EE3360 光電元件 HW#1 Solution

No.
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1. (a) Maxwell's eq under source free ($\rho=0$, $\vec{J}=0$)

$$\Rightarrow \begin{cases} \nabla\cdot\vec{E}=0 & -(1) \\ \nabla\times\vec{E}=-\dfrac{\partial\mu\vec{H}}{\partial t} & -(2) \\ \nabla\times\vec{H}=\dfrac{\partial\epsilon\vec{E}}{\partial t} & -(3) \\ \nabla\cdot\vec{H}=0 & -(4) \end{cases}$$

Vector identity

$$\boxed{\nabla\times(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}}$$

Curl of (3):

$$\nabla\times(\nabla\times\vec{H})=\nabla\times\left(\frac{\partial\epsilon\vec{E}}{\partial t}\right)$$

$$\Rightarrow \underbrace{\nabla(\nabla\cdot\vec{H})}_{\text{from (4)}=0}-\nabla^2\vec{H}=\frac{\partial}{\partial t}\underbrace{(\nabla\times\epsilon\vec{E})}_{\text{from (2)}=-\mu\epsilon\frac{\partial\vec{H}}{\partial t}}$$

(b) $\Rightarrow \underline{-\nabla^2 H=-\mu\epsilon\dfrac{\partial^2\vec{H}}{\partial t^2}}$ ※ Similarly, we can obtain: $-\nabla^2\vec{E}=-\mu\epsilon\dfrac{\partial^2\vec{E}}{\partial t^2}$ — (*)

The electric field intensity of a uniform plane EM wave:

$\vec{E}=E_0\cos(wt-kz)\hat{a}_x$ (or you can use phasor notation)

Substitute $\vec{E}$ into equation (*):

$$k^2E_0\cos(wt-kz)=\mu\epsilon w^2E_0\cos(wt-kz)$$

$$\frac{w^2}{k^2}=\frac{1}{\mu\epsilon}=V^2$$: wave velocity ⇒ Verified.

2. Assume $\vec{E}=(E_x\hat{a}_x+E_y\hat{a}_y+E_z\hat{a}_z)e^{jwt}\,\underline{e^{-jkz}}$ ⤳ propagation in z direction.

$\vec{H}=(H_x\hat{a}_x+H_y\hat{a}_y+H_z\hat{a}_z)e^{jwt}\,e^{-jkz}$

(1) $\nabla\cdot\vec{E}=0$

$$\Rightarrow(-jk)E_z e^{jwt}e^{-jkz}=0 \Rightarrow E_z=0$$

(2) $\nabla\times\vec{E}=-\mu(jw)\vec{H}$

$$\Rightarrow(jkE_y\hat{a}_x-jkE_x\hat{a}_y)e^{jwt}e^{-jkz}=-\mu(jw)(H_x\hat{a}_x+H_y\hat{a}_y+H_z\hat{a}_z)e^{jwt}e^{-jkz}$$

$$\Rightarrow\begin{cases} kE_y=-\mu wH_x \longrightarrow \dfrac{E_y}{H_x}=-\dfrac{\mu w}{k}=-\sqrt{\dfrac{\mu}{\epsilon}} \\ -kE_x=-\mu wH_y \longrightarrow \dfrac{E_x}{H_y}=\sqrt{\dfrac{\mu}{\epsilon}} \\ H_z=0 \end{cases}$$

(3) $\nabla \times \vec{H} = \epsilon(j\omega)\vec{E}$

$$\begin{cases} kH_y = \epsilon \omega E_x \longrightarrow \dfrac{E_x}{H_y} = \dfrac{k}{\epsilon \omega} = \sqrt{\dfrac{\mu}{\epsilon}} \\ -kH_x = \epsilon \omega H_y \longrightarrow \dfrac{E_y}{H_x} = -\sqrt{\dfrac{\mu}{\epsilon}} \\ E_z = 0 \end{cases}$$

(4) $\nabla \cdot \vec{H} = 0$

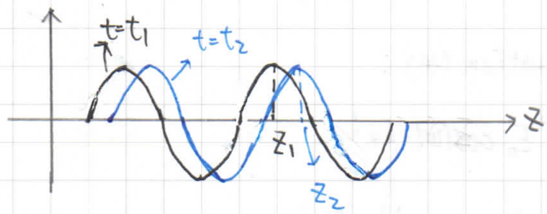
$\Rightarrow H_z = 0$

Therefore, the extra constraints for $\vec{E}$ and $\vec{H}$:

$\underline{E_z = H_z = 0}$ ※, and $\underline{\dfrac{E_x}{H_y} = -\dfrac{E_y}{H_x} = \sqrt{\dfrac{\mu}{\epsilon}}}$ ※

3. The definition of a wavefront is the locus of points <u>having the same phase</u>.

$E = E_0 \cos(\omega t - kz + \phi) \Rightarrow$ Phase $= \omega t - kz + \phi$



At time $t = t_1$ a certain point at $z = z_1$

phase $= \omega t_1 - kz_1 + \phi = \phi_1$

At time $t = t_2$, the point at $z = z_2$ has the same phase

$\phi_1 = \omega t_2 - kz_2 + \phi$

$\Rightarrow \omega t_1 - kz_1 + \phi = \omega t_2 - kz_2 + \phi$

$\Rightarrow k(z_2 - z_1) = \omega(t_2 - t_1)$

$\Rightarrow \dfrac{z_2 - z_1}{t_2 - t_1} = \underline{\dfrac{\omega}{k} = V}$: speed of wavefront. ※

4. (a) $\nabla\cdot\vec{E}_1 = -jke^{-jkz} \neq 0$

$\Rightarrow \because \vec{E}_1$ violates the Maxwell's eq. $\nabla\cdot\vec{E}=0$

$\therefore \vec{E}_1$ doesn't represent an EM wave.

