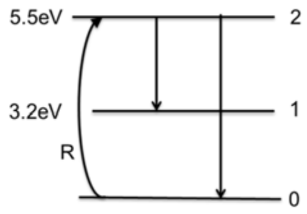
NTHU Electrical Engineering Department
EE3360 Optoelectronic Devices Spring 2019
HW #3

1. Consider the ideal laser medium shown below. The pump excites the atoms to energy level 2 at a rate R (per second per volume), which then decays to level 1 at a rate τ_{21}^{-1} and back to level 0 at a rate τ_{20}^{-1}. Level 1 decays back to 0 so fast that the approximate N_1=0 is appropriate. The radiative rate for 2→ 1 transition A_{21} is 5×10^{6} sec^{-1}, and its width is 9GHz. (Assume a Lorentzian lineshape and steady state.)

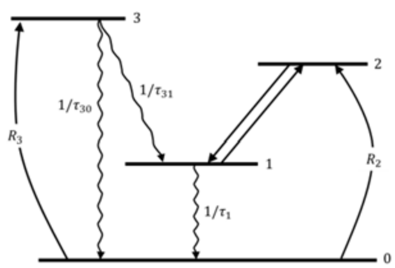


(a) What is the stimulated emission cross section?

(b) What must be the pump rate R in order to obtain a gain coefficient of $0.02cm^{-1}$?

2. (a) Please write down the rate equation for each energy level of a 2 level atomic system. Define each variable clearly. (b) Please show that population inversion can NOT be achieved in a 2 level atomic system.

3. The following energy level diagram shown illustrates a situation that can occur in gas lasers. The upper state of 3 → 1 transition is pumped at a rate R_3, and if the lifetime ratio τ_3/τ_1 is favorable, then we can obtain gain on the 3 → 1 transition. However, state 2 can also be excited at a rate R_2, and if sufficient feedback is provided, oscillation can take place on the 2 → 1 transition, which raises the population of state 1 and lowers the gain at λ_{31}. Analyze this case by answering the following two questions. Assume steady state and neglect degeneracy and spontaneous decay from 2 → 1. Define $\tau_3^{-1} = \tau_{30}^{-1} + \tau_{31}^{-1}$.



(a) Assume $R_2 = 0$ and thus no oscillation in the 2 → 1 transition. Find an expression of the gain coefficient on 3 → 1 in terms of the lifetimes, the pumping rates, and the stimulated emission cross section.

(b) Assume that $R_2 \neq 0$ and that there is a strong saturating signal at the 2 → 1 wavelength such that $N_1 = N_2$. (Neglect spontaneous decay from 2 → 1.) Find a new expression for the gain coefficient at λ_{31}.

4. (a) In class, we derive the relationship between Einstein A and B, and gain coefficients, assuming that each state has a different energy. In fact, there are many ways that an atom can have the same energy, described by "degeneracy". The degeneracy number g_i represents the number of states that an atom can have the energy E_i. If taking degeneracy into account, Boltzmann distribution describing the ratio of two populations in E_2 and E_1 would be $N_2/N_1 = (g_2/g_1)e^{-h\nu/kT}$. Please derive the relationship between A and B coefficients, and gain coefficient, including degeneracy.

(b) There are several ways of specifying fundamental quantities. For instance, the frequency ν is in Hz, but equally informative is wavelength in nm or μm, wavenumber $\bar{\upsilon}$ (=1/λ) in cm^{-1}, or in energy units of hν in Joules or hν/e in volts. The spontaneous emission profile from a certain transition can be approximated by the shape shown below, represented in wavenumber. What is the stimulated emission cross section and the absorption cross section? (Given degeneracy g_2=3, g_1=5, A_{21}=10 sec^{-1})