No.
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OE device 2019 spring Hw03. solution.

1. (a)

$$h\nu_0 = 5.5 - 3.2 = 2.3\,(eV) \Rightarrow \nu_0 = \frac{2.3\times1.6\times10^{-19}}{6.626\times10^{-34}} = 5.554\times10^{14}\ (Hz)$$

For Lorentzian line shape. $g(\nu) = \frac{1}{\pi}\times\frac{\Delta\nu\times\frac{1}{2}}{\left(\frac{\Delta\nu}{2}\right)^2} = \frac{2}{\pi\Delta\nu}$

Stimulated emission cross section :

$$\sigma_{st} = A_{21}\times\frac{\lambda^2}{8\pi n^2}g(\nu) = A_{21}\times\frac{\left(\frac{c}{\nu_0}\right)^2}{8\pi n^2}\times\frac{2}{\pi\Delta\nu} = 5\times10^6\times\frac{(3\times10^8)^2}{1\times8\pi\times(5.554\times10^{14})^2}\times\frac{2}{\pi\times9\times10^9}$$

$$= 4.106\times10^{-18}$$ ※

(b)

In steady state : $\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - \frac{N_2}{\tau_{20}} = 0$

$$\Rightarrow R - \frac{(\tau_{20}+\tau_{21})}{\tau_{20}\tau_{21}}N_2 = 0 \quad \Rightarrow N_2 = \frac{\tau_{20}\tau_{21}R}{\tau_{20}+\tau_{21}}$$

$\tau_{21} = \frac{1}{A_{21}}$

For $N_1 \sim 0 \Rightarrow \Delta N = N_2 - N_1 = N_2$

gain coefficient $\gamma = \sigma\times\Delta N = \sigma\times N_2 = \sigma\times\frac{\tau_{20}\tau_{21}R}{\tau_{20}+\tau_{21}}$

For $\gamma = 0.02\,(cm^{-1}) \Rightarrow R = \frac{\gamma}{\sigma}\times\frac{\tau_{20}+\tau_{21}}{\tau_{20}\times\tau_{21}} = \frac{0.02\times100}{4.106\times10^{-18}}\times\frac{(200+100)\times10^{-9}}{200\times10^{-9}\times100\times10^{-9}}$

$$= 7.306\times10^{24}\,(m^{-3}s^{-1})$$

2. (a)

E_2 ———— N_2 For level 2 : $\frac{dN_2}{dt} = -A_{21}N_2 + B_{12}\rho N_1 - B_{21}\rho N_2$

E_1 ———— N_1 For level 1 : $\frac{dN_1}{dt} = A_{21}N_2 - B_{12}\rho N_1 + B_{21}\rho N_2$

- N_2, N_1 : population density of level 2 and 1.
- A_{21} : decay rate ($1/s$)
- B_{21}, B_{12} : transition rate ($m^3/J\cdot s$)
- ρ : energy density (J/m^3)

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(b) Assume the total population density $N = N_1 + N_2$

the population difference $\Delta N = N_1 - N_2$

$$\begin{cases} \dfrac{dN_2}{dt} = B_{12}\rho(N_1 - N_2) - A_{21}N_2 \\ \dfrac{dN_1}{dt} = A_{21}N_2 + B_{12}\rho(N_2 - N_1) \end{cases}$$

$$\begin{cases} N = N_1 + N_2 \\ \Delta N = N_1 - N_2 \end{cases} \Rightarrow 2N_2 = N - \Delta N$$

$$\frac{d\Delta N}{dt} = \frac{d}{dt}(N_1 - N_2) = 2B_{12}\rho(N_2 - N_1) + 2A_{21}N_2$$

$$= -2B_{12}\rho\Delta N + A_{21}(N - \Delta N)$$

In steady state, $\dfrac{d\Delta N}{dt} = 0 \Rightarrow (2B_{12}\rho + A_{21})\Delta N = A_{21}N$

$$\Rightarrow \Delta N = \frac{A_{21}}{2B_{12}\rho + A_{21}}N$$

$\because$ ΔN is always positive

$\Rightarrow$ population inversion cannot be achieved in 2 levels system.