$\nabla\cdot\vec{E}_2 = 0 \Rightarrow \vec{E}_2$ represents an EM wave.

$\nabla\cdot\vec{E}_3 = 0 \Rightarrow \vec{E}_3$ represents an EM wave.

(b) $\nabla\times\vec{E} = -\mu(j\omega)\vec{H}$

$\nabla\times\vec{E}_2 = \underbrace{k}_{=\omega\sqrt{\mu\epsilon}}(\hat{a}_x + j\hat{a}_z)e^{-jky} = -\mu(j\omega)\vec{H}_2$

$\Rightarrow \vec{H}_2 = \sqrt{\frac{\epsilon}{\mu}}(j\hat{a}_x - \hat{a}_z)e^{-jky}$ ※

$\nabla\times\vec{E}_3 = \sqrt{2}jke^{-jk(x-z)/\sqrt{2}}\hat{a}_y = -\mu(j\omega)\vec{H}_3$

$\Rightarrow \vec{H}_3 = -\sqrt{\frac{2\epsilon}{\mu}}e^{-jk(x-z)/\sqrt{2}}\hat{a}_y$ ※

(c) $\vec{E}_2 = \hat{a}_x\cos(\omega t-ky) + \hat{a}_z\cos(\omega t-ky+\frac{\pi}{2})$

$= \hat{a}_x\cos(\omega t-ky) - \hat{a}_z\sin(\omega t-ky)$ ※

$\vec{H}_2 = -\sqrt{\frac{\epsilon}{\mu}}[\hat{a}_x\sin(\omega t-ky) + \hat{a}_z\cos(\omega t-ky)]$ ※

$\vec{E}_3 = (\hat{a}_x+\hat{a}_z)\cos[\omega t-k(x-z)/\sqrt{2}]$ ※

$\vec{H}_3 = -\hat{a}_y\sqrt{\frac{2\epsilon}{\mu}}\cos[\omega t-k(x-z)/\sqrt{2}]$ ※

5. (a) $\vec{E} = E_0\cos(\omega t-ky)\hat{a}_z = 10\cos(10^8t+y)\hat{a}_z$

$\omega = 10^8$ rad/s $\quad \vec{k} = -1\hat{a}_y$

direction of propagation : $-y$

time period : $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10^8} \simeq 62.832$(ns)

wavelength : $|k| = 1 = \frac{2\pi}{\lambda} \Rightarrow \lambda \simeq 6.283$(m)

phase velocity : $v_p = \frac{\omega}{k} = 10^8$(m/s)

(b) $\epsilon_r = \dfrac{\left(\frac{1}{\sqrt{\epsilon_0 \mu_0}}\right)^2}{\left(\frac{1}{\sqrt{\epsilon \mu_0}}\right)^2} = \left(\dfrac{c}{V_p}\right)^2 = 9.$

(c) $\dfrac{\partial E_z}{\partial y} = -\dfrac{\partial B_x}{\partial t} \Rightarrow \vec{B} = \hat{a}_x\, 10^{-7} \cos(10^8 t + y) \ (T)$

$\vec{H} = \hat{a}_x \dfrac{1}{4\pi} \cos(10^8 t + y) \quad (\frac{A}{m})$

phasor notation:

$\vec{E} = 10\, e^{jy}\, e^{10^8 t}\, \hat{a}_z \ (V/m)$

$\vec{H} = \frac{1}{4\pi} e^{jy} e^{10^8 t}\, \hat{a}_x \ (A/m)$

6. (a)

$f = 300\,MHz \Rightarrow k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c} = 2\pi \Rightarrow k_x = k_y = k\cos 45^\circ = \sqrt{2}\pi$

$\therefore \vec{E} = E_0\, e^{-j\sqrt{2}\pi(x+y)}\, \hat{a}_z$

$= (1+j)\, e^{-j\sqrt{2}\pi(x+y)}\, \hat{a}_z$

$= \sqrt{2}\, e^{-j[\sqrt{2}\pi(x+y) - \frac{\pi}{4}]}\, \hat{a}_z \ (\frac{V}{m})$

$\vec{B} = \dfrac{1}{c}\, \vec{a}_k \times \vec{E}$

$= \dfrac{1}{c}\, e^{-j[\sqrt{2}\pi(x+y) - \frac{\pi}{4}]}\, (\hat{a}_x - \hat{a}_y)$

$= 3.33 \times 10^{-9}\, e^{-j[\sqrt{2}\pi(x+y) - \frac{\pi}{4}]}\, (\hat{a}_x - \hat{a}_y)$

(b) $\vec{E}(x.y.z,t) = \sqrt{2}\cos[6\pi\times 10^8 t - \sqrt{2}\pi(x+y) + \frac{\pi}{4}]\,\hat{a}_z$

$\vec{B}(x.y.z,t) = 3.33\times 10^{-9} \cos[6\pi\times 10^8 t - \sqrt{2}\pi(x+y) + \frac{\pi}{4}]\,(\hat{a}_x - \hat{a}_y)$

At $(x.y.z) = (10, 1, 5)$

$\vec{E} = \sqrt{2}\cos(6\pi\times 10^8 t - 15.306\pi)\,\hat{a}_z \ (\frac{V}{m})$

$\vec{B} = 3.33\times 10^{-9} \cos(6\pi\times 10^8 t - 15.306\pi)(\hat{a}_x - \hat{a}_y)\ (T)$