Slab Waveguides Fundamentals

Integrated Optics

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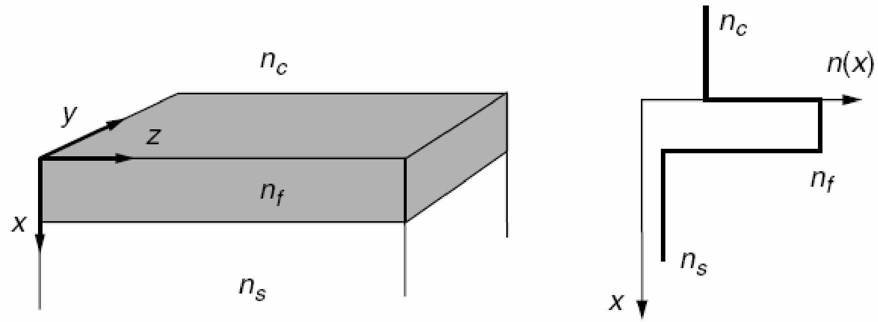
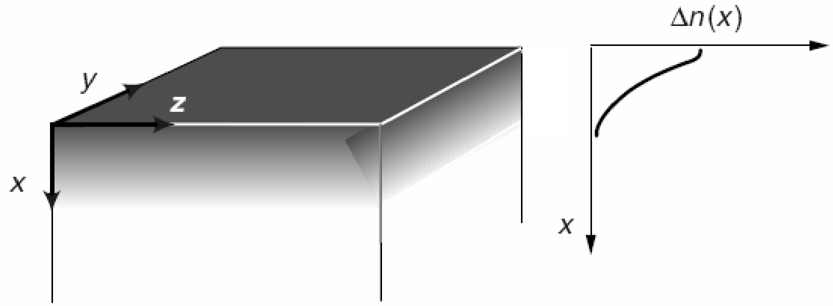
SLAB WAVEGUIDES



Step-index

Figure 3.4 Asymmetric step index planar waveguide. Right: refractive index profile, where $n_f < n_s \leq n_c$

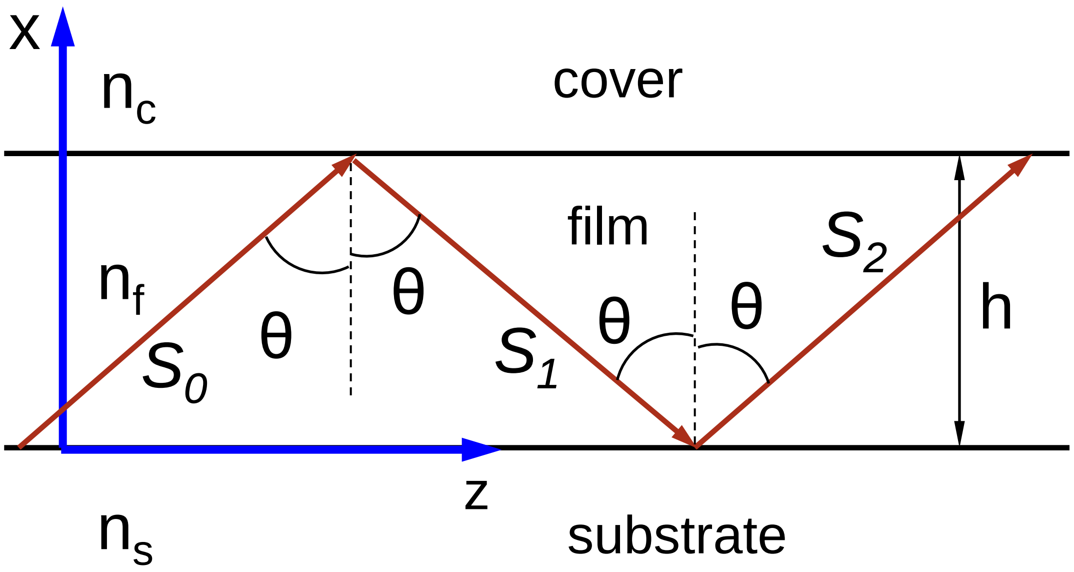


Graded-index

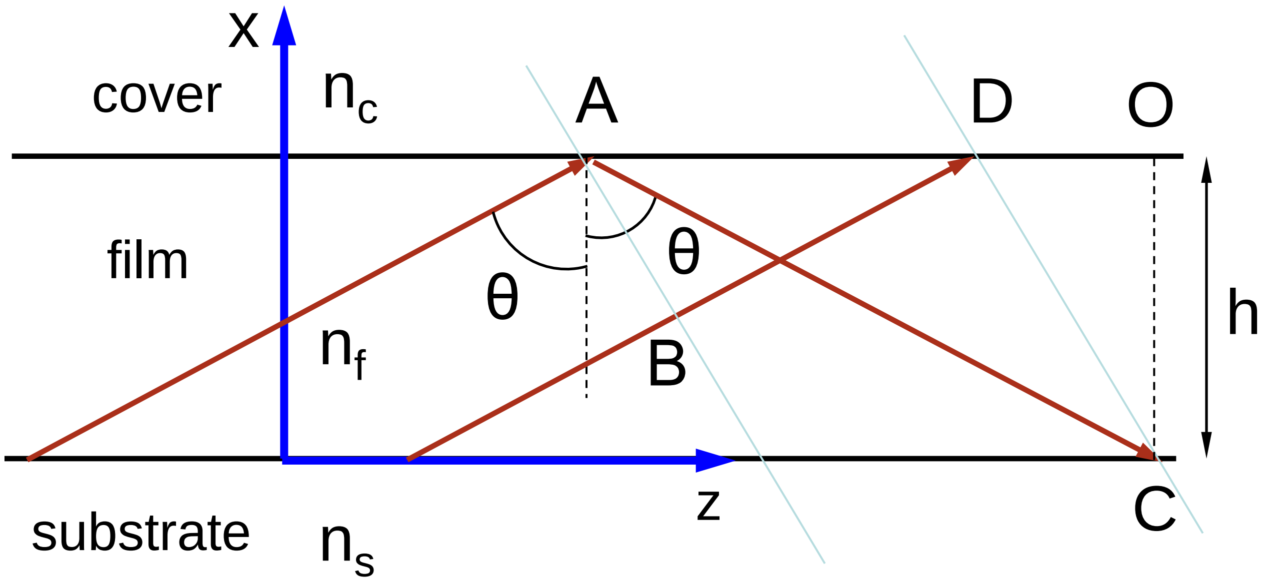
Figure 3.5 Graded index planar waveguide

G. Lifante, Integrated Photonics Fundamentals, Wiley 2003

SLAB WAVEGUIDE GEOMETRY

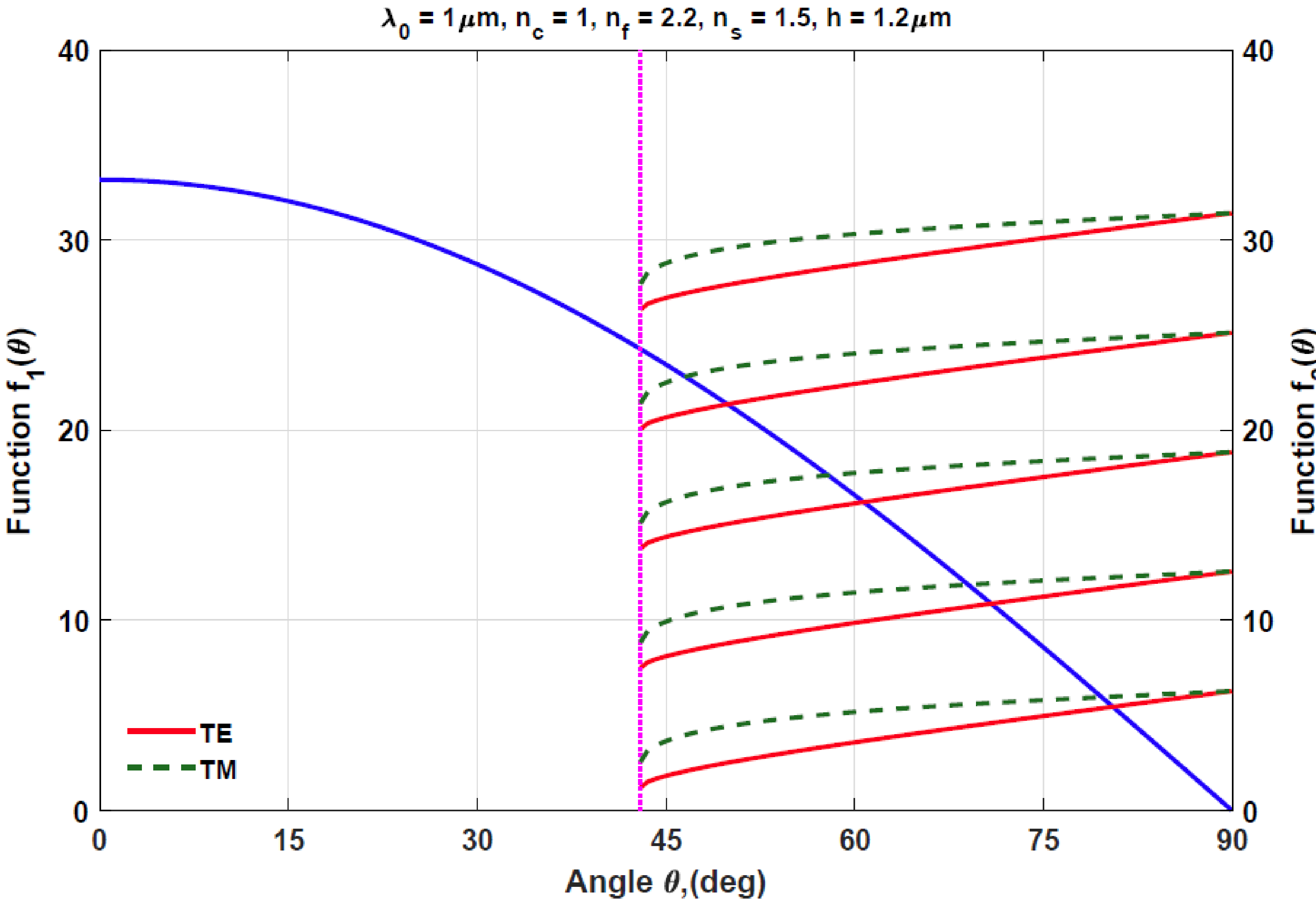


SLAB WAVEGUIDE SELF-CONSISTENCY CONDITION
(waveguiding condition)

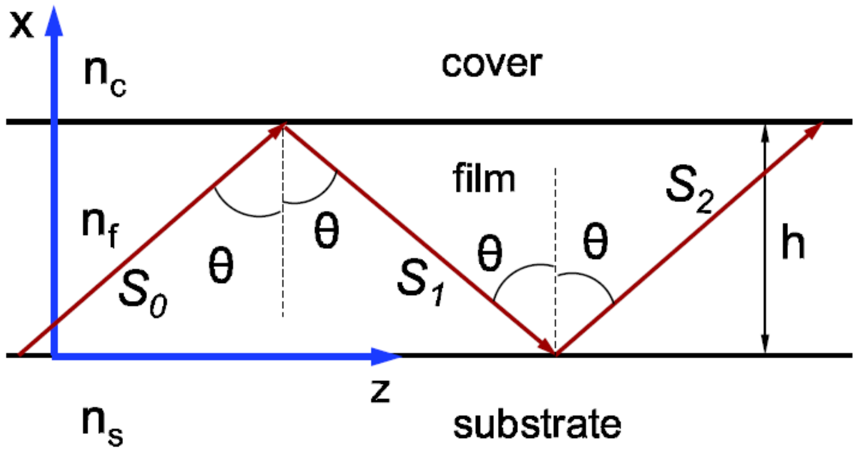


$$2k_0 n_f h \cos\theta - 2\phi^p_{fs}(\theta) - 2\phi^p_{fc}(\theta) = 2\pi\nu, \qquad \nu = 0, 1, 2, \cdots$$

SLAB WAVEGUIDE SELF-CONSISTENCY CONDITION
(graphical interpretation)



SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH



$$\vec{E} = [E_x(x)\hat{x} + E_y(x)\hat{y} + E_z(x)\hat{z}]\exp(-j\beta z),$$

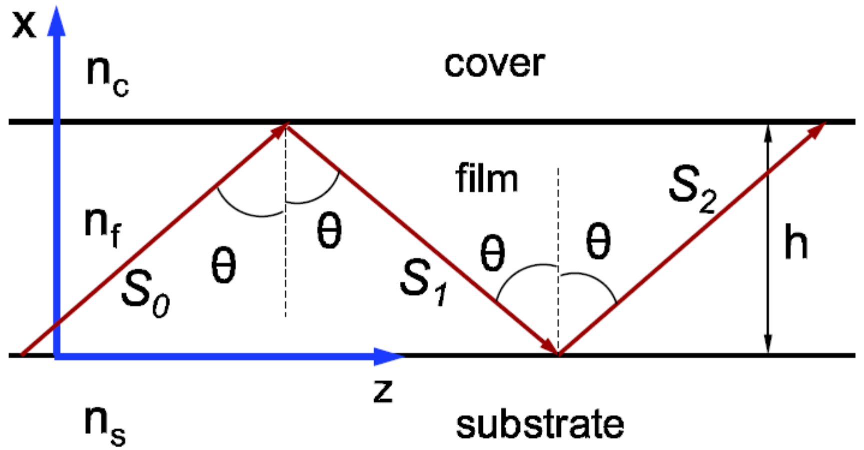
$$\vec{H} = [H_x(x)\hat{x} + H_y(x)\hat{y} + H_z(x)\hat{z}]\exp(-j\beta z),$$

Helmholtz Equation:

$$\frac{d^2\vec{U}}{dx^2} + (k_0^2 n^2 - \beta^2)\vec{U} = 0$$

$$\vec{U} = \vec{U}_+ \exp(-j\vec{k}_+ \cdot \vec{r}) + \vec{U}_- \exp(-j\vec{k}_- \cdot \vec{r})$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH



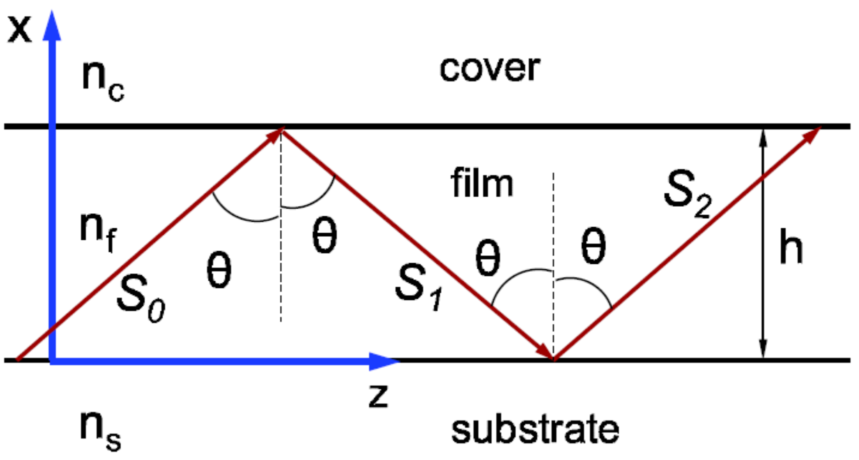
$$k_{cx}^2 = k_0^2 n_c^2 - \beta^2 < 0 \quad \Rightarrow \quad k_{cx} = \pm j\sqrt{\beta^2 - k_0^2 n_c^2} = \pm j\gamma_c,$$

$$k_{fx}^2 = k_0^2 n_f^2 - \beta^2 > 0 \quad \Rightarrow \quad k_{fx} = \pm\sqrt{k_0^2 n_f^2 - \beta^2},$$

$$k_{sx}^2 = k_0^2 n_s^2 - \beta^2 < 0 \quad \Rightarrow \quad k_{sx} = \pm j\sqrt{\beta^2 - k_0^2 n_s^2} = \pm j\gamma_s,$$

$$\vec{U} = \begin{cases} \vec{U}_c e^{-\gamma_c(x-h)} e^{-j\beta z}, & x > h, \\ \left[\vec{U}_{f1} e^{-jk_{fx}x} + \vec{U}_{f2} e^{+jk_{fx}x}\right] e^{-j\beta z}, & 0 < x < h, \\ \vec{U}_s e^{\gamma_s x} e^{-j\beta z}, & x < 0, \end{cases}$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH



$$\frac{d}{dx}\begin{bmatrix} E_y \\ H_z \\ \hdashline H_y \\ E_z \end{bmatrix} = \left[\begin{array}{cc:cc} 0 & -j\omega\mu_0 & 0 & 0 \\ -j\omega\epsilon + j\dfrac{\beta^2}{\omega\mu_0} & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & j\omega\epsilon \\ 0 & 0 & j\omega\mu_0 - j\dfrac{\beta^2}{\omega\epsilon} & 0 \end{array}\right] \begin{bmatrix} E_y \\ H_z \\ \hdashline H_y \\ E_z \end{bmatrix},$$

$$\begin{bmatrix} H_x \\ \hdashline E_x \end{bmatrix} = \left[\begin{array}{c:c} -\dfrac{\beta}{\omega\mu_0} & 0 \\ \hdashline 0 & \dfrac{\beta}{\omega\epsilon} \end{array}\right] \begin{bmatrix} E_y \\ \hdashline H_y \end{bmatrix}.$$

