# 2019. spring 光電元件 HW#2 solution.

1. (a)

$$\lambda = \frac{c}{\nu} \Rightarrow d\lambda = -c \times \frac{1}{\nu^2} d\nu \Rightarrow -\frac{\lambda}{\nu}\frac{d\nu}{d\lambda} = 1$$

$$\because \Delta\nu << \nu_0 \text{ and } \Delta\lambda << \lambda_0 \quad \therefore \left|\frac{\lambda_0}{\nu_0}\frac{\Delta\nu}{\Delta\lambda}\right| = 1 \Rightarrow \Delta\lambda = \frac{\lambda_0}{\nu_0}\Delta\nu = \Delta\nu\frac{\lambda_0^2}{c}$$

$$l_c = c\Delta t = c\frac{1}{\Delta\nu} = c\frac{\lambda_0^2}{\Delta\lambda} \times \frac{1}{c} = \frac{\lambda_0^2}{\Delta\lambda}$$

(b)

$$\Delta\lambda = \Delta\nu\frac{\lambda_0^2}{c} = 2.1\times10^{12} \times (532\times10^{-9})^2/(3\times10^8) = 1.981 \text{ (nm)}$$

$$t_c = \Delta t = \frac{1}{\Delta\nu} = 4.762\times10^{-13} \text{ (s)}$$

(c)

$$l_c = \frac{\lambda_0^2}{\Delta\lambda} = (1310\times10^{-9})^2/(90\times10^{-9}) = 19.068 \text{ (}\mu m\text{)}$$

(d)

$$l_c = \frac{c}{\Delta\nu} = 0.2 m$$

2. (a)

$$W_0 = \frac{\lambda}{\theta\pi} = (1064\times10^{-9})/(\pi\times0.5\times10^{-3}) = 6.774\times10^{-4} \text{ (m)}$$

Rayleigh length $= z_0 = \frac{\pi W_0^2}{\lambda} = 1.388 \text{ (m)}$

$$I_0 = \frac{2P}{\pi W_0^2} = 1.387\times10^6 \text{ (W/m}^2\text{)}$$

$$W(z=100cm) = W_0 \times \sqrt{1+(z/z_0)^2} = 8.419\times10^{-4} \text{ (m)}$$

$$I(z=100cm) = \frac{2P}{\pi W^2(z)} = 8.98\times10^5 \text{ (W/m}^2\text{)}$$

(b)

at $z = z_0$

for laser: $I = \frac{1}{2}I_0 = 6.935\times10^5$ (W/m²)

for spherical wave, $I = \frac{P}{4\pi z_0^2} = 0.0413$ (W/m²).

3. $\lambda = 1064$ nm

divergence angle $\theta = \frac{W_2 - W_1}{z_2 - z_1} = \frac{\lambda}{\pi W_0} = \frac{(3.38 - 1.699)mm}{10 cm}$

$$\Rightarrow W_0 = \frac{\lambda(z_2 - z_1)}{\pi(W_2 - W_1)} = 2.015\times10^{-5} \text{ (m)}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}} \Rightarrow z_0 = 1.199\times10^{-3} \text{ (m)}$$

$$W = W_0\sqrt{1+(z/z_0)^2} \Rightarrow z = z_0\sqrt{\left(\frac{W}{W_0}\right)^2 - 1}$$

$\Rightarrow z_1 = 0.101$ m, beam waist 距 location1 0.101 m.

4. $TEM_{00}$

$$P_{00}(w) = \int_0^{2\pi}\int_0^{r} |A_{00}|^2 \left(\frac{w_0}{w}\right)^2 e^{-\frac{2\rho^2}{w^2}} \rho\, d\rho\, d\theta = |A_{00}|^2 \left(\frac{w_0}{w}\right)^2 \left(-\frac{w^2}{2}\right) e^{-\frac{2\rho^2}{w^2}} \pi \Big|_0^{\rho = r}$$

$$= |A_{00}|^2 \frac{\pi w_0^2}{2}\left(1 - e^{\frac{-2r^2}{w^2}}\right)$$

$$\frac{P_{00}(r \to w)}{P_{00}(r \to \infty)} = 1 - e^{-2}$$

$$\frac{P_{01,w}}{P_{01,tot}} = \frac{\int_0^{2\pi}\int_0^{w} |A_{01}|^2 \left(\frac{w_0}{w}\right)^2 e^{\frac{-2\rho^2}{w^2}} \frac{4 \times 2y^2}{w^2} \rho\, d\rho\, d\theta}{\int_0^{2\pi}\int_0^{\infty} |A_{01}|^2 \left(\frac{w_0}{w}\right) e^{\frac{-2\rho^2}{w^2}} \frac{4 \times 2y^2}{w^2} \rho\, d\rho\, d\theta} = \frac{\int_0^{2\pi}\int_0^{w} e^{\frac{-2\rho^2}{w^2}} \frac{\rho^2\cos^2\theta}{w^2} \rho\, d\rho\, d\theta}{\int_0^{2\pi}\int_0^{\infty} e^{\frac{-2\rho^2}{w^2}} \frac{\rho^2\cos^2\theta}{w^2} \rho\, d\rho\, d\theta}$$

$$= \frac{\int_0^{w} \rho^2 e^{\frac{-2\rho^2}{w^2}} \rho\, d\rho}{\int_0^{\infty} \rho^2 e^{\frac{-2\rho^2}{w^2}} \rho\, d\rho} = \frac{\int_0^{w^2} u e^{-\frac{2u}{w^2}} du}{\int_0^{\infty} u e^{\frac{-2u}{w^2}} du} = \frac{w^2 e^{-2} + \frac{w^2}{2} e^{-2} - \frac{w^2}{2}}{0 - \frac{w^2}{2}} = 1 - 3e^{-2}$$

5.