3, (a) In steady state
$$\begin{cases} \dfrac{dN_3}{dt} = R_3 - \dfrac{N_3}{\tau_{30}} - \dfrac{N_3}{\tau_{31}} = 0 & \text{— ①} \\ \dfrac{dN_1}{dt} = \dfrac{N_3}{\tau_{31}} - \dfrac{N_1}{\tau_1} = 0 & \text{— ②} \end{cases}$$

From ①: $R_3 - \left(\frac{1}{\tau_{30}} + \frac{1}{\tau_{31}}\right)N_3 = R_3 - \frac{1}{\tau_3}N_3 = 0 \Rightarrow N_3 = R_3\tau_3$

Substitute $N_3 = R_3\tau_3$ into ②: $\frac{R_3\tau_3}{\tau_{31}} - \frac{N_1}{\tau_1} = 0 \Rightarrow N_1 = \frac{\tau_1\tau_3}{\tau_{31}}R_3$

Gain coefficient $\gamma = \sigma\Delta N = \sigma(N_3 - N_1) = \sigma\left(R_3\tau_3 - R_3\frac{\tau_1\tau_3}{\tau_{31}}\right)$

$$= R_3\tau_3\sigma\left(1 - \frac{\tau_1}{\tau_{31}}\right)_{\#}$$

(b)
$$\begin{cases} \frac{dN_3}{dt} = R_3 - \frac{N_3}{\tau_{30}} - \frac{N_3}{\tau_{31}} = 0 & \text{— ①} \\ \frac{dN_1}{dt} = R_2 + \frac{N_3}{\tau_{31}} - \frac{N_1}{\tau_1} = 0 & \text{— ②} \end{cases}$$

Substitute $N_3 = R_3 \tau_3$ into ②. $R_2 + \frac{R_3\tau_3}{\tau_{31}} - \frac{N_1}{\tau_1} = 0 \Rightarrow N_1 = \tau_1 (R_2 + \frac{\tau_3}{\tau_{31}} R_3)$

Gain coefficient $\gamma = \sigma \times \Delta N = \sigma(N_3 - N_1) = \sigma(R_3\tau_3 - \tau_1(R_2 + \frac{\tau_3}{\tau_{31}} R_3))$

$= \sigma[\tau_3 R_3 (1 - \frac{\tau_1}{\tau_{31}}) - \tau_1 R_2]$ ※

4.

(a) at equilibrium $\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)}$

Boltzmann $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-h\nu/kT} = \frac{B_{12}\rho(\nu)}{A_{21}+B_{21}\rho(\nu)}$

$\Rightarrow \rho(\nu) = \frac{A_{21}\frac{g_2}{g_1}e^{-h\nu/kT}}{B_{12} - B_{21}\frac{g_2}{g_1}e^{-h\nu/kT}} = \frac{A_{21}}{B_{21}} \frac{1}{\frac{B_{12}g_1}{B_{21}g_2}e^{h\nu/kT} - 1}$

$\because \rho(\nu) = \frac{8\pi n^3 \nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$ (plank formula)

$\therefore \frac{B_{12}}{B_{21}} = \frac{g_2}{g_1}, \quad \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$

gain coefficient $= \gamma(\nu) = \frac{h\nu}{c/n}(B_{21}N_2 - B_{12}N_1) = A_{21}\frac{\lambda^2}{8\pi n^2}(N_2 - \frac{g_2}{g_1}N_1)$

(b)

$\int_0^\infty g(\nu)d\nu = \frac{1}{2}\cdot\frac{k}{3} + \frac{1}{2}(\frac{k}{3}+k)\cdot 3 + \frac{1}{2}k = 1 \rightarrow$ probability 積分.

$\Rightarrow \frac{8}{3}(cm^{-1})k = 1 \Rightarrow (8\times10^{10})k = 1 \Rightarrow k = 1.25\times10^{-11}(s) = g(\nu_0)$

$\bar{\nu}_0 = 18340 - 2627 = 15713\ (cm^{-1}) \Rightarrow \lambda_0 = 0.636\ \mu m$

$\sigma_{SE} = \frac{\lambda^2}{8\pi}A_{21}g(\nu_0) = 2.012\times10^{-20}\ cm^2$

$\sigma_{abs} = \sigma_{SE}\times\frac{g_2}{g_1} = 1.206\times10^{-20}\ cm^2$