No.
Date: / /

OE Devices Hw7 2019 spring

1.

$$\Delta\phi = 4dh - 2\overset{\text{phase change of refl}}{\phi} = 2\pi m$$

$$h = k\cos\theta_m = \frac{2\pi n}{\lambda}\cos\theta_m$$

$$\Rightarrow \frac{2\pi n(2d)}{\lambda}\cos\theta_m - \phi_m = m\pi \quad ※$$

2.

(a)

$$\begin{cases} -d \le y \le d: \dfrac{d^2}{dy^2}E_z + \dfrac{(k_1^2-\beta^2)}{h^2}E_z = 0 \Rightarrow E_z(y) = A_1\cos(hy) \quad ※ \\ y>d,\ y<-d: \dfrac{d^2}{dy^2}E_z - \dfrac{(k_2^2-\beta^2)}{\alpha^2}E_z = 0 \Rightarrow E_z(y) = A_2 e^{-\alpha(y-d)} \quad ※ \end{cases}$$

$k_1 = \omega\sqrt{\mu_0\epsilon_1}$ $k_2 = \omega\sqrt{\mu_0\epsilon_2}$ $h = \sqrt{k_1^2-\beta^2}$, $\alpha = \sqrt{\beta^2-k_2^2}$

(b)

For $-d \le y \le d$:

$E_x = H_y = 0$

$$H_x = \frac{-1}{h^2}\left(-j\omega\epsilon_1\frac{\partial E_z}{\partial y}\right) = \frac{j\omega\epsilon_1}{h^2}A_1 h[-\sin(hy)] = -j\frac{\omega\epsilon_1 A_1}{h}\sin(hy) \quad ※$$

$$E_y = \frac{-1}{h^2}j\beta\frac{\partial E_z}{\partial y} = \frac{-1}{h^2}j\beta A_1 h[-\sin(hy)] = \frac{j\beta A_1}{h}\sin(hy) \quad ※$$

For $d>d$, $d<-d$:

$E_x = H_y = 0$

$$H_x = \frac{-1}{-\alpha^2}\left(j\omega\epsilon_2\frac{\partial E_z}{\partial y}\right) = \frac{j\omega\epsilon_2}{-\alpha^2}A_2(-\alpha e^{-\alpha(y-d)}) = j\frac{\omega\epsilon_2 A_2}{\alpha}e^{-\alpha(y-d)} \quad ※$$

$$E_y = \frac{-1}{-\alpha^2}j\beta\frac{\partial E_z}{\partial y} = \frac{1}{\alpha^2}j\beta A_2(-\alpha e^{-\alpha(y-d)}) = -j\frac{\beta A_2}{\alpha}e^{-\alpha(y-d)} \quad ※$$

(c) By B.C.

$$\begin{cases} E_t \text{ is continuous at } y=d \Rightarrow A_1\cos(hd) = A_2 \quad \text{①} \\ H_t \text{ is continuous at } y=d \Rightarrow -j\frac{\omega\epsilon_1 A_1}{h}\sin(hd) = j\frac{\omega\epsilon_2 A_2}{\alpha} \end{cases}$$

$$\Rightarrow \frac{-\epsilon_1 A_1}{h}\sin(hd) = \frac{\epsilon_2 A_2}{\alpha} \quad \text{②}$$

From ① and ② $\rightarrow \cot(hd) = -\frac{\epsilon_1}{\epsilon_2}\frac{\alpha}{h}$ #

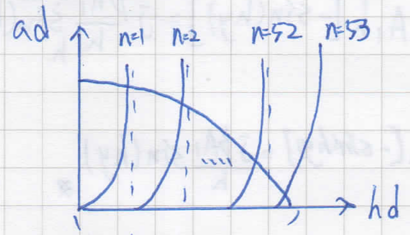
And $\begin{cases} k_1^2-\beta^2 = h^2 \\ \beta^2-k_2^2=\alpha^2 \end{cases} \Rightarrow k_1^2-k_2^2 = h^2+\alpha^2$ #

(d)

$$V = d\sqrt{k_1^2-k_2^2} = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2-n_2^2} = \frac{2\pi\times\frac{160}{2}}{1.5}\sqrt{1.51^2-1.49^2} = 26.128\pi < \frac{53}{2}\pi$$

The number of mode is 53

From $M = \mathrm{Int}\left(\frac{2V}{\pi}\right)+1 = 53$ modes



(e)

$$V < \frac{\pi}{2} \Rightarrow \frac{2\pi d}{\lambda_0}\sqrt{n_1^2-n_2^2} < \frac{\pi}{2} \Rightarrow \frac{d}{\lambda_0} < \frac{\pi}{2}\times\frac{1}{2\pi\sqrt{n_1^2-n_2^2}} \Rightarrow \frac{d}{\lambda_0} < 1.0206$$ #

No.
Date: / /

3.