TE Modes

$$\{E_y, H_x, H_z\}$$

TM Modes

$$\{H_y, E_x, E_z\}$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

TE Modes

$$k_0 \max\{n_c, n_s\} < \beta < k_0 n_f$$

$$\vec{E} = \hat{y} \begin{cases} E_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x > h, \\ \left[E_{f1} e^{-jk_{fx}x} + E_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 < x < h, \\ E_s e^{\gamma_s x} e^{-j\beta z}, & x < 0, \end{cases}$$

$$H_x = -\frac{\beta}{\omega\mu_0} \begin{cases} E_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x \geq h, \\ \left[E_{f1} e^{-jk_{fx}x} + E_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 \leq x \leq h, \\ E_s e^{\gamma_s x} e^{-j\beta z}, & x \leq 0, \end{cases}$$

$$H_z = \frac{1}{\omega\mu_0} \begin{cases} -j\gamma_c E_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x > h, \\ \left[k_{fx} E_{f1} e^{-jk_{fx}x} - k_{fx} E_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 < x < h, \\ j\gamma_s E_s e^{\gamma_s x} e^{-j\beta z}, & x < 0. \end{cases}$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

TE Modes

Dispersion Equation

$$k_0 \max\{n_c, n_s\} < \beta < k_0 n_f$$

$$\underbrace{\begin{bmatrix} -1 & e^{-jk_{fx}h} & e^{+jk_{fx}h} & 0 \\ j\gamma_c & k_{fx}e^{-jk_{fx}h} & -k_{fx}e^{+jk_{fx}h} & 0 \\ 0 & 1 & 1 & -1 \\ 0 & k_{fx} & -k_{fx} & -j\gamma_s \end{bmatrix}}_{\tilde{A}_{TE}(\beta^2)} \begin{bmatrix} E_c \\ E_{f1} \\ E_{f2} \\ E_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det\{\tilde{A}_{TE}(\beta^2)\} = 0 \Longrightarrow \tan(k_{fx}h) = \frac{\dfrac{\gamma_s}{k_{fx}} + \dfrac{\gamma_c}{k_{fx}}}{1 - \dfrac{\gamma_s}{k_{fx}}\dfrac{\gamma_c}{k_{fx}}}.$$

$$\vec{E}_\nu = \hat{y}E_0 \begin{cases} \cos(k_{fx}h - \phi_{fs})\; e^{-\gamma_c(x-h)}\; e^{-j\beta_\nu z}, & x > h, \\ \cos(k_{fx}x - \phi_{fs})\; e^{-j\beta_\nu z}, & 0 < x < h, \\ \cos\phi_{fs}\; e^{\gamma_s x}\; e^{-j\beta_\nu z}, & x < 0, \end{cases}$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

TM Modes

$$\vec{H} = \hat{y} \begin{cases} H_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x > h, \\ \left[H_{f1} e^{-jk_{fx}x} + H_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 < x < h, \\ H_s e^{\gamma_s x} e^{-j\beta z}, & x < 0, \end{cases}$$

$$k_0 \max\{n_c, n_s\} < \beta < k_0 n_f$$

$$E_x = \frac{\beta}{\omega \epsilon_0} \begin{cases} \frac{1}{n_c^2} H_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x > h, \\ \frac{1}{n_f^2} \left[H_{f1} e^{-jk_{fx}x} + H_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 < x < h, \\ \frac{1}{n_s^2} H_s e^{\gamma_s x} e^{-j\beta z}, & x < 0, \end{cases}$$

$$E_z = \frac{1}{\omega \epsilon_0} \begin{cases} +j \frac{\gamma_c}{n_c^2} H_c e^{-\gamma_c (x-h)} e^{-j\beta z}, & x > h, \\ \left[\frac{k_{fx}}{n_f^2} H_{f1} e^{-jk_{fx}x} - \frac{k_{fx}}{n_f^2} H_{f2} e^{+jk_{fx}x} \right] e^{-j\beta z}, & 0 < x < h, \\ -j \frac{\gamma_s}{n_s^2} H_s e^{\gamma_s x} e^{-j\beta z}, & x < 0. \end{cases}$$

SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

TM Modes

Dispersion Equation

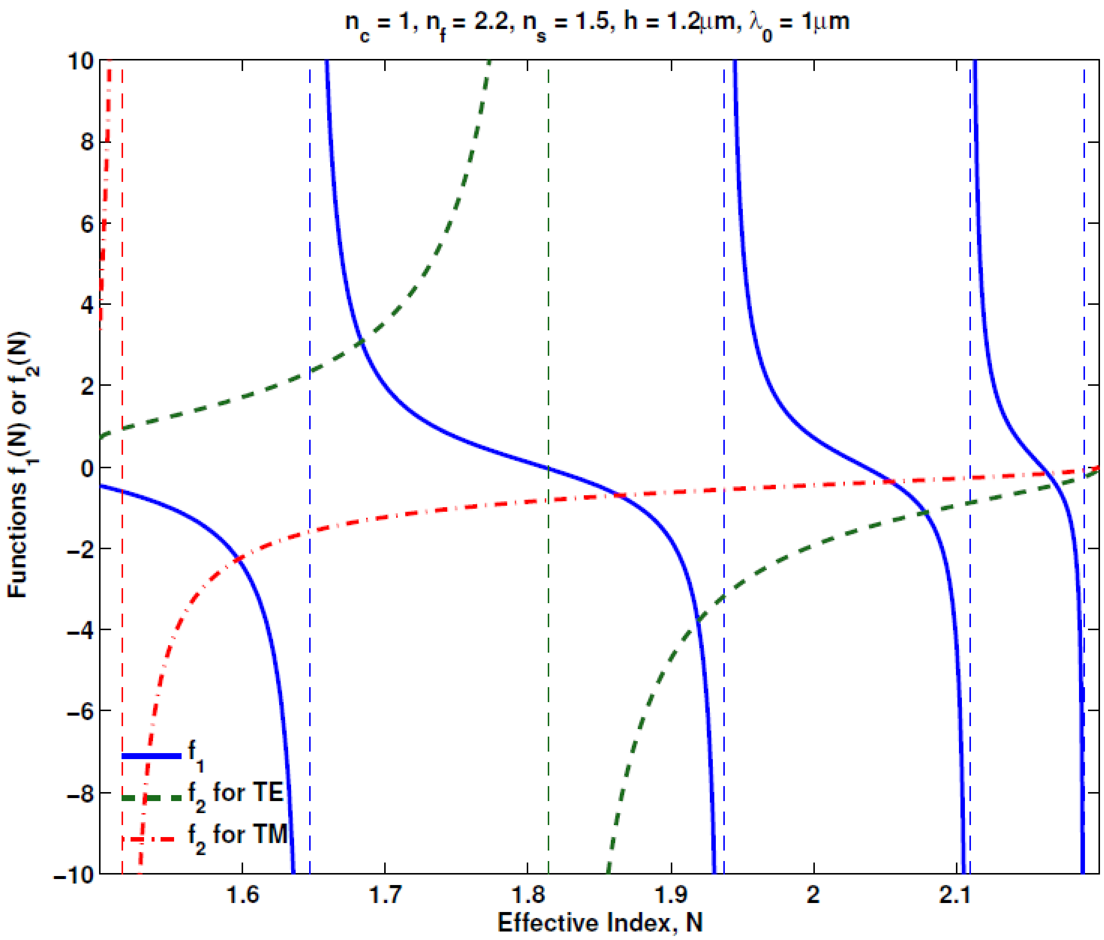
$$k_0 \max\{n_c, n_s\} < \beta < k_0 n_f$$

$$\underbrace{\begin{bmatrix} -1 & e^{-jk_{fx}h} & e^{+jk_{fx}h} & 0 \\ -j\dfrac{\gamma_c}{n_c^2} & -\dfrac{k_{fx}}{n_f^2}e^{-jk_{fx}h} & \dfrac{k_{fx}}{n_f^2}e^{+jk_{fx}h} & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -\dfrac{k_{fx}}{n_f^2} & \dfrac{k_{fx}}{n_f^2} & j\dfrac{\gamma_s}{n_s^2} \end{bmatrix}}_{\tilde{\mathcal{A}}_{TM}(\beta^2)} \begin{bmatrix} H_c \\ H_{f1} \\ H_{f2} \\ H_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det\{\tilde{\mathcal{A}}_{TM}(\beta^2)\} = 0 \Longrightarrow \tan(k_{fx}h) = \frac{\dfrac{n_f^2}{n_s^2}\dfrac{\gamma_s}{k_{fx}} + \dfrac{n_f^2}{n_c^2}\dfrac{\gamma_c}{k_{fx}}}{1 - \dfrac{n_f^4}{n_s^2 n_c^2}\dfrac{\gamma_c \gamma_s}{k_{fx}^2}}.$$

$$\vec{H}_\nu = \hat{y} H_0 \begin{cases} \cos(k_{fx}h - \phi_{fs})\; e^{-\gamma_c(x-h)}\; e^{-j\beta_\nu z}, & x > h, \\ \cos(k_{fx}x - \phi_{fs})\; e^{-j\beta_\nu z}, & 0 < x < h, \\ \cos\phi_{fs}\; e^{\gamma_s x}\; e^{-j\beta_\nu z}, & x < 0, \end{cases}$$

TE, TM Modes Graphical Solution

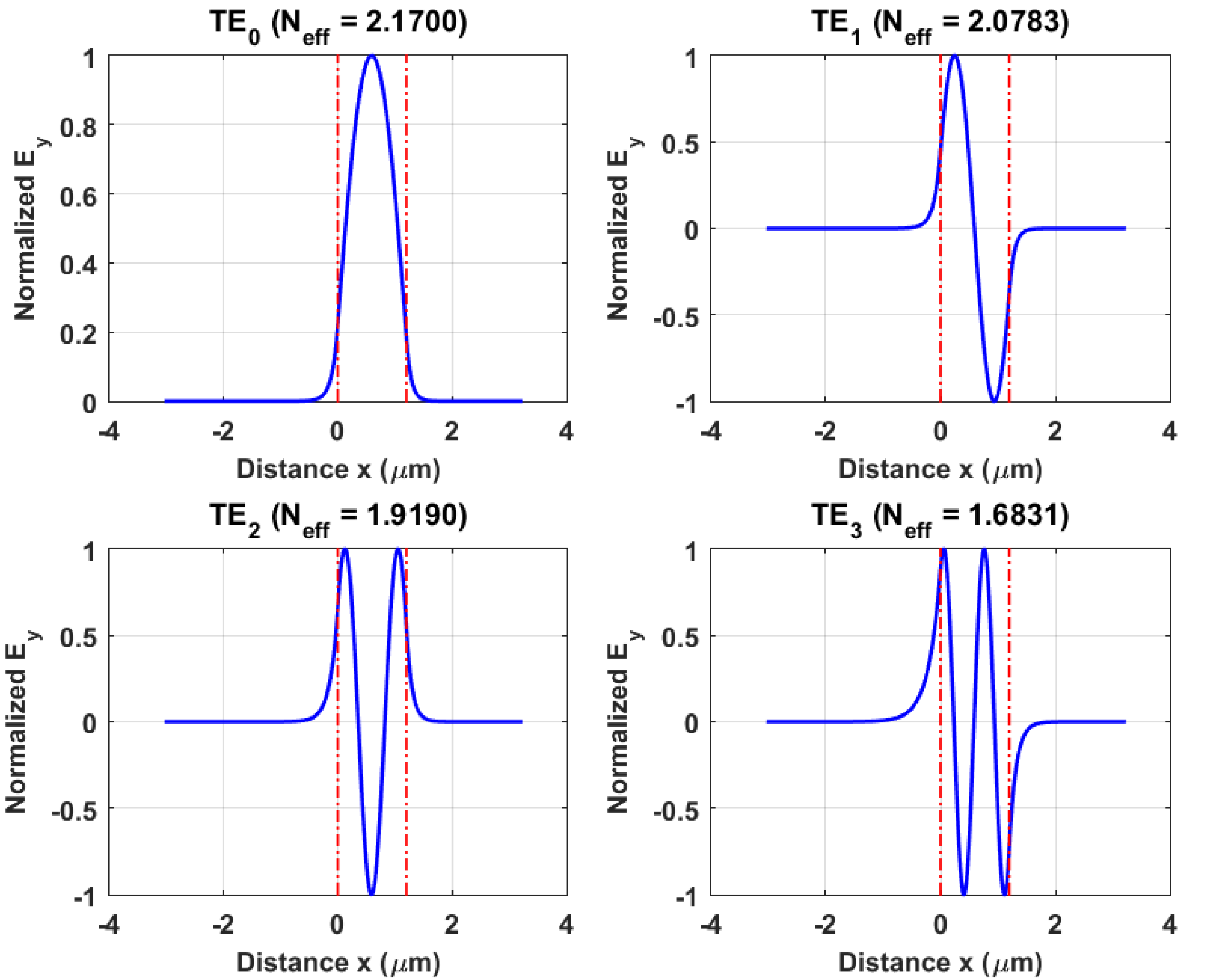


N_{eff} for TE Modes: 2.1700, 2.0783, 1.9190, 1.6831

N_{eff} for TM Modes: 2.1642, 2.0542, 1.8636, 1.5968

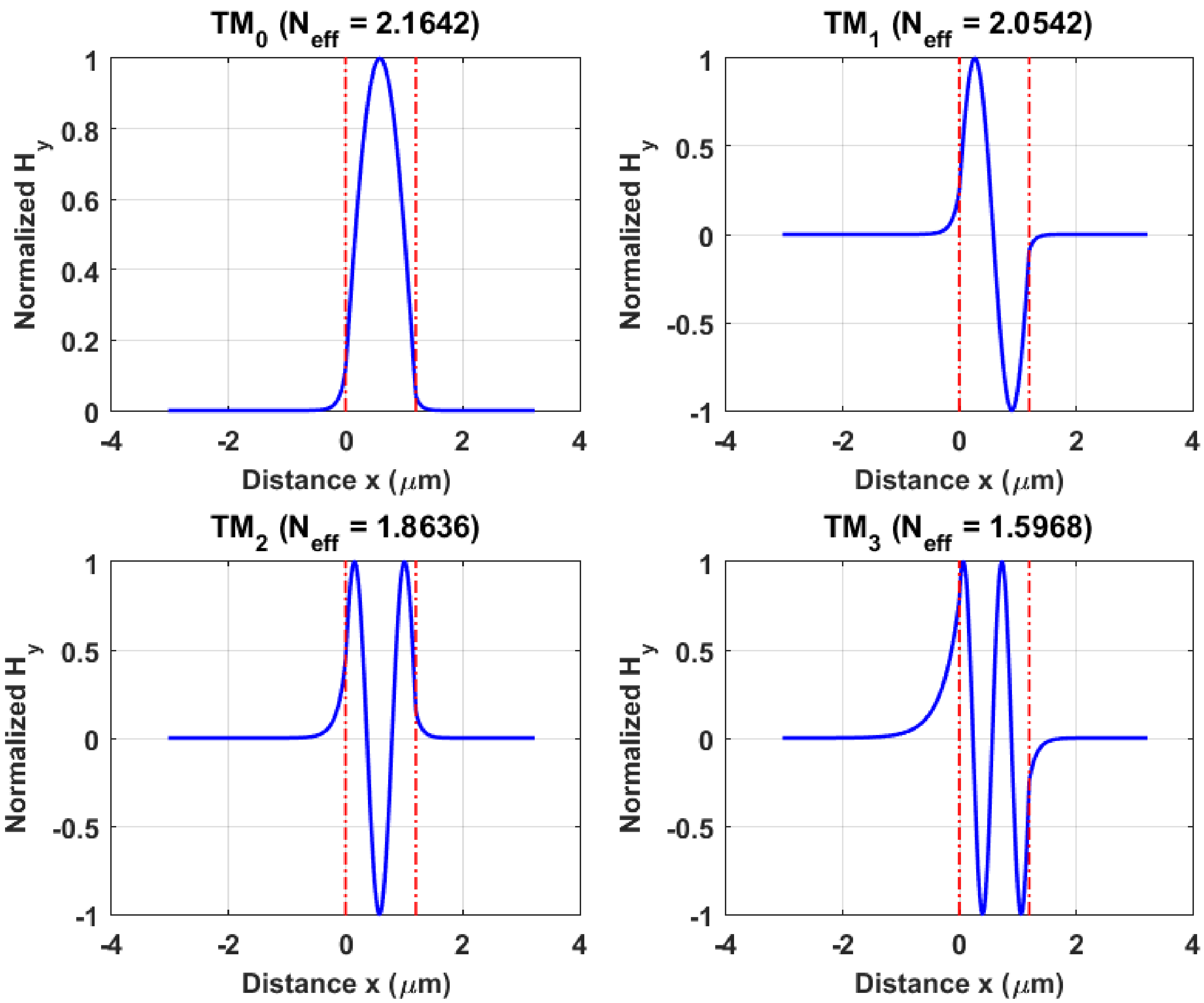
SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

