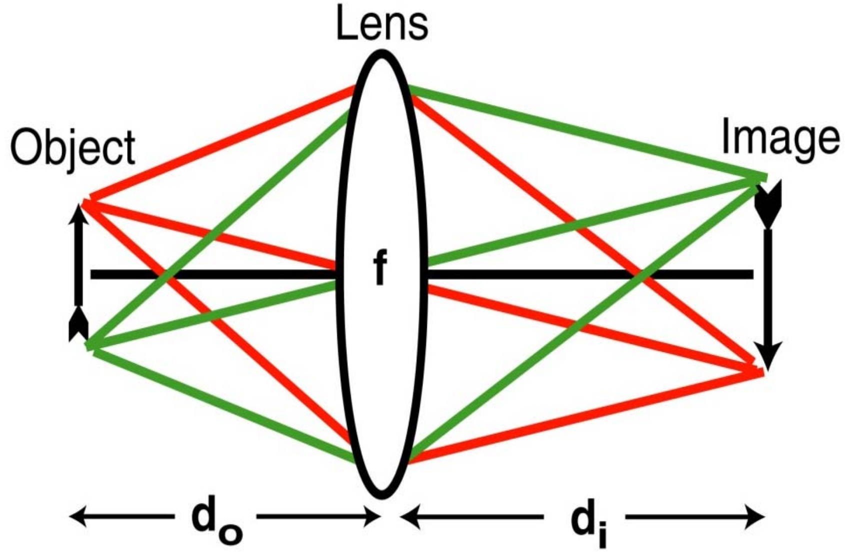




The Lens Law

From the object to the image:

1) A distance d_o
2) A lens of focal length f
3) A distance d_i



$$O = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_o \\ -1/f & 1 - d_o/f \end{bmatrix}$$

$$= \begin{bmatrix} 1 - d_i/f & d_o + d_i - d_o d_i / f \\ -1/f & 1 - d_o/f \end{bmatrix}$$

$B = d_o + d_i - d_o d_i / f =$

$d_o d_i \left[1/d_o + 1/d_i - 1/f \right] =$

0 if

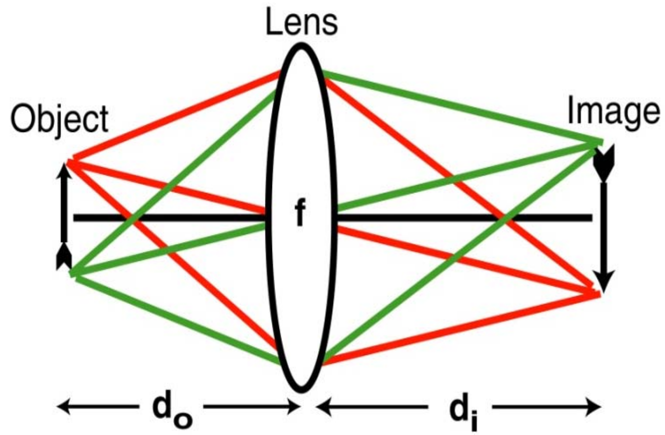
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The Lens Law

Imaging magnification

If the imaging condition is satisfied:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \;\rightarrow\; O=\begin{bmatrix} 1-d_i/f & 0 \\ -1/f & 1-d_o/f \end{bmatrix}$$