(a) $V < \frac{\pi}{2} \Rightarrow \frac{2\pi d}{\lambda_0}\sqrt{n_1^2-n_2^2} < \frac{\pi}{2} \Rightarrow \lambda_0 < 4d\sqrt{n_1^2-n_2^2}$

$\Rightarrow \lambda_0 < 4\times\frac{0.275}{2}\times\sqrt{3.6^2-3.4^2} \Rightarrow \lambda_0 < 0.6508$

(b) $V = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2-n_2^2} = \frac{2\pi\times\frac{0.275}{2}}{880\times10^{-3}}\sqrt{3.6^2-3.4^2} = 1.162$

$\alpha \sim \frac{V}{d} = \frac{1.162}{0.1375} = 8.451\ (\mu m^{-1})\quad \delta = \frac{1}{\alpha} = 0.118$

field width $= 2d + 2\delta = 2\times(0.1375+0.118) = 0.511\ (\mu m)$ #

(c) $\lambda\uparrow \Rightarrow V\downarrow \Rightarrow \delta\uparrow \Rightarrow$ field width $\uparrow$ #

4.

(a) $NA = \sqrt{n_1^2-n_2^2} = \sqrt{1.47^2-1.45^2} = 0.0136$ #

(b) $\Delta = \frac{n_1-n_2}{n_1} = \frac{1.47-1.45}{1.47} = 0.0136$ #

(c) $V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2-n_2^2} = \frac{2\pi\times\frac{60}{2}}{0.87}\times\sqrt{1.47^2-1.45^2} = 52.3586$

$M \approx \frac{V^2}{2} = 1370$ modes. #

(d) $V < 2.405 \Rightarrow \lambda > \frac{2\pi a\sqrt{n_1^2-n_2^2}}{2.405} \Rightarrow \lambda > \frac{2\pi(30\,\mu m)\times\sqrt{1.47^2-1.45^2}}{2.405} = 18.94\ (\mu m)$ #

(e) $\frac{\Delta\tau}{L} = \frac{n_1-n_2}{c} = 66.7\ (ns\cdot km^{-1})$ #

5.

(a)

$$10\log\left(\frac{P_{in}}{P_{out}}\right) = 10\log\left(\frac{100}{2}\right) = 16.99\ (dB)$$

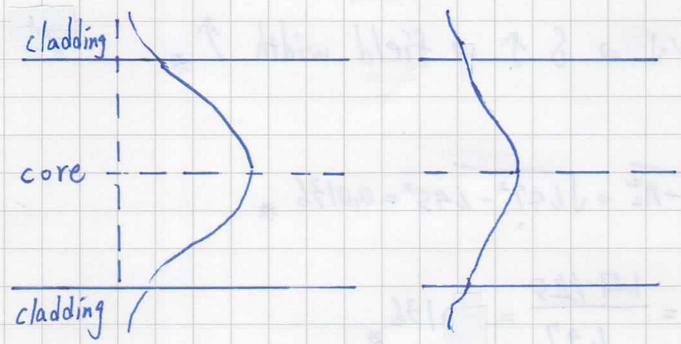
(b)

$$\frac{1}{L}\times 10\log\left(\frac{P_{in}}{P_{out}}\right) = 1.699\ (dB\cdot km^{-1})$$

(c)

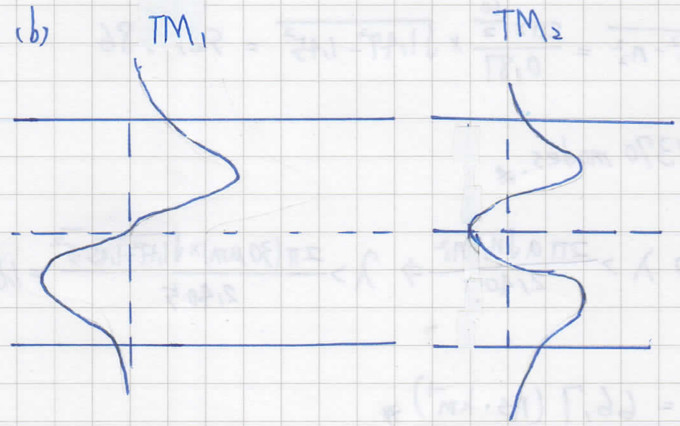
$$16.99 + \underbrace{9\times 1}_{\text{splice loss}} = 25.99\ (dB)$$

6. (a)



TE_0 confine more energy in the core.

(b)



TM_1 has only one node in the core, but TM_2 has two nodes in the core.