TE Modes Example (n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm

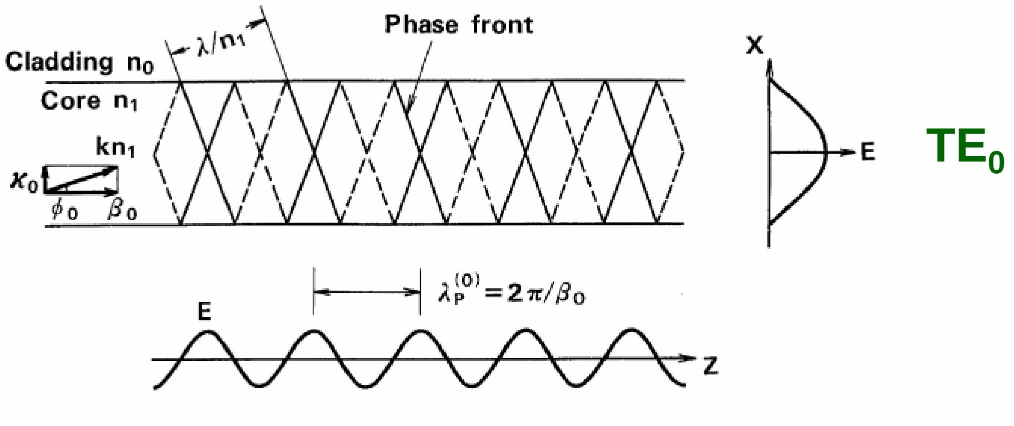


SLAB WAVEGUIDE ELECTROMAGNETIC APPROACH

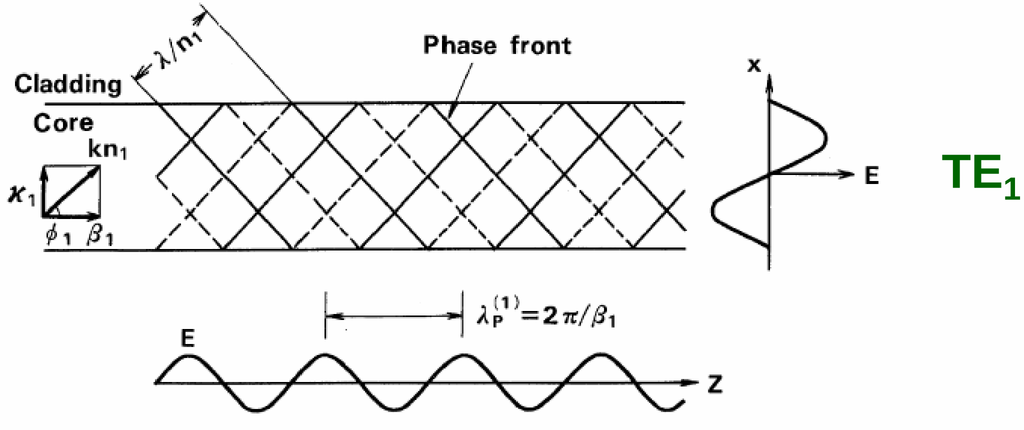
TM Modes Example (n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm



SLAB WAVEGUIDES MODES



(a) Fundamental mode (m=0)



(b) Higher-order mode (m=1)

Figure 1.4 Formation of modes: (a) Fundamental mode, (b) higher-order mode.

SLAB WAVEGUIDES MODES

TE_0

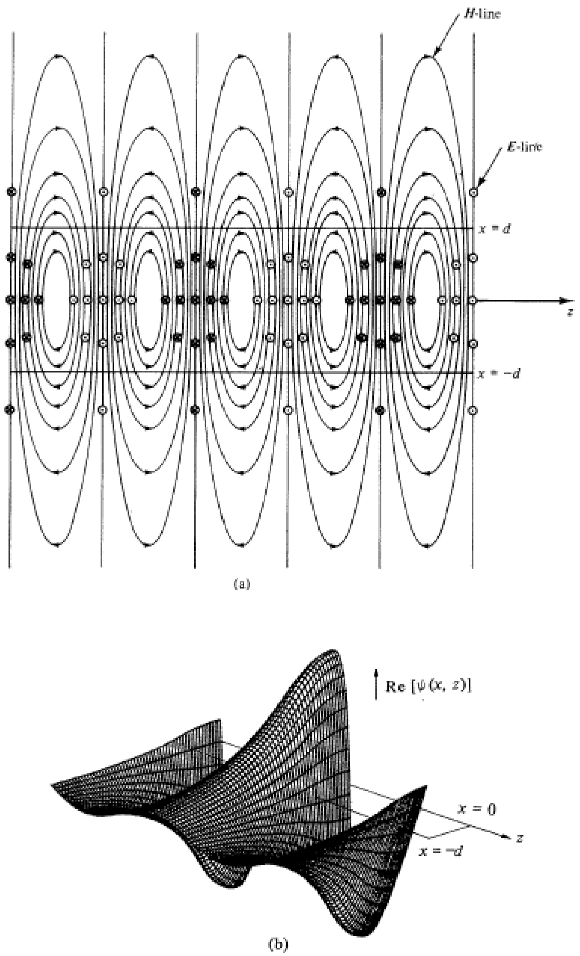


Figure 6.6 (a) Dominant TE mode ($m = 0$) of dielectric slab. With $\varepsilon = \varepsilon_0 n^2$, the wavelength λ for which graph is constructed is given by $nd/\lambda = 0.37$. (b) "Three-dimensional" plot of potential Re [$\psi(x, z)$] for dominant TE mode.

TE_1

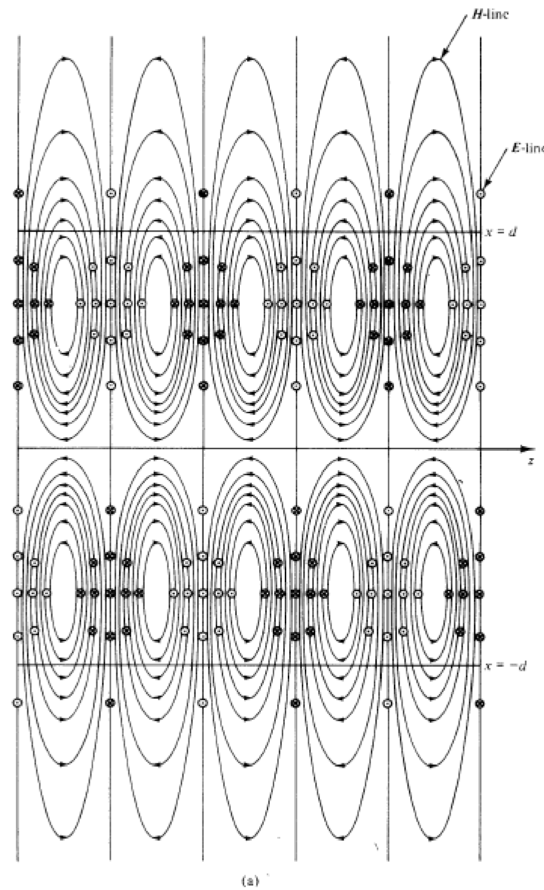


Figure 6.7 (a) First antisymmetric TE mode ($m = 1$) of dielectric slab. With $\varepsilon = \varepsilon_0 n^2$; the wavelength λ for which graph is constructed is given by $nd/\lambda = 1.4$. (b) "Three-dimensional" plot of potential Re [$\psi(x, z)$] for lowest-order antisymmetric TE mode.

H.A. Haus, Waves and Fields in Optoelectronics, Prentice-Hall 1984

SLAB WAVEGUIDES MODES

TM_0

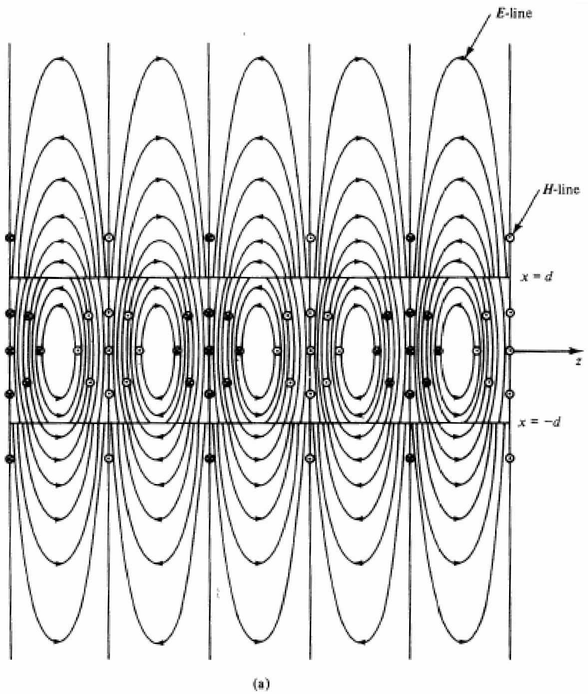


Figure 6.8 (a) Lowest-order TM mode of dielectric slab ($m = 0$). With $\varepsilon = \varepsilon_0 n^2$, the wavelength λ for which graph is constructed is given by $nd/\lambda = 0.35$. The transverse propagation constant k_x is the same as that of Fig. 6.6a. (b) "Three-dimensional" plot of potential Re $[\Xi(x, t)]$ for lowest-order TM mode.

H.A. Haus, Waves and Fields in Optoelectronics, Prentice-Hall 1984

TM_1

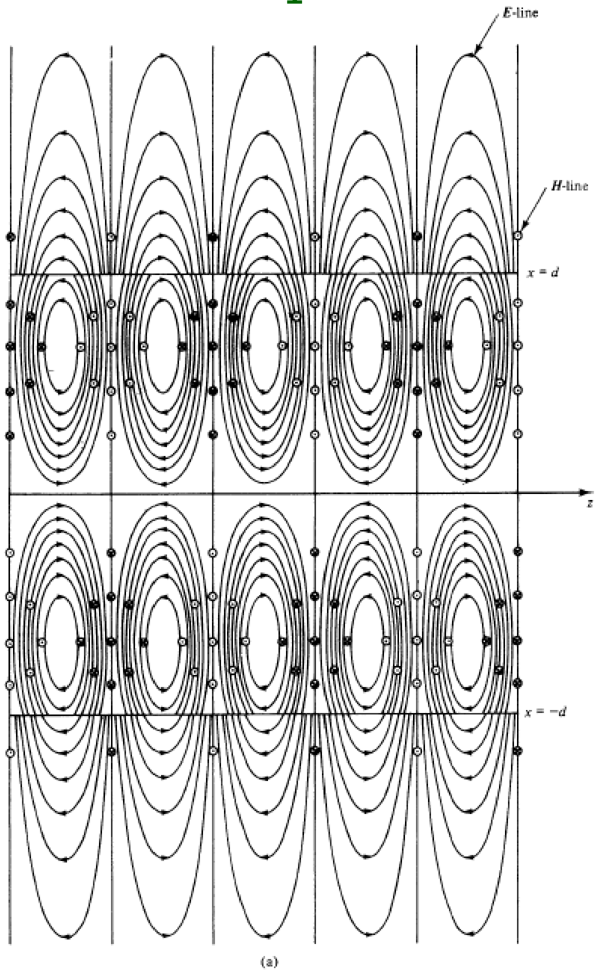


Figure 6.9 (a) First antisymmetric TM modes of dielectric slab. With $\varepsilon = \varepsilon_0 n^2$, the wavelength λ for which graph is constructed is given by $nd/\lambda = 1.18$. The transverse propagation constant k_x is the same as that of Fig. 6.7a. (b) "Three-dimensional" plot of potential Re $[\Xi(x, z)]$ for first antisymmetric TM mode.

SLAB WAVEGUIDES SUBSTRATE MODES

$$k_0 n_c < \beta < k_0 n_s$$

TE Substrate Modes

$$\vec{E}_\beta = \hat{y} E_0 \begin{cases} \cos\phi_{fc}^{TE}\ e^{-\gamma_c(x-h)}\ e^{-j\beta z}, & x > h, \\ \cos[k_{fx}(x-h) + \phi_{fc}^{TE}]\ e^{-j\beta z}, & 0 < x < h, \\ \left[\cos(k_{fx}h - \phi_{fc}^{TE})\cos(k_{sx}x) + \dfrac{k_{fx}}{k_{sx}}\sin(k_{fx}h - \phi_{fc}^{TE})\sin(k_{sx}x)\right]\ e^{-j\beta z}, & x < 0, \end{cases}$$

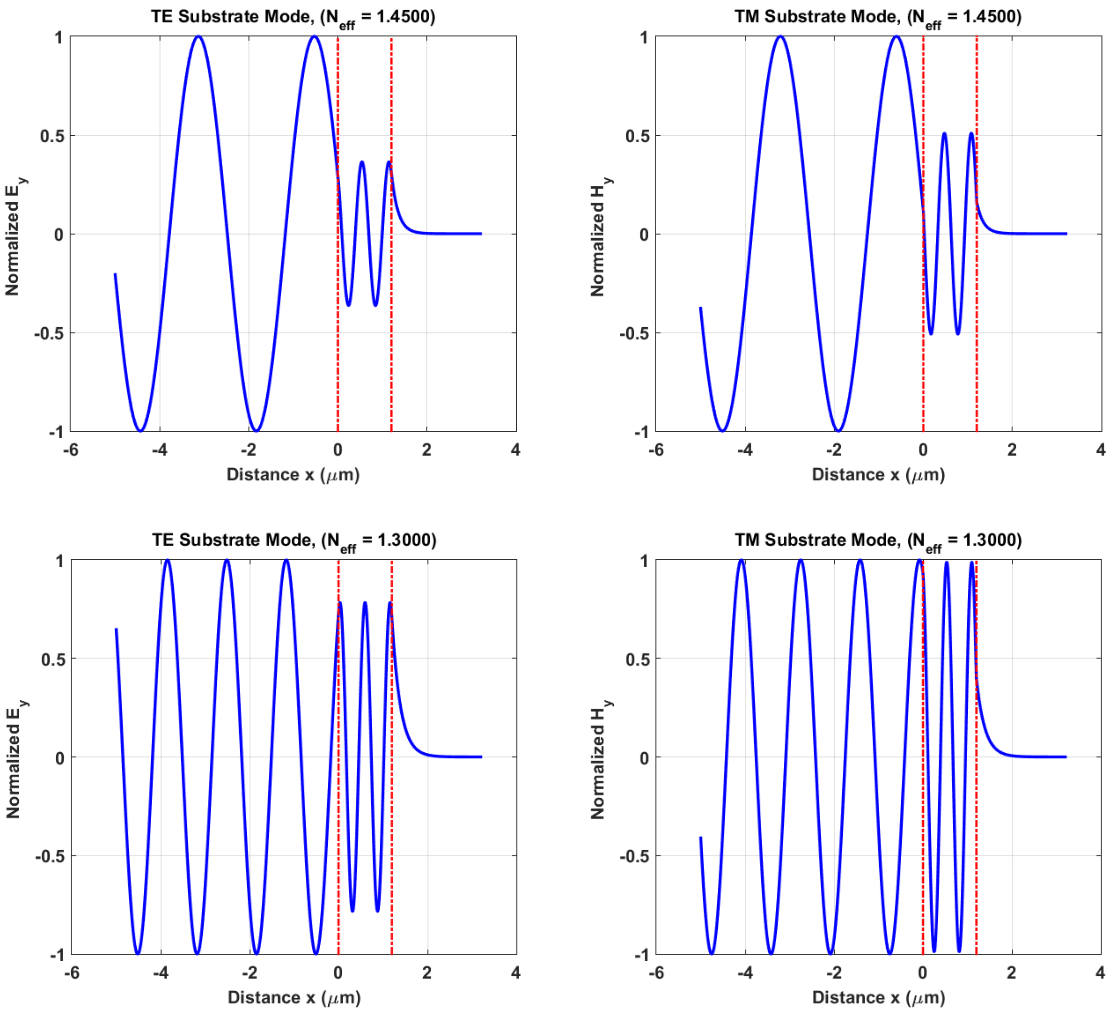
TM Substrate Modes

$$\vec{H}_\beta = \hat{y} H_0 \begin{cases} \cos\phi_{fc}^{TM}\ e^{-\gamma_c(x-h)}\ e^{-j\beta z}, & x > h, \\ \cos[k_{fx}(x-h) + \phi_{fc}^{TM}]\ e^{-j\beta z}, & 0 < x < h, \\ \left[\cos(k_{fx}h - \phi_{fc}^{TM})\cos(k_{sx}x) + \dfrac{k_{fx}/n_f^2}{k_{sx}/n_s^2}\sin(k_{fx}h - \phi_{fc}^{TM})\sin(k_{sx}x)\right]\ e^{-j\beta z}, & x < 0, \end{cases}$$

