Formula sheet

$$E_{(x,y,z)} = E_0 \frac{\omega_0}{\omega(z)} e^{-\frac{\gamma^2}{\omega^2(z)}} \cdot e^{-j\frac{k\gamma^2}{2R(z)}} \cdot e^{-j(kz - tan^{-1}(\frac{z}{z_0}))}$$

$$\omega^2(z) = \omega_0^2 \left(1 + \left(\frac{\lambda_0 z}{\pi n \omega_0^2}\right)^2\right) = \omega_0^2 (1 + (\frac{z}{z_0})^2) \qquad z_0 = \frac{n\pi\omega_0^2}{\lambda_0}$$

$$\mathrm{R(z)} = \mathrm{z}\left(1 + \left(\frac{\pi n \omega_0^2}{\lambda_0 z}\right)^2\right) = \mathrm{z}(1 + (\frac{z_0}{z})^2) \qquad \text{divergence angle } \theta = \frac{2\omega_0}{z_0}$$

$$E_{lm(x,y,z)} = E_0 \frac{\omega_0}{\omega(z)} e^{-\frac{\gamma^2}{\omega^2(z)}} H_l(\frac{\sqrt{2}x}{\omega(z)}) H_m(\frac{\sqrt{2}y}{\omega(z)}) \cdot e^{-j\frac{k\gamma^2}{2R(z)}} \cdot e^{-j(kz - (1+l+m)tan^{-1}(\frac{z}{z_0}))}$$

$$H_m(u) = (-1)^m e^{u^2} \frac{d^m}{du^m} e^{-u^2} \qquad \mathrm{FSR} = \frac{c}{2nd} \qquad \mathrm{T} = \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}sin^2kd}$$

$$\Delta v_{1/2} = \frac{c}{2nd} \cdot \frac{1-\sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}} \qquad \mathrm{Q} = \frac{v_0}{\Delta v_{1/2}} = \frac{\omega_0}{\Delta\omega_{1/2}} = \frac{\lambda_0}{\Delta\lambda_{1/2}} \qquad \mathrm{F} = \frac{FSR}{\Delta v_{1/2}}$$

$$\frac{dN_2}{dt} = -A_{21}N_2 \qquad \frac{dN_2}{dt} = B_{12}\rho N_1 \qquad \gamma = \sigma(N_2 - N_1) \qquad \frac{N_2}{N_1} = e^{-\frac{h\nu}{kT}}$$

$$\sigma = \frac{h\nu}{c/n} B_{21} \mathrm{g}(\nu) = \frac{h\nu}{c/n} B_{12} \mathrm{g}(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} \mathrm{g}(\nu) \qquad \mathrm{g}(\nu) = \frac{1}{2\pi} \cdot \frac{\Delta\nu}{(\nu - \nu_0)^2 + \left(\frac{\Delta\nu}{2}\right)^2}$$

$$\mathrm{g}(\nu) = \sqrt{\frac{4ln2}{\pi}} \frac{1}{\Delta v_D} e^{-4ln2(\frac{\nu - \nu_0}{\Delta v_D})^2}, \text{where } \Delta v_D = \sqrt{\frac{8K_BTln2}{Mc^2}} v_0$$

$$\gamma_{th} = \frac{1}{2l} ln \frac{1}{R_1R_2}$$