$$A=1-d_i/f=1-d_i\left[\frac{1}{d_o}+\frac{1}{d_i}\right]$$

$$\Rightarrow \boxed{M=-\frac{d_i}{d_o}}$$

$$D=1-d_o/f=1-d_o\left[\frac{1}{d_o}+\frac{1}{d_i}\right]$$

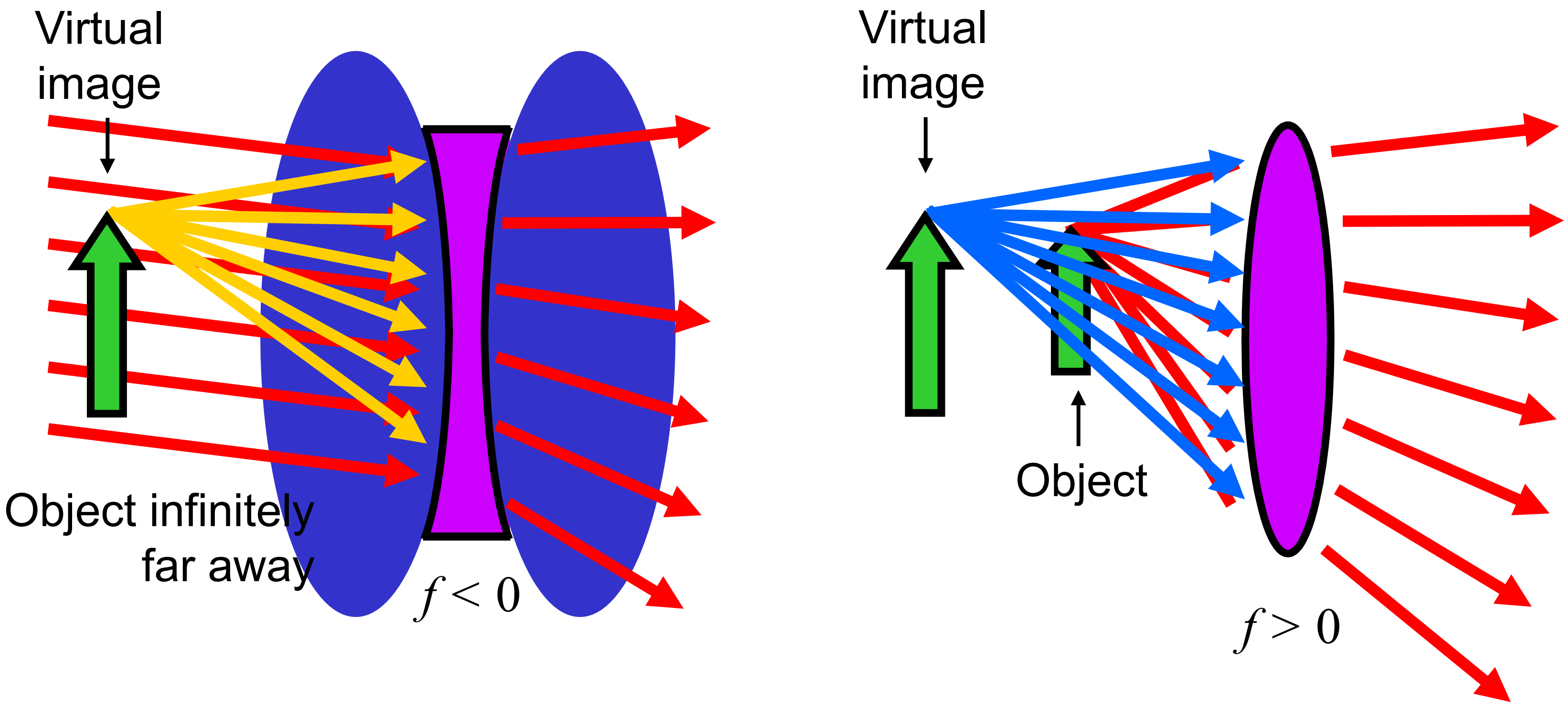
$$=-\frac{d_o}{d_i}=1/M$$

So: $$\boxed{O=\begin{bmatrix} M & 0 \\ -1/f & 1/M \end{bmatrix}}$$

Virtual Images

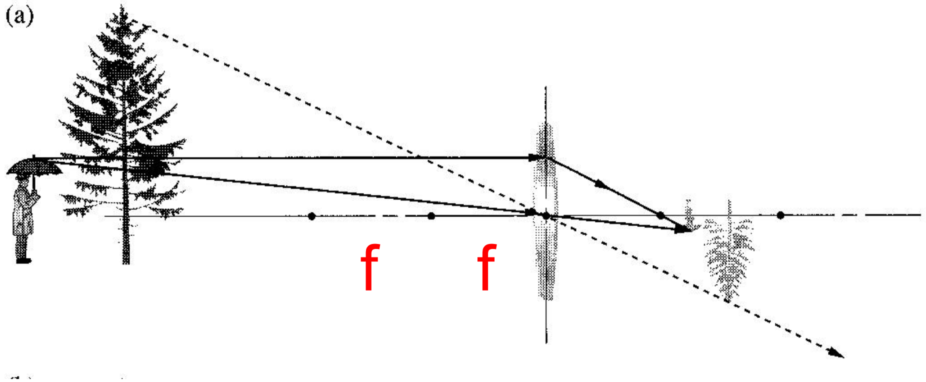
The outgoing rays from a point on the object never actually intersect at a point but can be traced backwards to one.