SLAB WAVEGUIDES SUBSTRATE MODES

TE/TM Substrate Modes Example (n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm

$$k_0 n_c < \beta < k_0 n_s$$



SLAB WAVEGUIDES RADIATION MODES

$$0 < \beta < k_0 n_c$$

TE Radiation Modes

$$\vec{E}_\beta = \hat{y}E_0 \begin{cases} \frac{1}{2}\Big[(1+\frac{k_{fx}}{k_{cx}})\cos[k_{cx}(x-h)+k_{fx}h-\phi] + \\ \qquad (1-\frac{k_{fx}}{k_{cx}})\cos[k_{cx}(x-h)-k_{fx}h+\phi]\Big]\ e^{-j\beta z}, & x>h, \\ \cos(k_{fx}x-\phi)\ e^{-j\beta z}, & 0<x<h, \\ \frac{1}{2}\Big[(1+\frac{k_{fx}}{k_{sx}})\cos[k_{sx}x-\phi]+(1-\frac{k_{fx}}{k_{sx}})\cos[k_{sx}x+\phi]\Big]\ e^{-j\beta z}, & x<0, \end{cases}$$

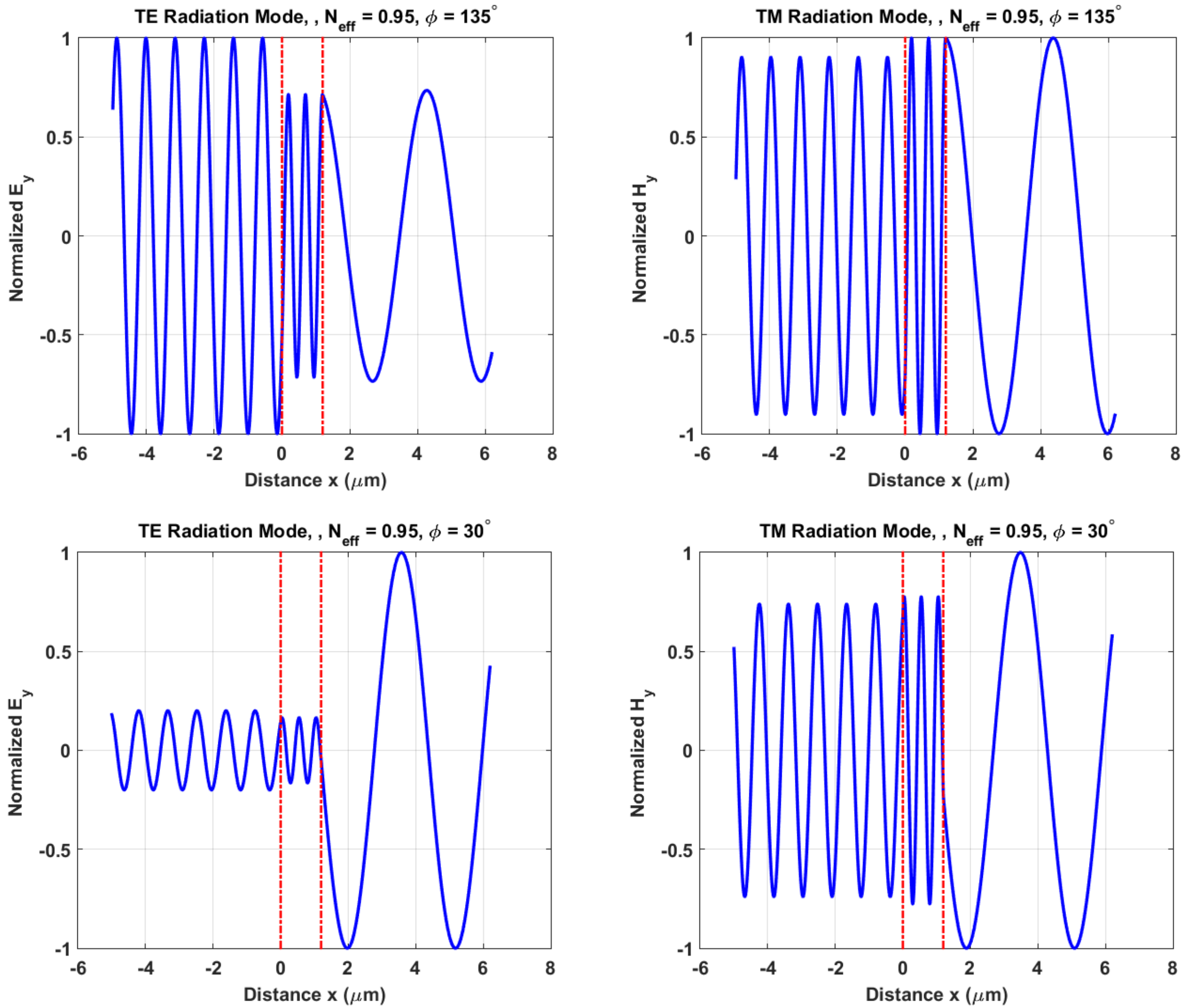
TM Radiation Modes

$$\vec{H}_\beta = \hat{y}E_0 \begin{cases} \frac{1}{2}\Big[(1+\frac{k_{fx}/n_f^2}{k_{cx}/n_c^2})\cos[k_{cx}(x-h)+k_{fx}h-\phi] + \\ \qquad (1-\frac{k_{fx}/n_f^2}{k_{cx}/n_c^2})\cos[k_{cx}(x-h)-k_{fx}h+\phi]\Big]\ e^{-j\beta z}, & x>h, \\ \cos(k_{fx}x-\phi)\ e^{-j\beta z}, & 0<x<h, \\ \frac{1}{2}\Big[(1+\frac{k_{fx}/n_f^2}{k_{sx}/n_s^2})\cos[k_{sx}x-\phi]+(1-\frac{k_{fx}/n_f^2}{k_{sx}/n_s^2})\cos[k_{sx}x+\phi]\Big]\ e^{-j\beta z}, & x<0, \end{cases}$$

SLAB WAVEGUIDES RADIATION MODES

TE/TM Radiation Modes Example (n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm

$$0 < \beta < k_0 n_c$$



SLAB WAVEGUIDES UNPHYSICAL MODES

$$k_0 n_f < \beta < \infty$$

TE Unphysical Modes

$$\vec{E}_\beta = \hat{y} E_0 \begin{cases} \frac{1}{2}\Big[(1+\frac{\gamma_f}{\gamma_c})\cosh[\gamma_c(x-h)+\gamma_f h-\phi] + \\ \qquad (1-\frac{\gamma_f}{\gamma_c})\cosh[\gamma_c(x-h)-\gamma_f h+\phi]\Big]\ e^{-j\beta z}, & x>h, \\ \cosh(\gamma_f x-\phi)\ e^{-j\beta z}, & 0<x<h, \\ \frac{1}{2}\Big[(1+\frac{\gamma_f}{\gamma_s})\cosh[\gamma_s x-\phi] + (1-\frac{\gamma_f}{\gamma_s})\cosh[\gamma_s x+\phi]\Big]\ e^{-j\beta z}, & x<0, \end{cases}$$

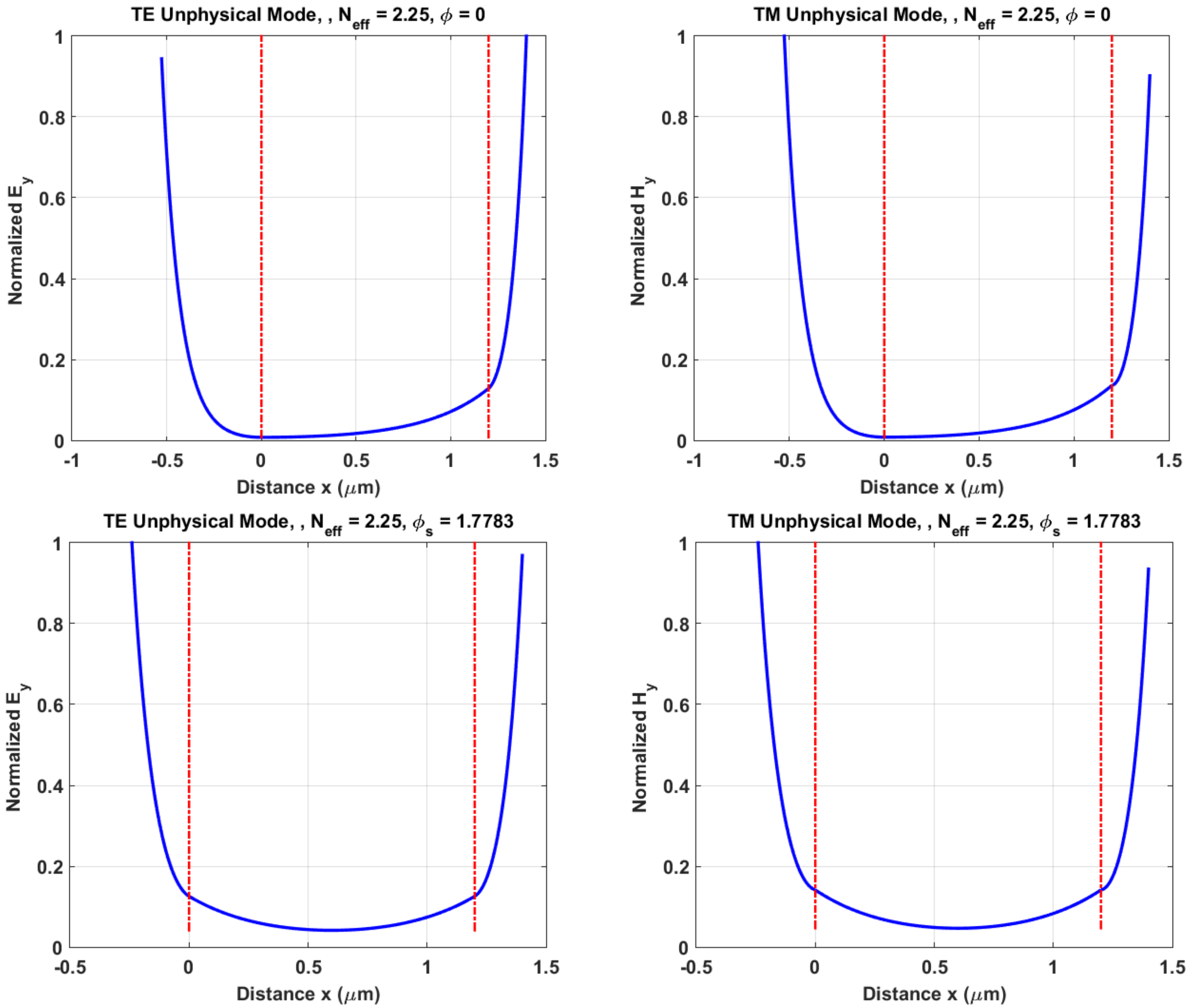
TM Unphysical Modes

$$\vec{H}_\beta = \hat{y} E_0 \begin{cases} \frac{1}{2}\Big[(1+\frac{\gamma_f/n_f^2}{\gamma_c/n_c^2})\cosh[\gamma_c(x-h)+\gamma_f h-\phi] + \\ \qquad (1-\frac{\gamma_f/n_f^2}{\gamma_c/n_c^2})\cosh[\gamma_c(x-h)-\gamma_f h+\phi]\Big]\ e^{-j\beta z}, & x>h, \\ \cosh(\gamma_f x-\phi)\ e^{-j\beta z}, & 0<x<h, \\ \frac{1}{2}\Big[(1+\frac{\gamma_f/n_f^2}{\gamma_s/n_s^2})\cosh[\gamma_s x-\phi] + (1-\frac{\gamma_f/n_f^2}{\gamma_s/n_s^2})\cosh[\gamma_s x+\phi]\Big]\ e^{-j\beta z}, & x<0, \end{cases}$$

SLAB WAVEGUIDES UNPHYSICAL MODES

TE/TM Radiation Modes Example (n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm

$$k_0 n_f < \beta < \infty$$



SLAB WAVEGUIDES MODE CLASSIFICATION

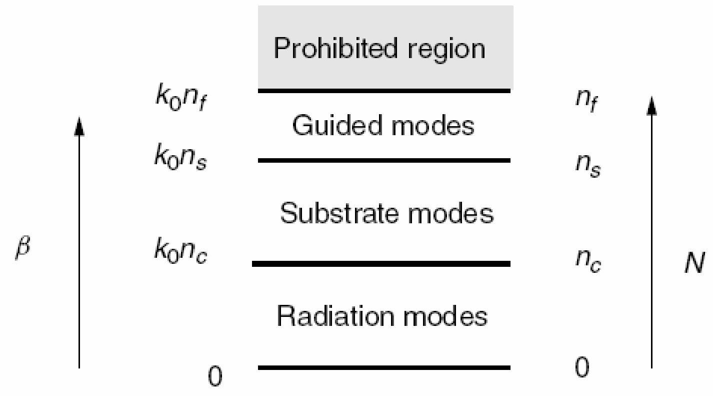


Figure 3.18 Range of values for the propagation constant β and the effective refractive index N for guided modes, substrate modes and radiation modes

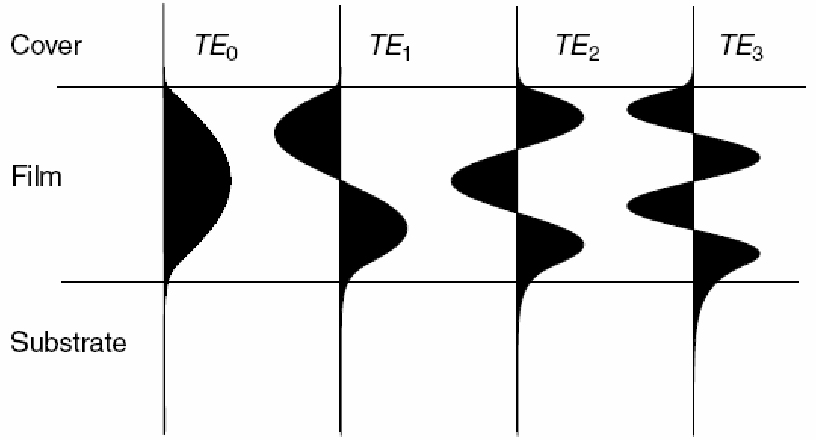


Figure 3.20 TE modes in an asymmetric step-index planar waveguide. The structure parameters are the following: $n_c = 1.00$, $n_f = 1.50$, $n_s = 1.43$, $d = 3.0$ µm, $\lambda = 633$ nm

G. Lifante, Integrated Photonics Fundamentals, Wiley 2003

SLAB WAVEGUIDES RAY PICTURE

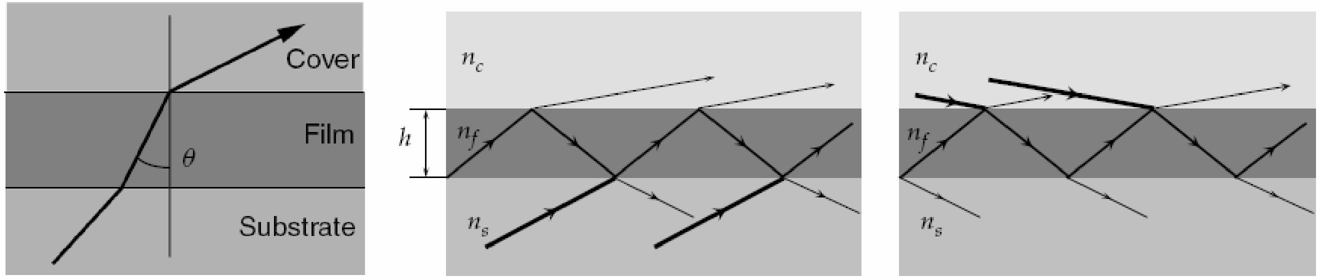


Figure 3.10 Radiation mode in an asymmetric step-index planar waveguide

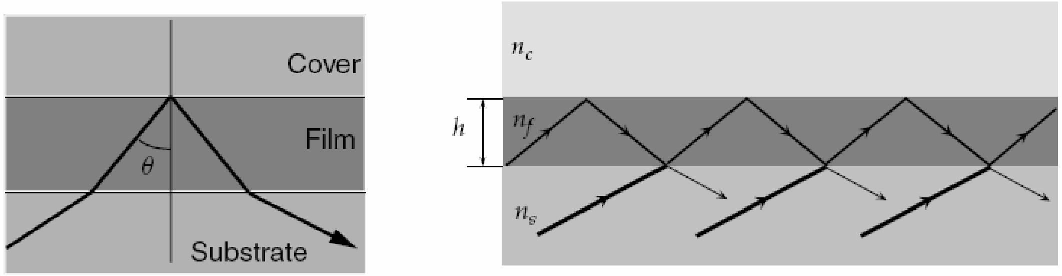


Figure 3.11 Ray path followed by a substrate radiation mode

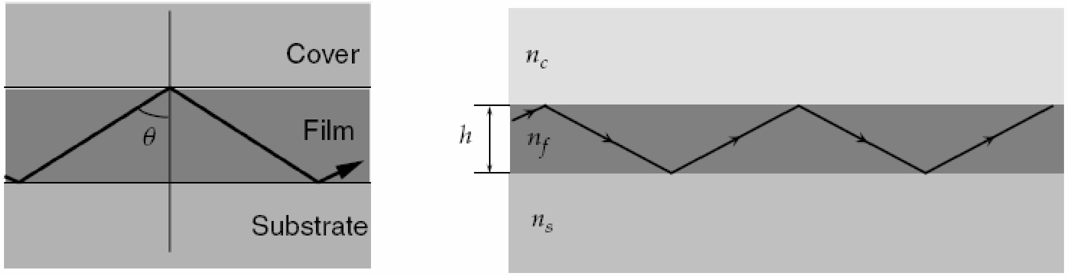


Figure 3.12 Guided mode in an asymmetric planar waveguide, showing the zig-zag path traced by the ray