(a) The phase of Gaussian beam is $\phi(w, z) = kz - \tan^{-1}\left(\frac{z}{z_0}\right) + \frac{kr^2}{2R(z)}$

at axis-points $\phi(0, z) = kz - \tan^{-1}\left(\frac{z}{z_0}\right)$, and at the location of mirrors $z_1, z_2$

$\phi(0, z_1) = kz_1 - \tan^{-1}\left(\frac{z_1}{z_0}\right)$

$\phi(0, z_2) = kz_2 - \tan^{-1}\left(\frac{z_2}{z_0}\right)$

As the beam propagates from mirror 1 to mirror 2, its phase change by

$\phi(0, z_2) - \phi(0, z_1) = k(z_2 - z_1) - \left[\tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)\right]$

In order that the beam truly retrace itself

$\Delta\phi(0, z) = 2\pi q$, $q = 0, \pm 1, \pm 2 \ldots$

substitute $k = \frac{2\pi\nu}{c}$ and $\nu_F = \frac{c}{2d}$

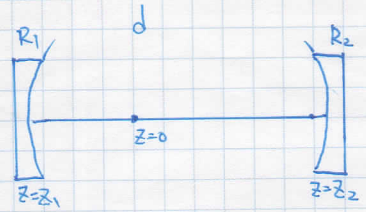
$\Rightarrow \nu_q = 2q\nu_F + \frac{\tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)}{\pi}\nu_F$

(b) $FSR = \frac{c}{2d} = \frac{3 \times 10^8}{2 \times 0.6} = 0.25\ GHz$

(c) $\Delta\nu = \nu_F \frac{1-R}{\pi\sqrt{R}}$, $Q = \frac{\nu_0}{\Delta\nu}$ ; $\mathcal{F} = \frac{\nu_F}{\Delta\nu} = \frac{\pi\sqrt{R}}{1-R}$

| R | $\Delta\nu$ | Q | $\mathcal{F}$ |
|---|---|---|---|
| 0.97 | 2.424 MHz | $2.063 \times 10^8$ | 103.135 |
| 0.85 | 12.947 MHz | $3.862 \times 10^7$ | 19.309 |
| 0.7 | 28.534 MHz | $1.752 \times 10^7$ | 8.761 |
| 0.6 | 41.094 MHz | $1.217 \times 10^7$ | 6.084 |

6.




$$z_2 - z_1 = d$$

$$R(z_1) = z_1\left[1+\left(\frac{z_0}{z_1}\right)^2\right] = -R_1$$ use this three equation we can find.

$$R(z_2) = z_2\left[1+\left(\frac{z_0}{z_2}\right)^2\right] = R_2$$

$$z_0^2 = \frac{d(R_1-d)(R_2-d)(R_1+R_2-d)}{(R_1+R_2-2d)^2}$$

$$z_1 = \frac{-d(R_2-d)}{R_1+R_2-2d}$$

$$z_2 = \frac{d(R_1-d)}{R_1+R_2-2d}$$

7.

(a) $\begin{cases} R = d\left(1+\frac{z_0^2}{d^2}\right) \\ R = 3d\left(1+\frac{z_0^2}{9d^2}\right) \end{cases} \Rightarrow \begin{cases} Rd - d^2 = z_0^2 \\ 3Rd - 9d^2 = z_0^2 \end{cases} \Rightarrow R = 2d \quad z_0^2 = d^2$

$\frac{d}{R} = \frac{1}{2}$

(b) $\phi(0, M_1) = kd - (1+m+p)\tan^{-1}\left(\frac{d}{z_0}\right)$, $\phi(0, M_2) = 2kd - (1+m+p)\tan^{-1}\left(\frac{2d}{z_0}\right)$

$$\phi(0, M_2) - \phi(0, M_1) = kd - (1+m+p)\left[\tan^{-1}\left(\frac{2d}{z_0}\right) - \tan^{-1}\left(\frac{d}{z_0}\right)\right] = 2\pi q$$

$k = \frac{2\pi\nu}{c} \Rightarrow \nu = \frac{c}{2d}\left\{1 + \frac{1+m+p}{\pi}\left[\tan^{-1}\left(\frac{2d}{z_0}\right) - \tan^{-1}\left(\frac{d}{z_0}\right)\right]\right\}$

(c) $\nu_{1,0,q} - \nu_{0,0,q} = \frac{c}{2\pi d}\left[\tan^{-1}\left(\frac{2d}{z_0}\right) - \tan^{-1}\left(\frac{d}{z_0}\right)\right] = 16.2\ MHz$