8. (a)

For $\frac{n_1-n_2}{n_1}=2.5\times10^{-3} \Rightarrow n_2=(1-2.5\times10^{-3})n_1$

$$V=\frac{2\pi a}{\lambda_0}\sqrt{n_1^2-n_2^2}=\frac{2\pi a}{\lambda}\sqrt{n_1^2-(1-2.5\times10^{-3})^2 n_1^2}=\frac{2\pi a}{\lambda}n_1\sqrt{1-(1-2.5\times10^{-3})^2}$$

$$=\frac{2\pi\times4}{1.55}\times1.45\times\sqrt{1-(1-2.5\times10^{-3})^2}=1.661 \Rightarrow \text{single mode fiber.}_{\#}$$

(b) For multimode $\Rightarrow V>2.405 \Rightarrow \lambda<1.071(\mu m)_{\#}$

(c) $NA=\sqrt{n_1^2-n_2^2}=0.102_{\#}$

(d) $2\alpha=2\times\sin^{-1}\left(\frac{NA}{1}\right)=0.204(rad)_{\#}$

(e)

$$\frac{\Delta\tau_m}{L}=|D_m|\Delta\lambda=12\times3=36(ps\cdot km^{-1})_{\#}$$

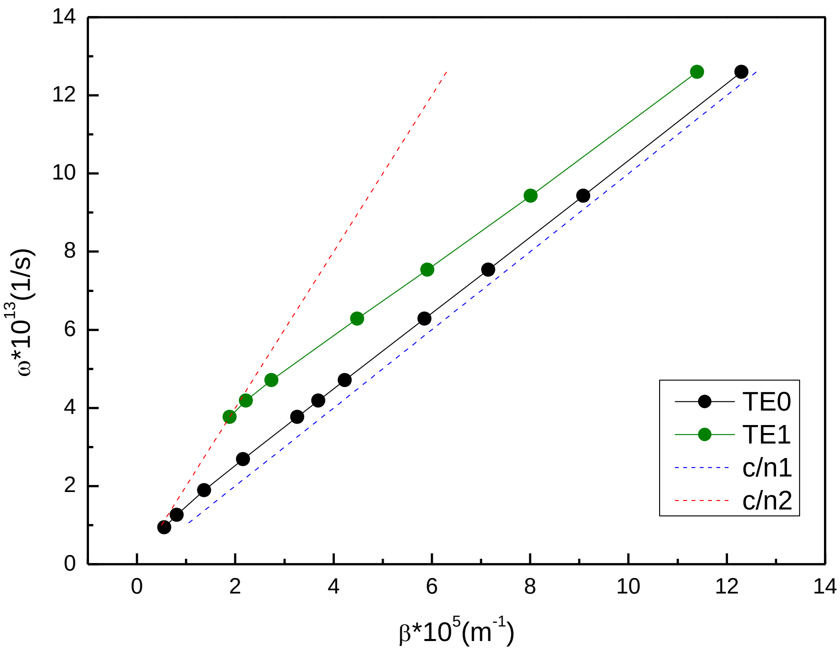
$$\frac{\Delta\tau_w}{L}=|D_w|\times\Delta\lambda=|-6|\times3=18(ps\cdot km^{-1})_{\#}$$

7.

$$\omega = 2\pi f = \frac{2\pi c}{\lambda_0}$$

$$\beta = k_1 \sin\theta = \frac{2\pi n_1}{\lambda_0} \sin\theta$$

$\lambda(\mu m)$	15	20	25	30	40	45
θ_0	77.8	74.52	71.5	68.7	63.9	61.7
θ_1	65.2	58.15	51.6	45.5	35.5	32.02
$\omega \times 10^{13} s^{-1}$	12.6	9.43	7.54	6.28	4.71	4.19
$\beta_0 \times 10^5 m^{-1}$	12.3	9.08	7.15	5.85	4.23	3.69
$\beta_1 \times 10^5 m^{-1}$	11.4	8.01	5.91	4.48	2.74	2.22
$\lambda(\mu m)$	50	70	100	150	200	
θ_0	59.74	53.2	46.4	39.9	36.45	
θ_1	30.17					
$\omega \times 10^{13} s^{-1}$	3.77	2.69	1.89	1.26	0.94	
$\beta_0 \times 10^5 m^{-1}$	3.26	2.16	1.37	0.81	0.56	
$\beta_1 \times 10^5 m^{-1}$	1.89					



$$Vg = \frac{d\omega}{d\beta}$$

$\omega \times 10^{13} s^{-1}$	TE_0 $(V_g * 10^8 (ms^{-1}))$	$TE_1(V_g * 10^8 (ms^{-1}))$
12.6	1.02	1.11
9.43	1.04	1.18
7.54	1.05	1.28
6.28	1.07	1.40
4.71	1.11	1.72
4.19	1.14	1.89
3.77	1.16	1.99
2.69	1.25	
1.89	1.38	
1.26	1.56	
0.94	1.68