G. Lifante, Integrated Photonics Fundamentals, Wiley 2003

SLAB WAVEGUIDES SUBSTRATE AND RADIATION MODES

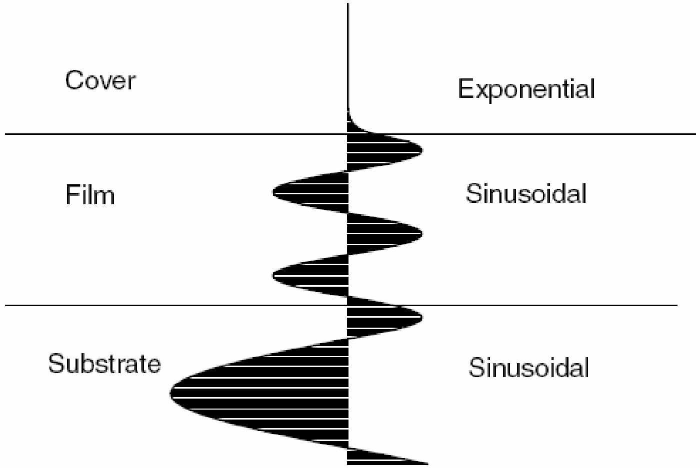


Figure 3.22 Substrate radiation mode in an asymmetric step-index planar waveguide

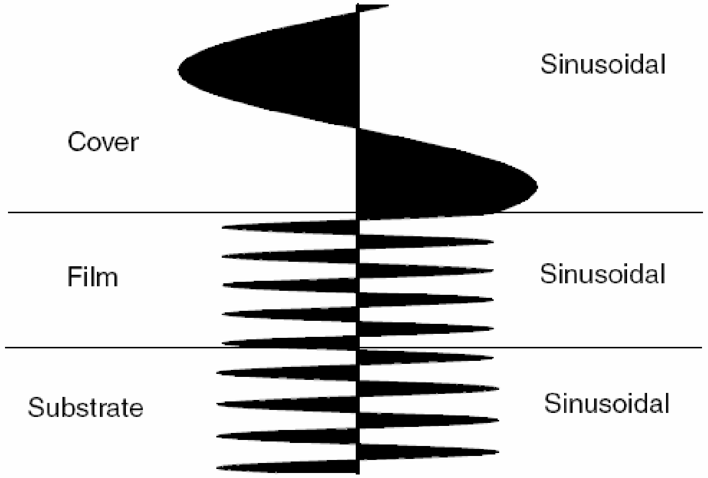


Figure 3.23 Radiation mode in an asymmetric step-index planar waveguide

G. Lifante, Integrated Photonics Fundamentals, Wiley 2003

SLAB WAVEGUIDES EVANESCENT MODES

$$\tilde{\beta} = \pm j\beta \text{ (where } \beta > 0)$$

$$k'_{cx} = (k_0^2 n_c^2 + \beta^2)^{1/2} > 0,\ k'_{fx} = (k_0^2 n_f^2 + \beta^2)^{1/2} > 0, \text{ and } k'_{sx} = (k_0^2 n_s^2 + \beta^2)^{1/2} > 0,$$

TE Forward Evanescent Modes

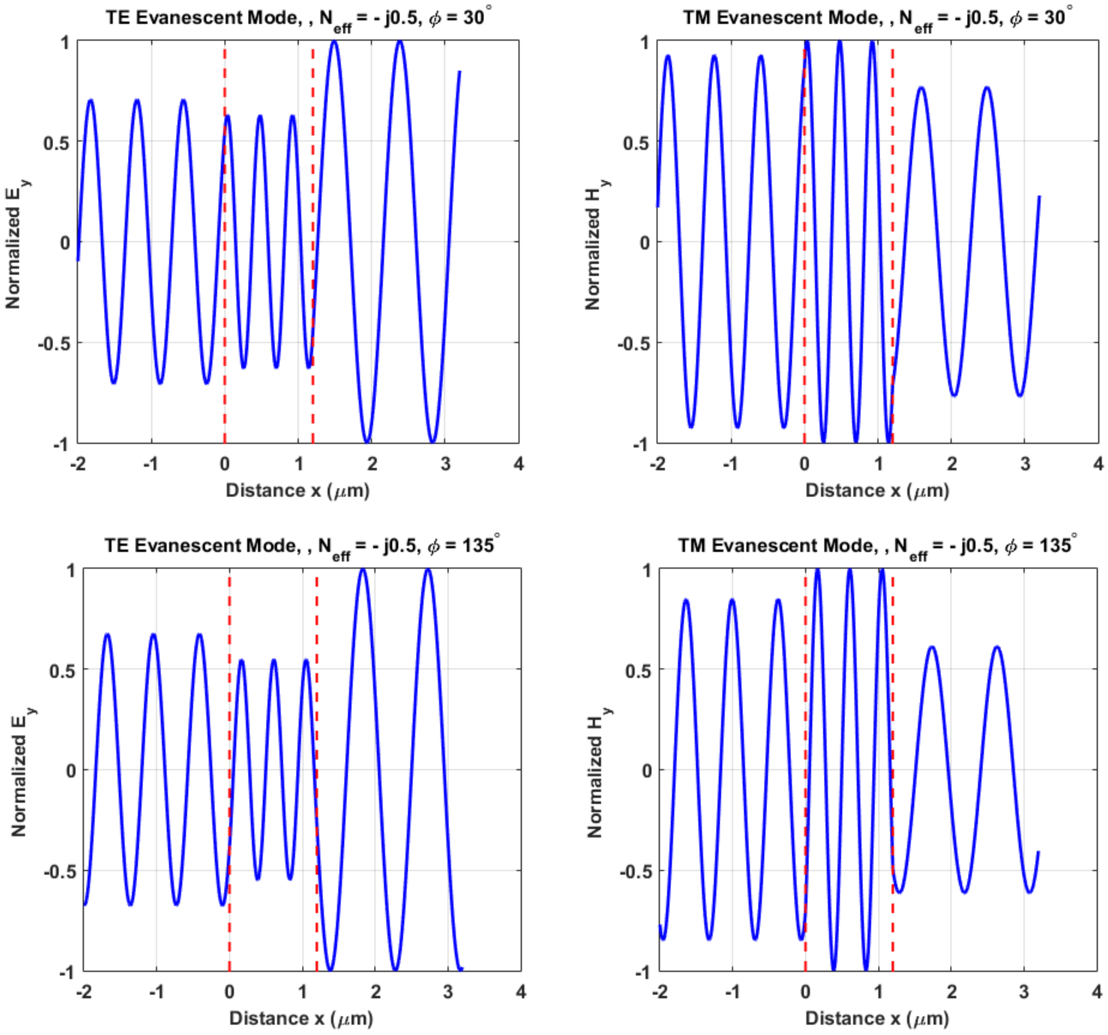
$$\vec{E}_\beta = \hat{y}E_0 \begin{cases} \frac{1}{2}\Big[(1 + \frac{k'_{fx}}{k'_{cx}})\cos[k'_{cx}(x-h) + k'_{fx}h - \phi] + \\ \qquad (1 - \frac{k'_{fx}}{k'_{cx}})\cos[k'_{cx}(x-h) - k'_{fx}h + \phi]\Big]\ e^{-\beta z}, & x > h, \\ \cos(k'_{fx}x - \phi)\ e^{-\beta z}, & 0 < x < h, \\ \frac{1}{2}\Big[(1 + \frac{k'_{fx}}{k'_{sx}})\cos[k'_{sx}x - \phi] + (1 - \frac{k'_{fx}}{k'_{sx}})\cos[k'_{sx}x + \phi]\Big]\ e^{-\beta z}, & x < 0, \end{cases}$$

TM Forward Evanescent Modes

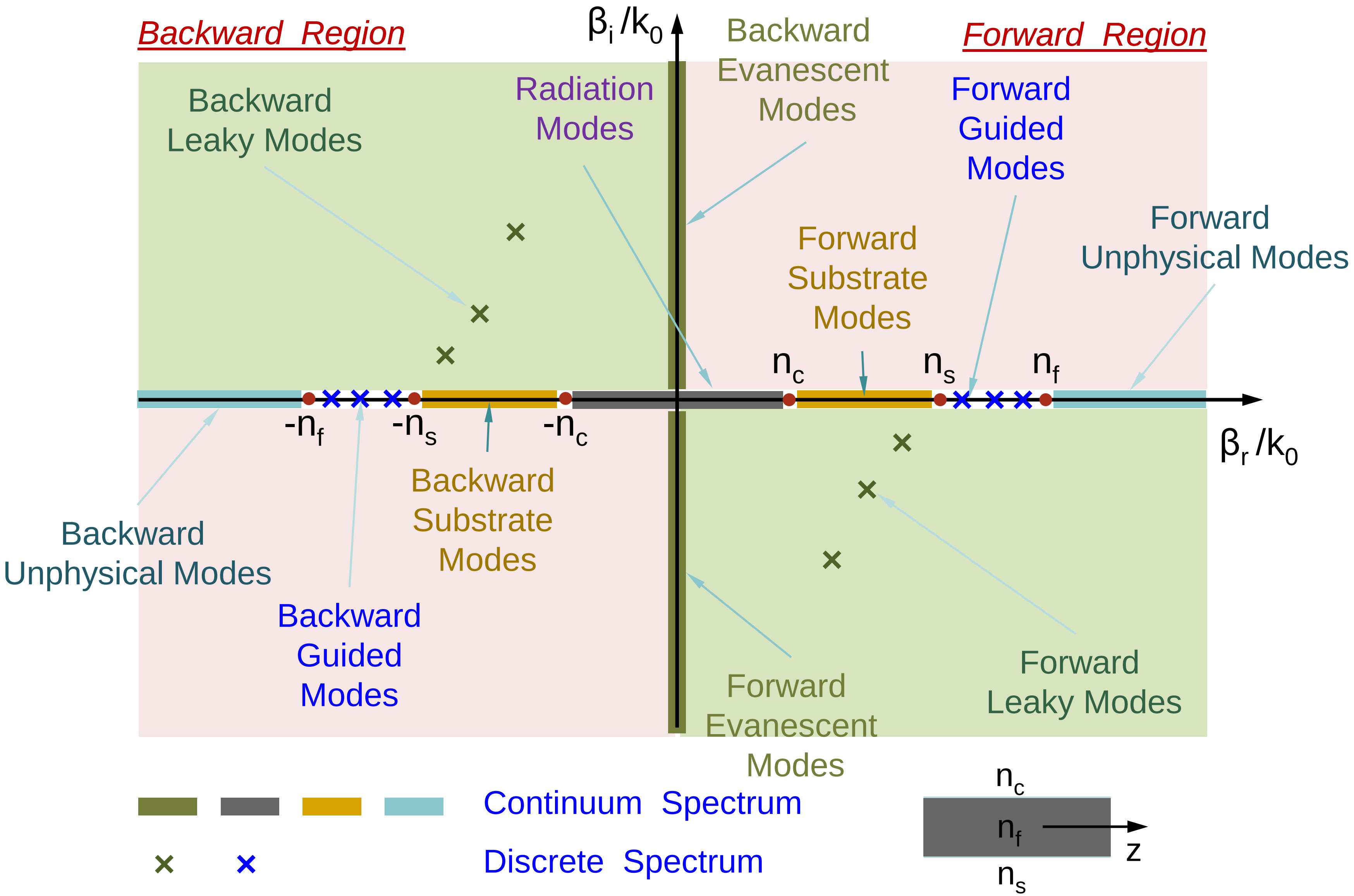
$$\vec{H}_\beta = \hat{y}E_0 \begin{cases} \frac{1}{2}\Big[(1 + \frac{k'_{fx}/n_f^2}{k'_{cx}/n_c^2})\cos[k'_{cx}(x-h) + k'_{fx}h - \phi] + \\ \qquad (1 - \frac{k'_{fx}/n_f^2}{k'_{cx}/n_c^2})\cos[k'_{cx}(x-h) - k'_{fx}h + \phi]\Big]\ e^{-\beta z}, & x > h, \\ \cos(k'_{fx}x - \phi)\ e^{-\beta z}, & 0 < x < h, \\ \frac{1}{2}\Big[(1 + \frac{k'_{fx}/n_f^2}{k'_{sx}/n_s^2})\cos[k'_{sx}x - \phi] + (1 - \frac{k'_{fx}/n_f^2}{k'_{sx}/n_s^2})\cos[k'_{sx}x + \phi]\Big]\ e^{-\beta z}, & x < 0, \end{cases}$$

TE/TM Forward Evanescent Modes Example

(n_c= 1, n_f =2.2, n_s=1.5, h=1.2μm, λ_0=1.0μm)

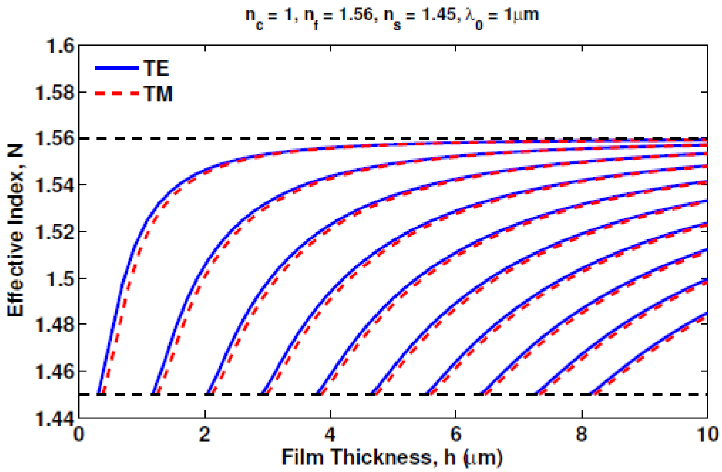


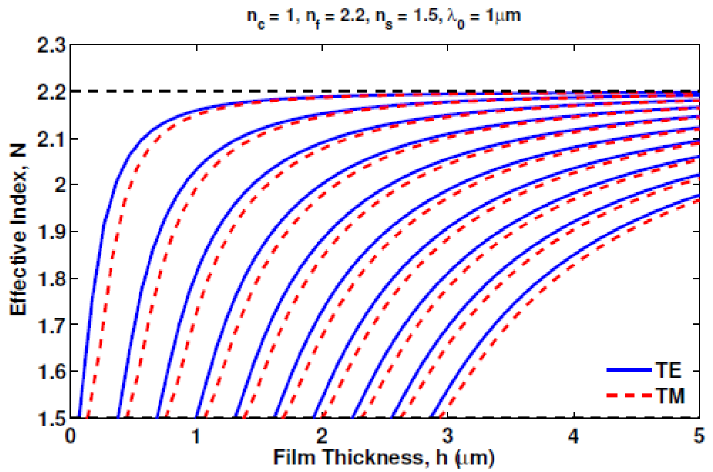
SLAB WAVEGUIDE MODE DIAGRAM



SLAB WAVEGUIDE CUTOFF CONDITIONS

$$k_0 h\sqrt{n_f^2 - n_s^2} - \tan^{-1}(\sqrt{a_w}) = \nu\pi, \qquad w = TE,\ TM.$$