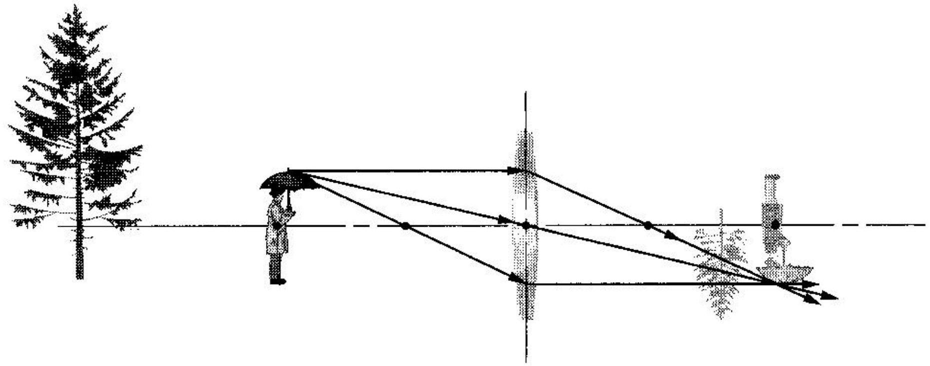
Negative-f lenses have virtual images, and positive-f lenses do also if the object is less than one focal length away.

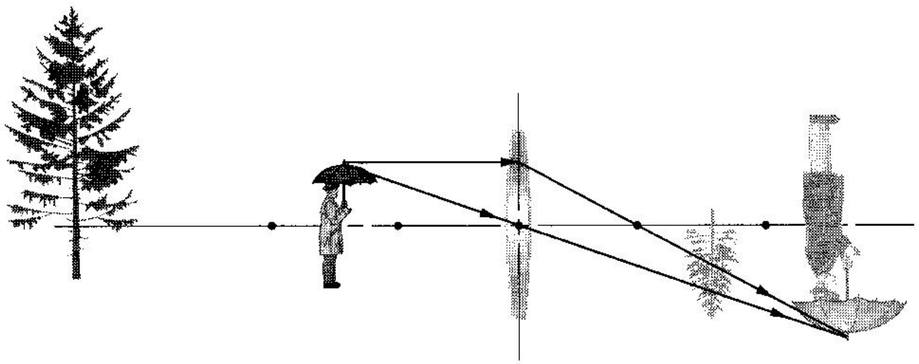


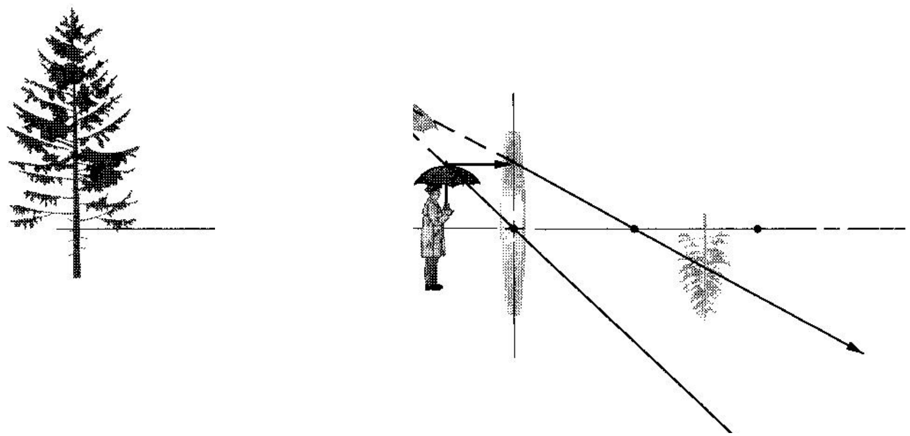
Simply looking at a flat mirror yields a virtual image.

