OE Device 2019 spring Hw 4. sol.

1.

(a) For threshold gain coefficient, round trip gain = 1.

$$\Rightarrow I = I_0 e^{\gamma_{th}L} e^{-\alpha_s L} R_2 e^{\gamma_{th}L} e^{-\alpha_s L} R_1 = I_0$$

$$\Rightarrow R_1 R_2 e^{2L(\gamma_{th}-\alpha_s)} = 1 \Rightarrow \gamma_{th} = \frac{1}{2L} \ln \frac{1}{R_1 R_2} + \alpha_s$$

(b) $$\gamma_{th} = \frac{1}{2\times 1} \ln \frac{1}{0.92\times 0.98} + 0.15 = 0.202 \ (m^{-1})$$

2. (a) In thermal equilibrum, $$\frac{N_2}{N_1} = e^{-\frac{h\nu}{kT}} = e^{\left(-\frac{6.626\times10^{-34}\times\frac{3\times10^8}{694.3\times10^{-9}}}{1.38\times10^{-23}\times 300}\right)}$$

$$= 9.2\times10^{-31} \simeq 0 \Rightarrow N_2 \simeq 0$$

$N_1 + N_2 = 10^{25}$ $\quad N_2 - N_1 = -N_2 = -10^{25} \ (cm^{-3})$

$$\gamma = \sigma \Delta N = A_{21} \times \frac{\lambda^2}{8\pi n^2} g(\nu) \Delta N = \frac{1}{2.4\times10^{-3}} \times \frac{(694.3\times10^{-9})^2}{8\pi\times(1.76)^2} \times \frac{2}{\pi(250\times10^9)} \times -10^{25}$$

$$= -65.7 \ (m^{-1})$$

(b)

$$\gamma = \sigma \Delta N \Rightarrow \Delta N = \frac{\gamma}{\sigma} = \frac{150}{\frac{1}{2.4\times10^{-3}} \times \frac{(694.3\times10^{-9})^2}{8\pi\times1.76^2} \times \frac{2}{\pi(250\times10^9)}} = 2.283\times10^{24} \ (m^{-3})$$

(c) $$\text{Gain} = e^{(2\gamma l)} > 2.5 \Rightarrow e^{(2\times150\times l)} > 2.5 \Rightarrow 300 l > \ln(2.5)$$

$$\Rightarrow l > \frac{\ln(2.5)}{300} = 3.054\times10^{-3} \ (m)$$

(d) $$\alpha_c = \frac{1}{2l} \ln \frac{1}{R_1 R_2} = \frac{1}{2\times 0.05} \ln \frac{1}{(0.95)^2} = 1.026 \ (m^{-1}) = \gamma_{th}$$

(e) $$\Delta N_{th} = \frac{\gamma_{th}}{\sigma} = \frac{1.026}{\frac{1}{2.4\times10^{-3}} \times \frac{(694.3\times10^{-9})^2}{8\pi\times(1.76)^2} \times \frac{2}{\pi(250\times10^9)}} = 1.562\times10^{23} \ (m^{-3})$$

3. (a) For $\gamma(\nu) = \gamma(\nu_0)\, e^{\left[-4\ln 2\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]} = \alpha_r$

$$\Rightarrow \frac{\alpha_r}{\gamma(\nu_0)} = e^{-4\ln 2\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2}$$

$$\ln\left(\frac{\alpha_r}{\gamma(\nu_0)}\right) = (-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2$$

$$\Rightarrow (\nu-\nu_0) = \Delta\nu_0 \sqrt{\frac{\ln\left(\frac{\alpha_r}{\gamma(\nu_0)}\right)}{\ln 2^{-4}}}$$

(b)

$$FSR = \frac{c}{2nd} = \frac{3\times 10^8}{2\times 1\times 1} = 1.5\times 10^8 \,(Hz)$$

$$\alpha = \frac{1}{2l}\ln\frac{1}{R_1R_2} = \frac{1}{2\times 1}\ln\frac{1}{1\times 0.97} = 0.015\,(m^{-1})$$

$$B = 2\nu_0\sqrt{\frac{\ln\left(\frac{\alpha_r}{\gamma(\nu_0)}\right)}{\ln 2^{-4}}} = 2\times(1.5\times 10^9)\times\sqrt{\frac{\ln\left(\frac{0.015}{2\times 10^{-1}}\right)}{\ln 2^{-4}}} = 2.9\times 10^9\,(Hz)$$

$$M = \frac{B}{FSR} = 19.33 \Rightarrow \text{There are } 19 \text{ modes}$$

4.

(a) $\gamma(\nu_0) = N\sigma(\nu_0) \simeq N_a\sigma(\nu_0) = 1.4\times 10^{20}\times 2\times 10^{-20} = 2.8\,(cm^{-1})$

$\alpha(\nu_0) \equiv -\gamma(\nu_0) = -2.8\,(cm^{-1})$

(b)

$$\alpha_r = \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = 0.0186\,(cm^{-1})$$

(c) $N_t \equiv \frac{\alpha_r}{\sigma} = \frac{0.0186}{2\times 10^{-20}} = 9.3\times 10^{17}\,(cm^{-3})$

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5.

$$\int_0^\infty g(\nu)d\nu = \frac{1}{2}k \times (3\times10^9) = 1 \Rightarrow k = 6{,}67\times10^{-10}\,(s) = g(\nu_0)$$

$$\lambda_0 = \frac{1}{\nu_0} = 8\times10^{-6}\,(cm)$$

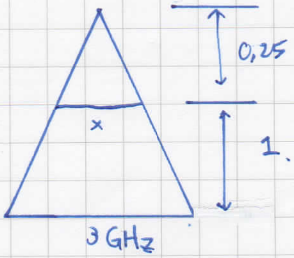
(a)

$$\sigma_{SE} = A_{21} \times \frac{\lambda^2}{8\pi n^2} g(\nu_0) = 2{,}5\times10^6 \times \frac{(8\times10^{-6})^2}{8\pi\times(2{,}75)^2} \times 6{,}67\times10^{-10} = 5{,}6\times10^{-16}\,(cm^2)$$

(b)

$$\gamma_0 = \frac{1}{2L_g} \ln\frac{1}{R_1R_2} = \frac{1}{16}\ln\left(\frac{1}{0{,}8\times0{,}75}\right) = 3{,}19\times10^{-2}\,(cm^{-1}) = \sigma\,\Delta N_{th}$$

$$\Rightarrow \Delta N_{th} = 5{,}696\times10^{13}\,(cm^{-3})$$



(c)

$$\frac{1{,}25}{3\times10^9} = \frac{0{,}25}{x} \Rightarrow x = 600\,(MHz)$$

$$\nu_F = \frac{c}{2\times(0{,}12+2{,}75\times0{,}08)} = 441{,}176\,(MHz)$$

$\therefore$ 1 mode ※

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