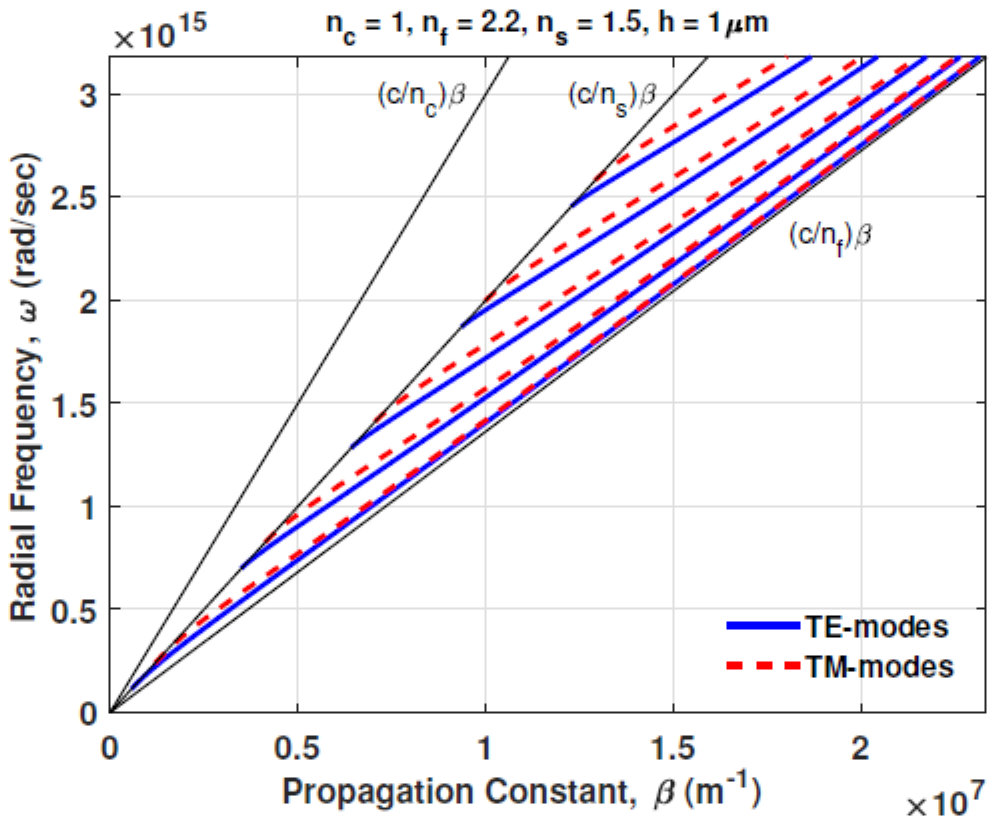


$$h_{cut,\nu}^{TE} = \frac{\nu\pi + \tan^{-1}(\sqrt{a_{TE}})}{k_0\sqrt{n_f^2 - n_s^2}},$$

$$h_{cut,\nu}^{TM} = \frac{\nu\pi + \tan^{-1}(\sqrt{a_{TM}})}{k_0\sqrt{n_f^2 - n_s^2}},$$

SLAB WAVEGUIDE CUTOFF CONDITIONS

$$k_0 h\sqrt{n_f^2 - n_s^2} - \tan^{-1}(\sqrt{a_w}) = \nu\pi, \qquad w = TE,\ TM.$$



$$\lambda_{0,cut,\nu}^{TE} = \frac{2\pi h\sqrt{n_f^2 - n_s^2}}{\nu\pi + \tan^{-1}(\sqrt{a_{TE}})},$$

$$\lambda_{0,cut,\nu}^{TM} = \frac{2\pi h\sqrt{n_f^2 - n_s^2}}{\nu\pi + \tan^{-1}(\sqrt{a_{TM}})},$$

$$\omega_{cut,\nu}^{TE} = c\frac{\nu\pi + \tan^{-1}(\sqrt{a_{TE}})}{h\sqrt{n_f^2 - n_s^2}},$$

$$\omega_{cut,\nu}^{TM} = c\frac{\nu\pi + \tan^{-1}(\sqrt{a_{TM}})}{h\sqrt{n_f^2 - n_s^2}},$$

SLAB WAVEGUIDE NORMALIZED PARAMETERS

$$
\begin{aligned}
V &= \frac{2\pi}{\lambda_0} h \sqrt{n_f^2 - n_s^2}, \\
b &= \frac{N^2 - n_s^2}{n_f^2 - n_s^2} = \frac{n_f^2 \sin^2\theta - n_s^2}{n_f^2 - n_s^2}, \\
\mathrm{a}_{TE} &= \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2}, \\
\mathrm{a}_{TM} &= \frac{n_f^4}{n_c^2} \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2},
\end{aligned}
$$

Dispersion for TE Guided Modes

$$
V\sqrt{1-b} - \tan^{-1}\left\{\sqrt{\frac{b}{1-b}}\right\} - \tan^{-1}\left\{\sqrt{\frac{b+\mathrm{a}_{TE}}{1-b}}\right\} = \nu\pi, \qquad \nu = 0, 1, \cdots
$$

Dispersion for TM Guided Modes

$$
V\sqrt{q}\frac{n_f}{n_s}\sqrt{1-b_{TM}} - \tan^{-1}\left\{\sqrt{\frac{b_{TM}}{1-b_{TM}}}\right\} - \tan^{-1}\left\{\sqrt{\frac{b_{TM}+\mathrm{a}_{TM}(1-b_{TM}d)}{1-b_{TM}}}\right\} = \nu\pi,
$$

SLAB WAVEGUIDE NORMALIZED DIAGRAM

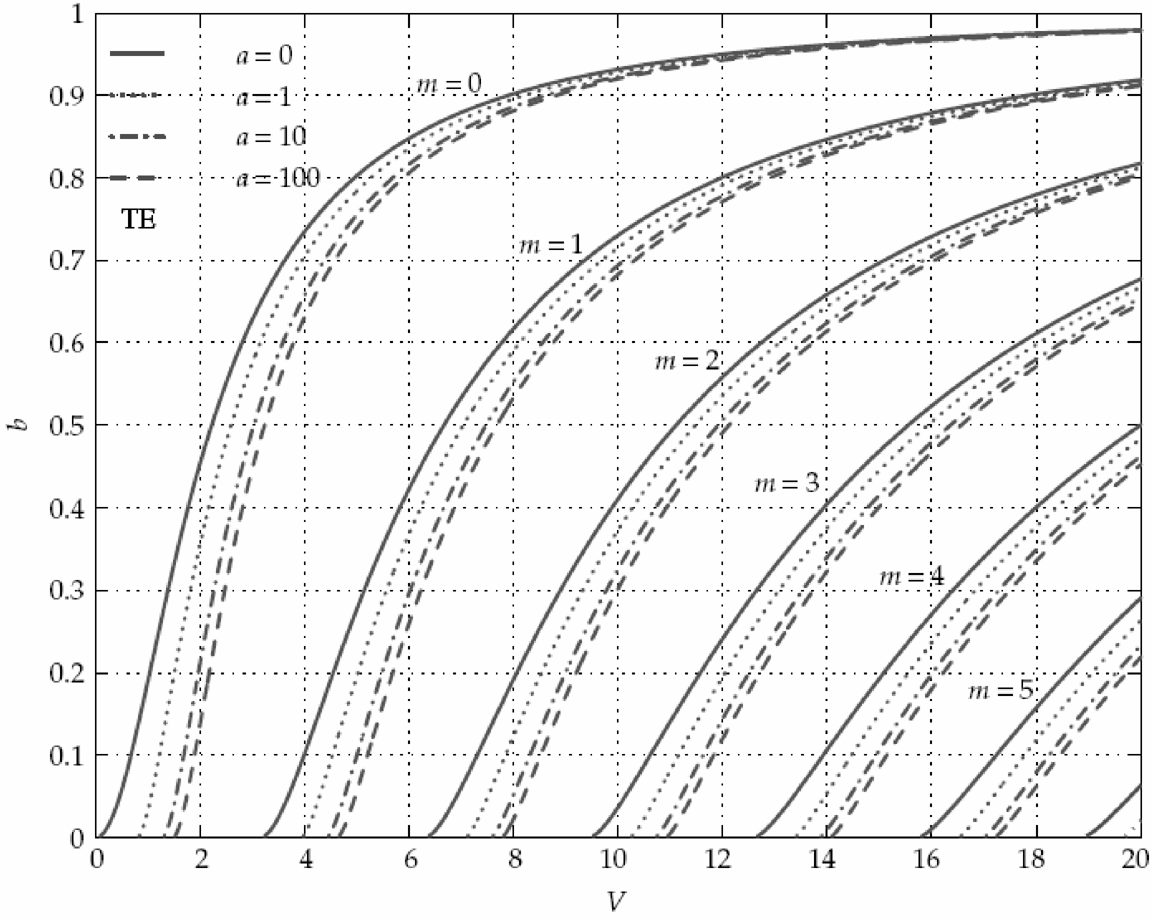
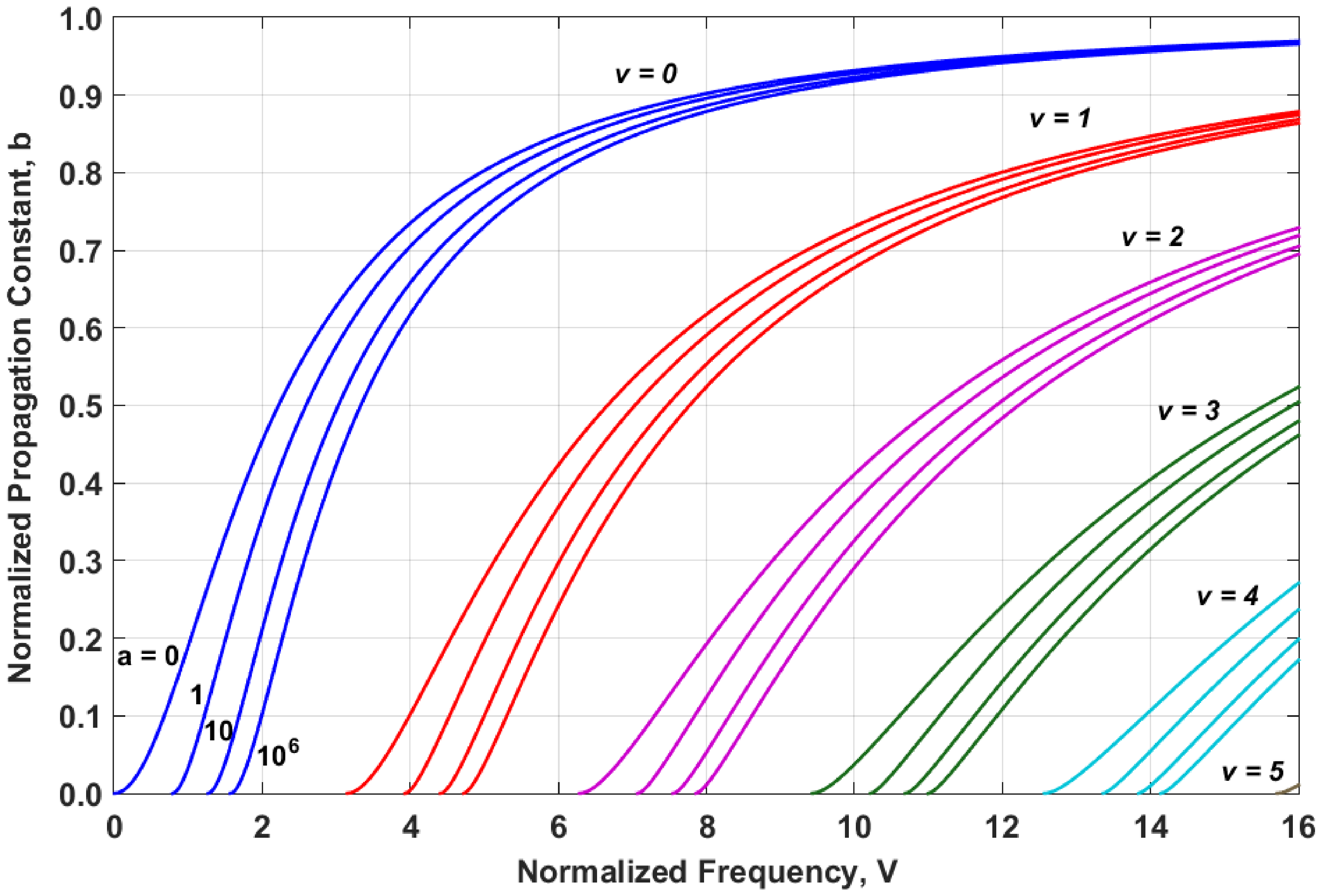


Figure 2.4 *bV* curves of TE modes guided by step-index thin-film waveguides.

SLAB WAVEGUIDE NORMALIZED DIAGRAM



POWER CONSIDERATIONS FOR SLAB WAVEGUIDES

$$
\begin{aligned}
P &= \int_{-\infty}^{+\infty} \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\}\cdot\hat{z}dx \\
&= \underbrace{\int_{-\infty}^{0} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_s} + \underbrace{\int_{0}^{h} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_f} + \underbrace{\int_{h}^{+\infty} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_c}
\end{aligned}
$$

TE Modes

$$
\vec{E}_\nu = \hat{y}E_0 \begin{cases} \cos(k_{fx}h - \phi_{fs}^{TE})\ e^{-\gamma_c(x-h)}\ e^{-j\beta_\nu z}, & x \geq h, \\ \cos(k_{fx}x - \phi_{fs}^{TE})\ e^{-j\beta_\nu z}, & 0 \leq x \leq h, \\ \cos\phi_{fs}^{TE}\ e^{\gamma_s x}\ e^{-j\beta_\nu z}, & x \leq 0, \end{cases}
\qquad H_x = (1/j\omega\mu_0)(dE_y/dz)
$$

$$
\begin{aligned}
P_s &= \frac{N_\nu^{TE}}{4}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_0|^2\frac{\cos^2(\phi_{fs})}{\gamma_s}, \\
P_f &= \frac{N_\nu^{TE}}{4}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_0|^2\left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}}\right], \\
P_c &= \frac{N_\nu^{TE}}{4}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_0|^2\frac{\cos^2(k_{fx}h - \phi_{fs})}{\gamma_c},
\end{aligned}
$$

$$
\begin{aligned}
\frac{P_s}{P} &= \frac{1}{h_{eff,\nu}^{TE}}\frac{\cos^2(\phi_{fs})}{\gamma_s}, \\
\frac{P_f}{P} &= \frac{1}{h_{eff,\nu}^{TE}}\left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}}\right], \\
\frac{P_c}{P} &= \frac{1}{h_{eff,\nu}^{TE}}\frac{\cos^2(k_{fx}h - \phi_{fs})}{\gamma_c}.
\end{aligned}
$$

$$
P = \frac{N_\nu^{TE}}{4}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_0|^2\left[h + \frac{1}{\gamma_s} + \frac{1}{\gamma_c}\right] = \frac{N_\nu^{TE}}{4}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_0|^2 h_{eff,\nu}^{TE},
$$

POWER CONSIDERATIONS FOR SLAB WAVEGUIDES

$$
\begin{aligned}
P &= \int_{-\infty}^{+\infty} \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\}\cdot\hat{z}dx \\
&= \underbrace{\int_{-\infty}^{0} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_s} + \underbrace{\int_{0}^{h} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_f} + \underbrace{\int_{h}^{+\infty} \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\hat{z}dx}_{P_c}
\end{aligned}
$$

TM Modes

$$
\vec{H}_\nu = \hat{y}H_0 \begin{cases} \cos(k_{fx}h - \phi_{fs}^{TM})\ e^{-\gamma_c(x-h)}\ e^{-j\beta_\nu z}, & x \geq h, \\ \cos(k_{fx}x - \phi_{fs}^{TM})\ e^{-j\beta_\nu z}, & 0 \leq x \leq h, \\ \cos\phi_{fs}^{TM}\ e^{\gamma_s x}\ e^{-j\beta_\nu z}, & x \leq 0, \end{cases} \qquad E_x = -(1/j\omega\epsilon)(dH_y/dz)
$$

$$
\begin{aligned}
P_s &= \frac{N_\nu^{TM}}{4}\sqrt{\frac{\mu_0}{\epsilon_0}}|H_0|^2\frac{\cos^2(\phi_{fs})}{n_s^2\gamma_s}, \\
P_f &= \frac{N_\nu^{TM}}{4}\sqrt{\frac{\mu_0}{\epsilon_0}}|H_0|^2\frac{1}{n_f^2}\left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}}\right], \\
P_c &= \frac{N_\nu^{TM}}{4}\sqrt{\frac{\mu_0}{\epsilon_0}}|H_0|^2\frac{\cos^2(k_{fx}h - \phi_{fs})}{n_c^2\gamma_c},
\end{aligned}
$$

$$
\begin{aligned}
\frac{P_s}{P} &= \frac{n_f^2}{h_{eff,\nu}^{TM}}\ \frac{\cos^2(\phi_{fs})}{n_s^2\gamma_s}, \\
\frac{P_f}{P} &= \frac{n_f^2}{h_{eff,\nu}^{TM}}\frac{1}{n_f^2}\left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}}\right], \\
\frac{P_c}{P} &= \frac{n_f^2}{h_{eff,\nu}^{TM}}\ \frac{\cos^2(k_{fx}h - \phi_{fs})}{n_c^2\gamma_c}.
\end{aligned}
$$

$$
P = \frac{N_\nu^{TM}}{4}\sqrt{\frac{\mu_0}{\epsilon_0}}|H_0|^2\frac{1}{n_f^2}\left[h + \frac{1}{q_s\gamma_s} + \frac{1}{q_c\gamma_c}\right] = \frac{N_\nu^{TM}}{4}\sqrt{\frac{\mu_0}{\epsilon_0}}|H_0|^2\frac{1}{n_f^2}h_{eff,\nu}^{TM},
$$

$$
\begin{aligned}
q_c &= \left(\frac{N_\nu^{TM}}{n_c}\right)^2 + \left(\frac{N_\nu^{TM}}{n_f}\right)^2 - 1 \\
q_s &= \left(\frac{N_\nu^{TM}}{n_s}\right)^2 + \left(\frac{N_\nu^{TM}}{n_f}\right)^2 - 1
\end{aligned}
$$