Summary: image magnification, location

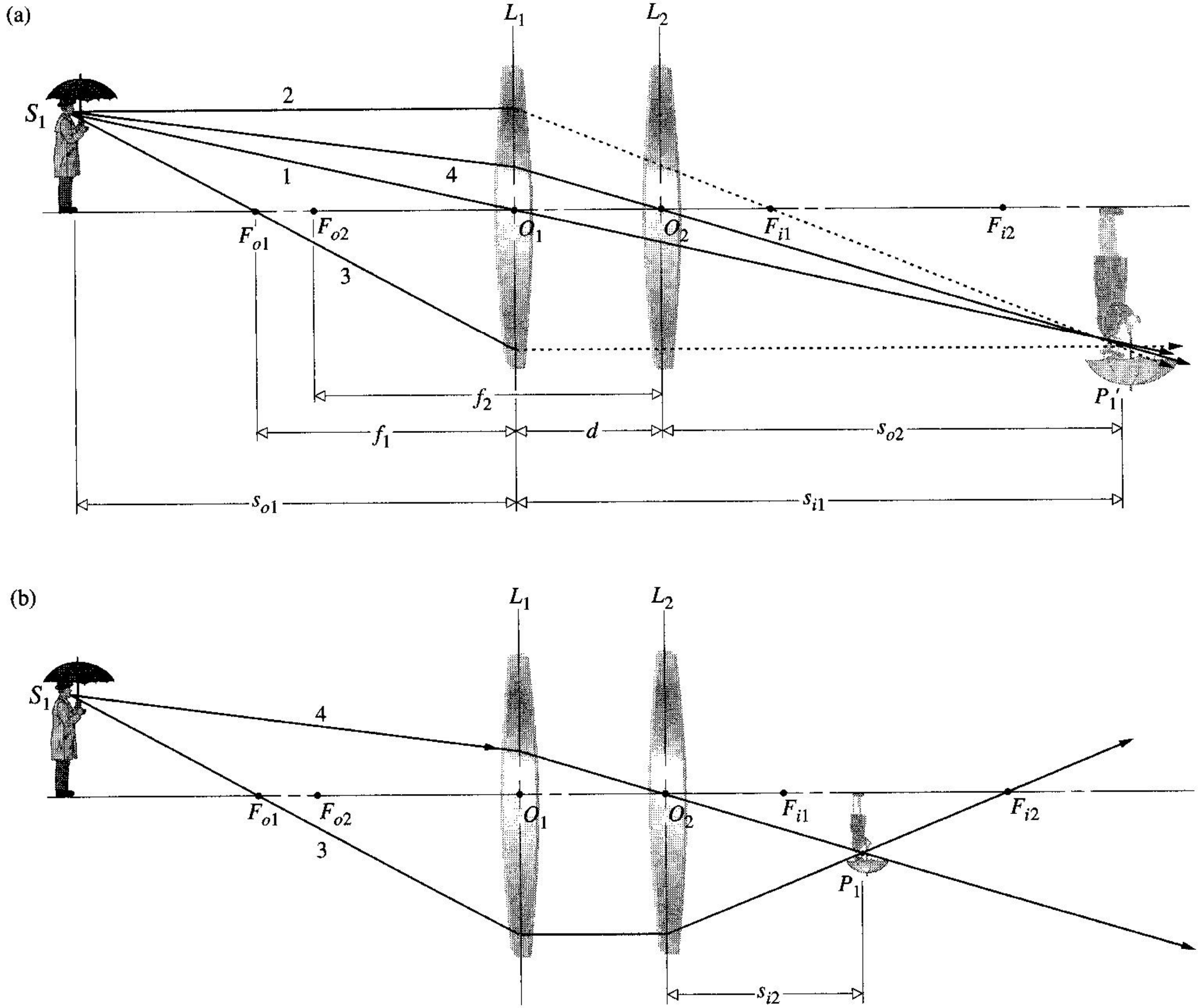






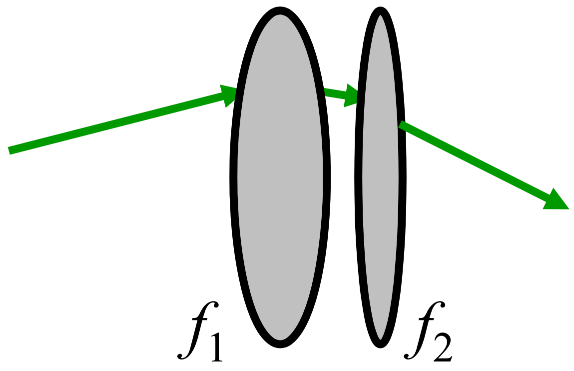


Lens sequence



Consecutive lenses

Suppose we have two lenses right next to each other (with no space in between)



$$O_{tot} = \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_1 - 1/f_2 & 1 \end{bmatrix}$$

$$1/f_{\text{tot}} = 1/f_1 + 1/f_2$$

So two consecutive lenses act as one whose focal length is computed by the resistive sum.