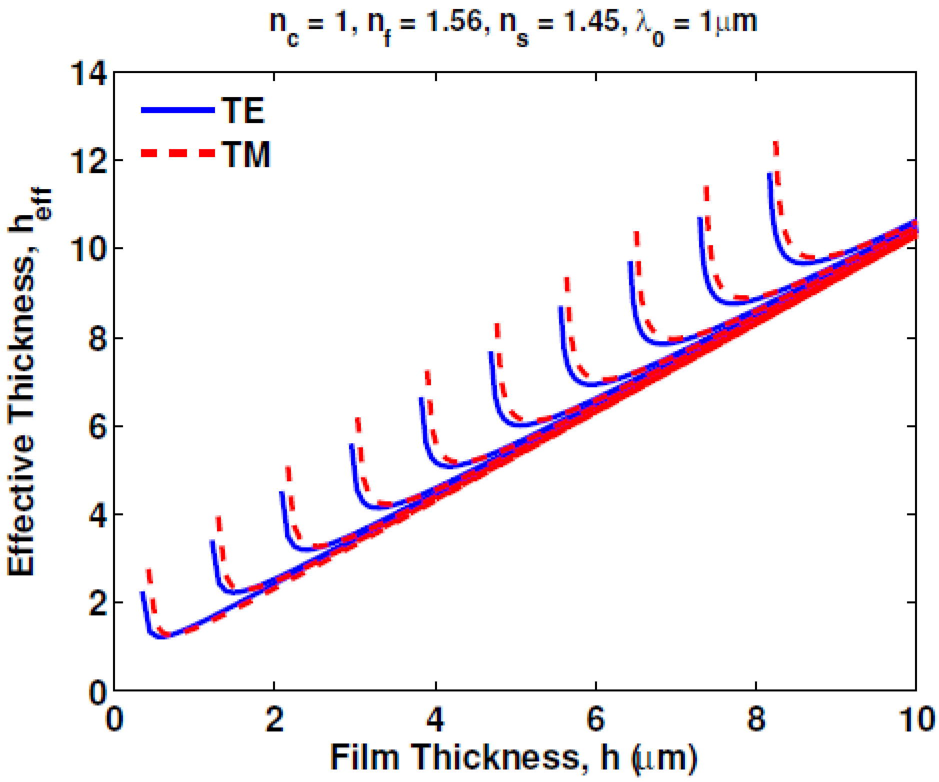
POWER CONSIDERATIONS FOR SLAB WAVEGUIDES

$$h_{eff}^{TE_\nu} = h + \frac{1}{\gamma_s} + \frac{1}{\gamma_c}$$

$$h_{eff}^{TM_\nu} = h + \frac{1}{q_s\gamma_s} + \frac{1}{q_c\gamma_c}$$

$$q_c = \left(\frac{N_\nu^{TM}}{n_c}\right)^2 + \left(\frac{N_\nu^{TM}}{n_f}\right)^2 - 1$$

$$q_s = \left(\frac{N_\nu^{TM}}{n_s}\right)^2 + \left(\frac{N_\nu^{TM}}{n_f}\right)^2 - 1$$



POWER CONSIDERATIONS FOR SLAB WAVEGUIDES

TE Modes

$$\frac{P_s}{P} = \frac{1}{h_{eff,\nu}^{TE}} \frac{\cos^2(\phi_{fs})}{\gamma_s},$$

$$\frac{P_f}{P} = \frac{1}{h_{eff,\nu}^{TE}} \left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}} \right],$$

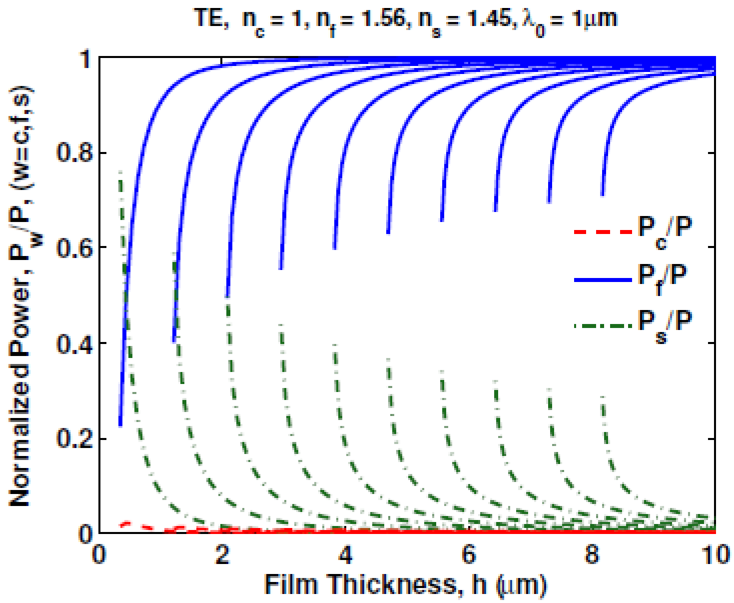
$$\frac{P_c}{P} = \frac{1}{h_{eff,\nu}^{TE}} \frac{\cos^2(k_{fx}h - \phi_{fs})}{\gamma_c}.$$

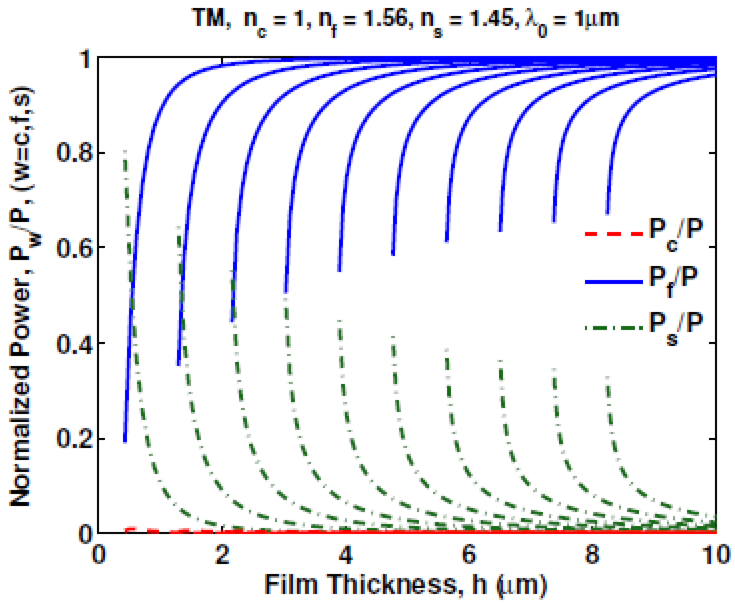
TM Modes

$$\frac{P_s}{P} = \frac{n_f^2}{h_{eff,\nu}^{TM}} \frac{\cos^2(\phi_{fs})}{n_s^2\gamma_s},$$

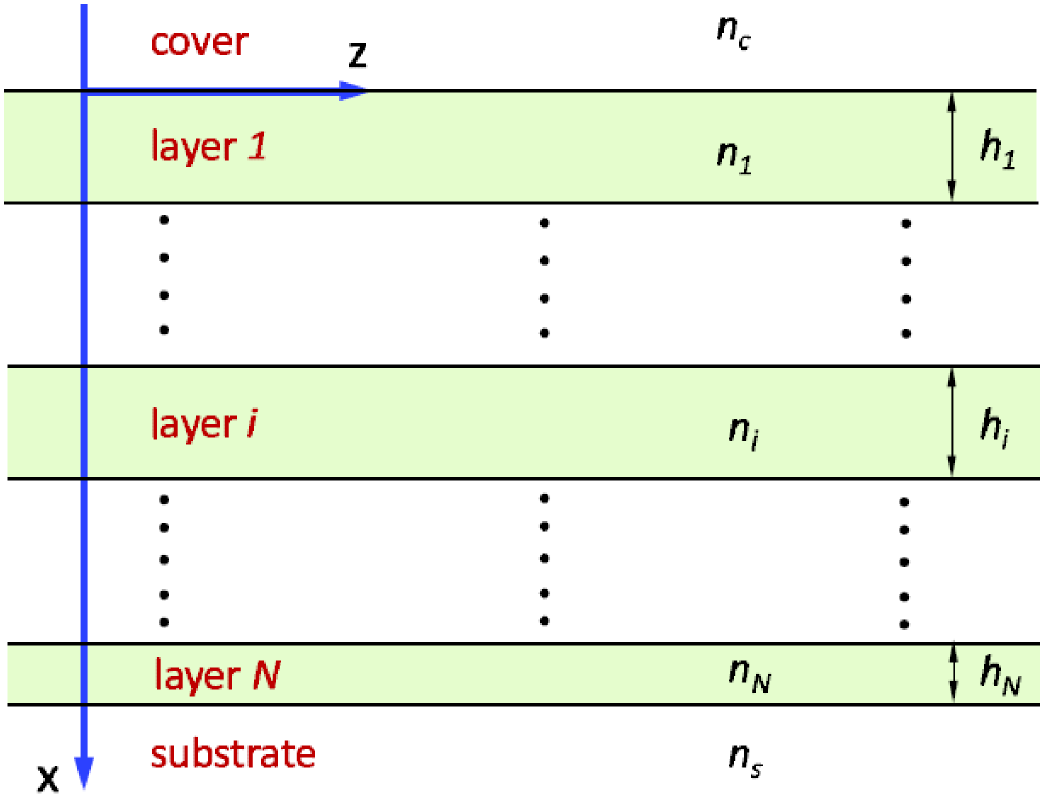
$$\frac{P_f}{P} = \frac{n_f^2}{h_{eff,\nu}^{TM}} \frac{1}{n_f^2} \left[h + \frac{\sin(2k_{fx}h - 2\phi_{fs}) + \sin(2\phi_{fs})}{2k_{fx}} \right],$$

$$\frac{P_c}{P} = \frac{n_f^2}{h_{eff,\nu}^{TM}} \frac{\cos^2(k_{fx}h - \phi_{fs})}{n_c^2\gamma_c}.$$





MULTILAYERED SLAB WAVEGUIDES

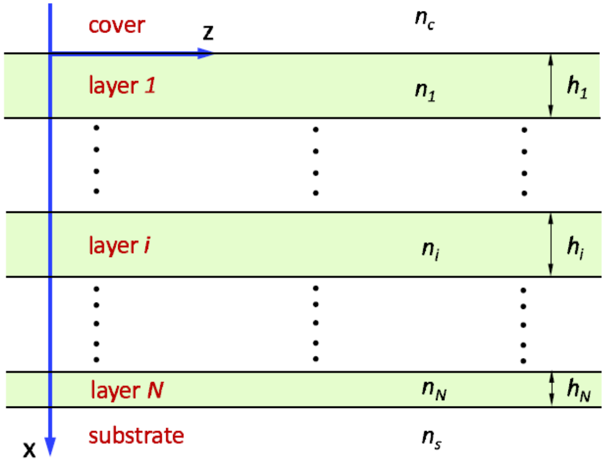


Range of propagation constant β possible values for guided modes

$$k_0 n_s < \beta_\nu < k_0 \max_i \{n_i\}$$

MULTILAYERED SLAB WAVEGUIDES

TE Modes



$$\frac{d}{dx}\begin{bmatrix} E_y \\ H_z \end{bmatrix} = \begin{bmatrix} 0 & -j\omega\mu_0 \\ -j\omega\epsilon + j\dfrac{\beta^2}{\omega\mu_0} & 0 \end{bmatrix}\begin{bmatrix} E_y \\ H_z \end{bmatrix},$$

$$H_x = -\frac{\beta}{\omega\mu_0}E_y.$$

$$D_i = \sum_{\ell=1}^{i-1} h_\ell, \qquad \text{with } D_1 = 0, \quad i = 1, 2, \cdots, N$$

$$\vec{E} = \hat{y}\begin{cases} E_c e^{\gamma_c x} e^{-j\beta z}, & x < 0, \\ \left[E_{i1} e^{-jk_{xi}(x-D_i)} + E_{i2} e^{+jk_{xi}(x-D_i)}\right] e^{-j\beta z}, & D_i < x < D_{i+1}, \\ E_s e^{-\gamma_s(x-D_{N+1})} e^{-j\beta z}, & x > D_{N+1}, \end{cases}$$

$$H_z = \frac{1}{-j\omega\mu_0}\begin{cases} \gamma_c E_c e^{\gamma_c x} e^{-j\beta z}, & x < 0, \\ \left[-jk_{xi} E_{i1} e^{-jk_{xi}(x-D_i)} + jk_{xi} E_{i2} e^{+jk_{xi}(x-D_i)}\right] e^{-j\beta z}, & D_i < x < D_{i+1}, \\ -\gamma_s E_s e^{-\gamma_s(x-D_{N+1})} e^{-j\beta z}, & x > D_{N+1}. \end{cases}$$

TE Modes

Boundary Conditions

$$\begin{bmatrix} 1 & 1 \\ -jk_{x1} & jk_{x1} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ \gamma_c \end{bmatrix} E_c, \qquad \text{at } x = 0,$$

$$\begin{bmatrix} e^{-jk_{xi}h_i} & e^{+jk_{xi}h_i} \\ -jk_{xi}e^{-jk_{xi}h_i} & jk_{xi}e^{+jk_{xi}h_i} \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -jk_{x,i+1} & jk_{x,i+1} \end{bmatrix} \begin{bmatrix} E_{i+1,1} \\ E_{i+1,2} \end{bmatrix}, \qquad \text{at } x = D_{i+1},$$

$$\begin{bmatrix} e^{-jk_{xN}h_N} & e^{+jk_{xN}h_N} \\ -jk_{xN}e^{-jk_{xN}h_N} & jk_{xN}e^{+jk_{xN}h_N} \end{bmatrix} \begin{bmatrix} E_{N1} \\ E_{N2} \end{bmatrix} = \begin{bmatrix} 1 \\ -\gamma_s \end{bmatrix} E_s, \qquad \text{at } x = D_{N+1}$$

MULTILAYERED SLAB WAVEGUIDES

TE Modes

$$\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} = \underbrace{\frac{1}{2jk_{x1}} \begin{bmatrix} jk_{x1} & -1 \\ jk_{x1} & 1 \end{bmatrix}}_{\tilde{M}_{1,0}} \begin{bmatrix} 1 \\ \gamma_c \end{bmatrix} E_c,$$

$$\begin{bmatrix} E_{i+1,1} \\ E_{i+1,2} \end{bmatrix} = \underbrace{\frac{1}{2} \begin{bmatrix} e^{-jk_{xi}h_i}\left(1+\frac{k_{xi}}{k_{x,i+1}}\right) & e^{+jk_{xi}h_i}\left(1-\frac{k_{xi}}{k_{x,i+1}}\right) \\ e^{-jk_{xi}h_i}\left(1-\frac{k_{xi}}{k_{x,i+1}}\right) & e^{+jk_{xi}h_i}\left(1+\frac{k_{xi}}{k_{x,i+1}}\right) \end{bmatrix}}_{\tilde{M}_{i+1,i}} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ -\gamma_s \end{bmatrix} E_s = \underbrace{\begin{bmatrix} e^{-jk_{xN}h_N} & e^{+jk_{xN}h_N} \\ -jk_{xN}e^{-jk_{xN}h_N} & jk_{xN}e^{+jk_{xN}h_N} \end{bmatrix}}_{\tilde{M}_{N+1,N}} \begin{bmatrix} E_{N1} \\ E_{N2} \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ -\gamma_s \end{bmatrix} E_s = \underbrace{\tilde{M}_{N+1,N}\tilde{M}_{N,N-1}\cdots\tilde{M}_{2,1}\tilde{M}_{1,0}}_{\tilde{M}} \begin{bmatrix} 1 \\ \gamma_c \end{bmatrix} E_c = \begin{bmatrix} m_{11} & m_{21} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 1 \\ \gamma_c \end{bmatrix} E_c.$$

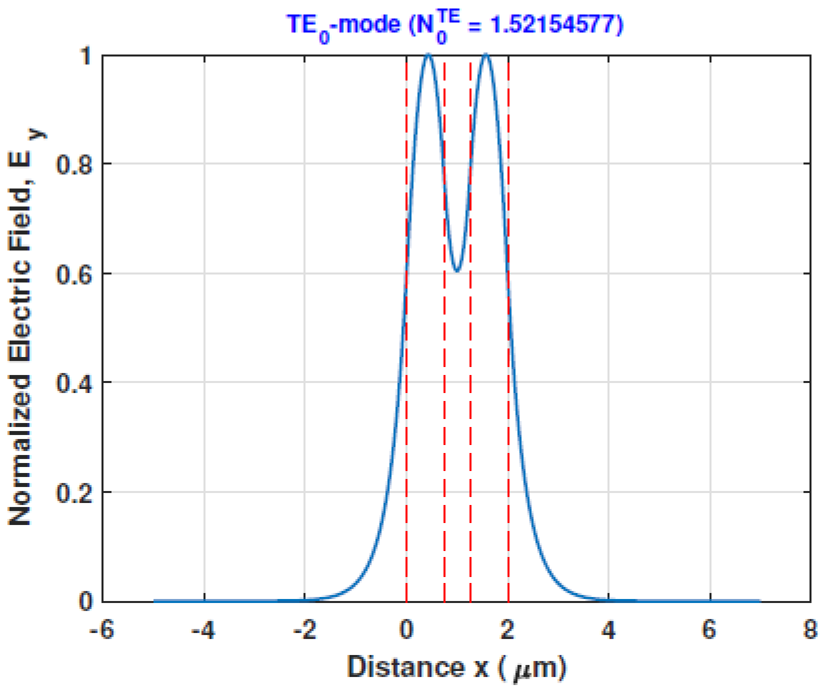
$$\underbrace{\begin{bmatrix} m_{11}+m_{12}\gamma_c & -1 \\ m_{21}+m_{22}\gamma_c & \gamma_s \end{bmatrix}}_{\tilde{A}_{TE}(\beta^2)} \begin{bmatrix} E_c \\ E_s \end{bmatrix} = 0 \implies \det\left\{\tilde{A}_{TE}(\beta^2)\right\} = 0,$$

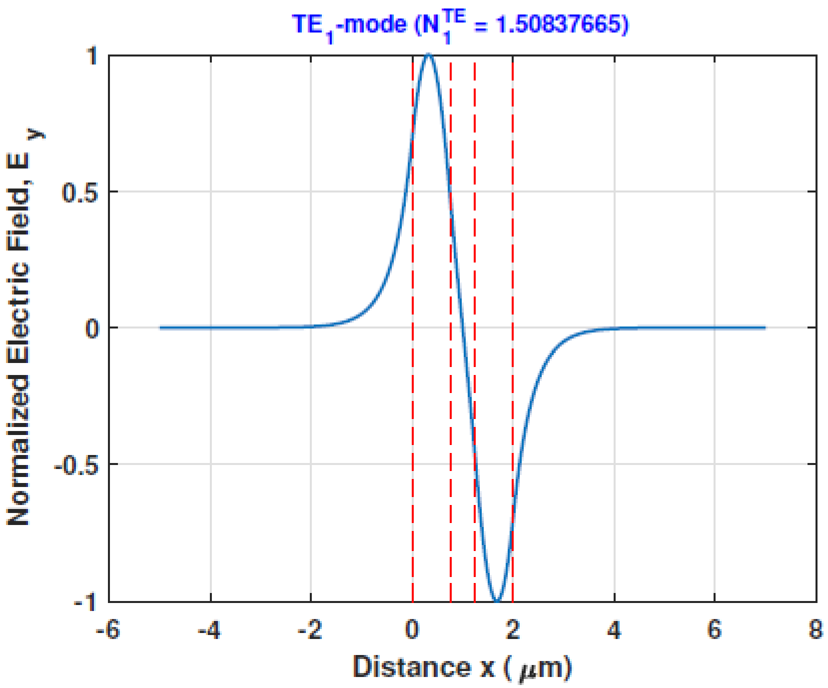
$$\boxed{\det\left\{\tilde{A}_{TE}(\beta^2)\right\} = 0 \Rightarrow \gamma_s m_{11} + \gamma_c m_{22} + \gamma_c\gamma_s m_{12} + m_{21} = 0.}$$

MULTILAYERED SLAB WAVEGUIDES

TE Guided Modes Example

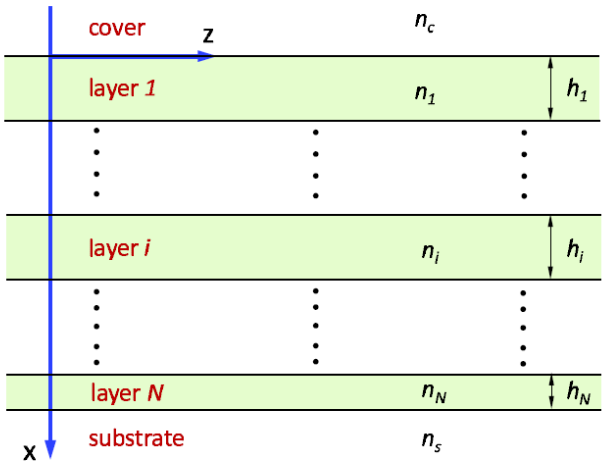
n_c= 1.45, n_1 =1.56, n_2 =1.45, n_3 =1.56 n_s=1.45,
h_1=0.75 μm, h_2=0.50, h_3=0.75 μm λ_0=1.0μm





MULTILAYERED SLAB WAVEGUIDES

TM Modes



$$\frac{d}{dx}\begin{bmatrix} H_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 & j\omega\epsilon \\ j\omega\mu_0 - j\dfrac{\beta^2}{\omega\epsilon} & 0 \end{bmatrix}\begin{bmatrix} H_y \\ E_z \end{bmatrix},$$

$$E_x = \frac{\beta}{\omega\epsilon} H_y.$$

$$= \hat{y}\begin{cases} H_c e^{\gamma_c x} e^{-j\beta z}, & x < 0, \\ \left[H_{i1} e^{-jk_{xi}(x-D_i)} + H_{i2} e^{+jk_{xi}(x-D_i)}\right] e^{-j\beta z}, & D_i < x < D_{i+1}, \\ H_s e^{-\gamma_s (x - D_{N+1})} e^{-j\beta z}, & x > D_{N+1}, \end{cases}$$

$$E_z = \frac{1}{j\omega\epsilon_0}\begin{cases} \dfrac{\gamma_c}{n_c^2} H_c e^{\gamma_c x} e^{-j\beta z}, & x < 0, \\ \left[-j\dfrac{k_{xi}}{n_i^2} H_{i1} e^{-jk_{xi}(x-D_i)} + j\dfrac{k_{xi}}{n_i^2} H_{i2} e^{+jk_{xi}(x-D_i)}\right] e^{-j\beta z}, & D_i < x < D_{i+1}, \\ -\dfrac{\gamma_s}{n_s^2} H_s e^{-\gamma_s(x-D_{N+1})} e^{-j\beta z}, & x > D_{N+1}. \end{cases}$$

MULTILAYERED SLAB WAVEGUIDES

TM Modes

Boundary Conditions

$$\begin{bmatrix} e^{-jk_{xi}h_i} & e^{+jk_{xi}h_i} \\ j\dfrac{k_{xi}}{n_i^2}e^{-jk_{xi}h_i} & -j\dfrac{k_{xi}}{n_i^2}e^{+jk_{xi}h_i} \end{bmatrix} \begin{bmatrix} H_{i1} \\ H_{i2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ j\dfrac{k_{x,i+1}}{n_{i+1}^2} & -j\dfrac{k_{x,i+1}}{n_{i+1}^2} \end{bmatrix} \begin{bmatrix} H_{i+1,1} \\ H_{i+1,2} \end{bmatrix}, \qquad \text{at } x = D_{i+1}.$$

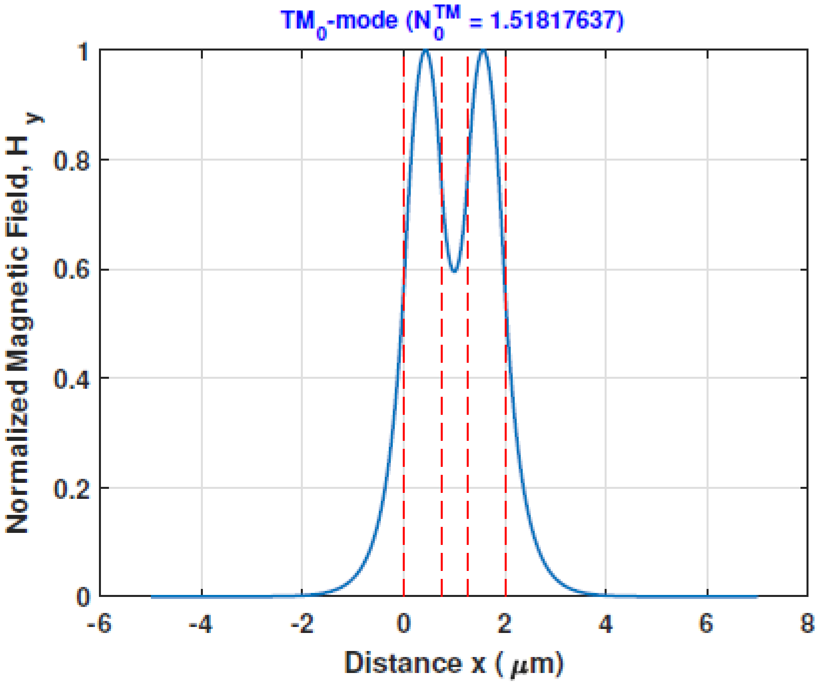
Dispersion Equation

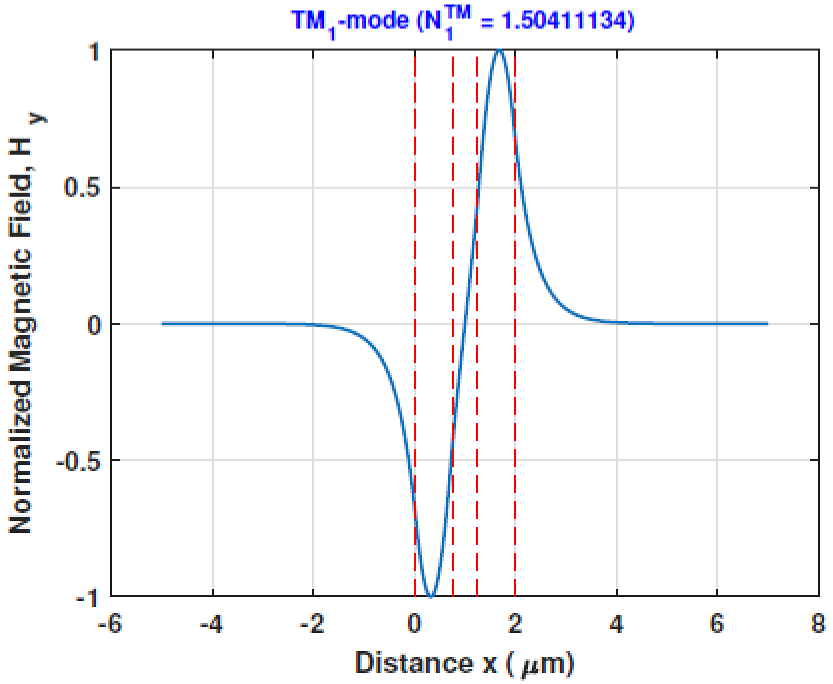
$$\det\left\{\tilde{\mathcal{A}}_{TM}(\beta^2)\right\} = 0 \Rightarrow -\frac{\gamma_s}{n_s^2}m_{11} - \frac{\gamma_c}{n_c^2}m_{22} + \frac{\gamma_c}{n_c^2}\frac{\gamma_s}{n_s^2}m_{12} + m_{21} = 0.$$

MULTILAYERED SLAB WAVEGUIDES

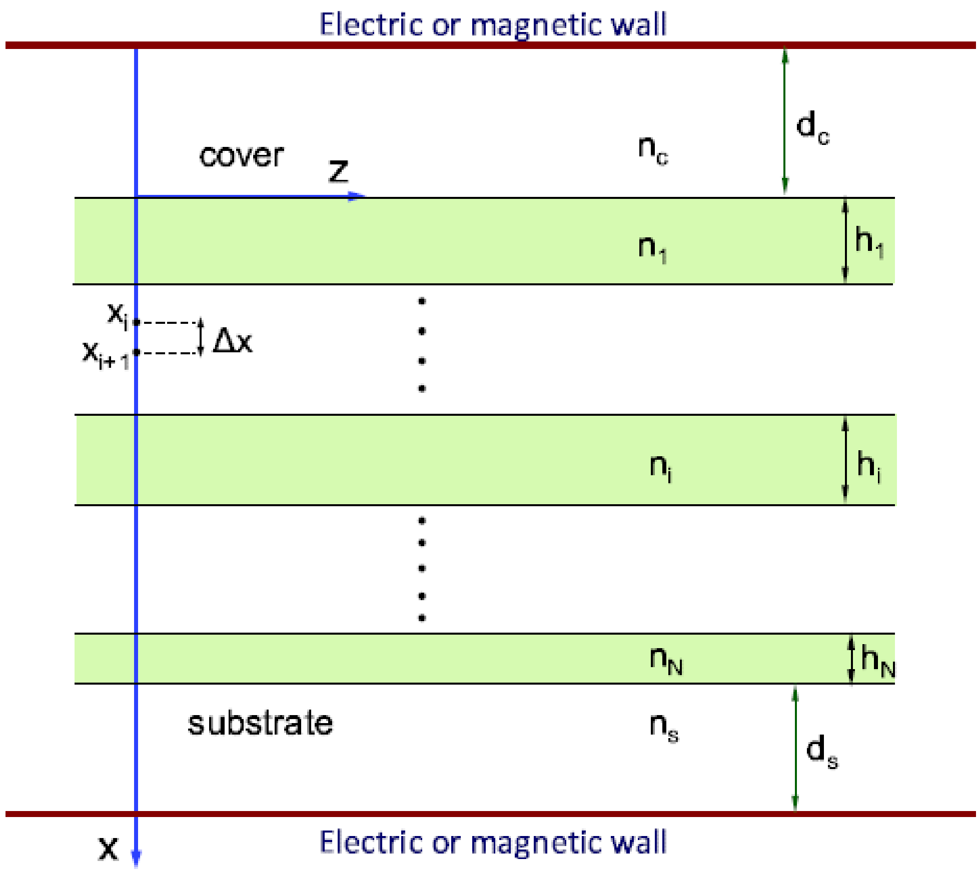
TM Guided Modes Example

n_c= 1.45, n_1 =1.56, n_2 =1.45, n_3 =1.56 n_s=1.45,
h_1=0.75 μm, h_2=0.50, h_3=0.75 μm λ_0=1.0μm





FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES



Helmholtz Equations

$$\nabla^2 \vec{E} + \vec{\nabla}\left[\frac{\vec{\nabla}\varepsilon}{\varepsilon}\cdot\vec{E}\right] + k_0^2\varepsilon\vec{E} = 0,$$

$$\nabla^2 \vec{H} + \left[\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right]\times\left(\vec{\nabla}\times\vec{H}\right) + k_0^2\varepsilon\vec{H} = 0,$$

TE Modes

$$\frac{d^2E_y}{dx^2} + \left[k_0^2 n^2(x) - \beta^2\right]E_y = 0,$$

TM Modes

$$\frac{d^2H_y}{dx^2} - \frac{1}{n^2(x)}\frac{dn^2(x)}{dx}\frac{dH_y}{dx} + \left[k_0^2 n^2(x) - \beta^2\right]H_y = 0,$$

$$n^2(x)\frac{d}{dx}\left(\frac{1}{n^2(x)}\frac{dH_y}{dx}\right) + \left[k_0^2 n^2(x) - \beta^2\right]H_y = 0,$$

L. A. Coldren et al., "Diode Lasers & Photonic Integrated Circuits," J. Wiley & Sons (2012 -2nd Ed.)

K. Kawano and T. Kitoh, "Introduction to Optical Waveguide Analysis," J. Wiley & Sons (2001)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES

$$U(x+\Delta x) \simeq U(x) + \frac{dU}{dx}\frac{\Delta x}{1!} + \frac{d^2U}{dx^2}\frac{(\Delta x)^2}{2!} + \frac{d^3U}{dx^3}\frac{(\Delta x)^3}{3!} + \frac{d^4U}{dx^4}\frac{(\Delta x)^4}{4!},$$

$$U(x-\Delta x) \simeq U(x) - \frac{dU}{dx}\frac{\Delta x}{1!} + \frac{d^2U}{dx^2}\frac{(\Delta x)^2}{2!} - \frac{d^3U}{dx^3}\frac{(\Delta x)^3}{3!} + \frac{d^4U}{dx^4}\frac{(\Delta x)^4}{4!},$$

$$\frac{d^2U}{dx^2} = \frac{U_{i-1} - 2U_i + U_{i+1}}{(\Delta x)^2}.$$

$$\frac{U_{i-1} - 2U_i + U_{i+1}}{(\Delta x)^2} + k_0^2 n_i^2 U_i = \beta^2 U_i, \qquad i = 0, 1, \cdots, M+1.$$

TE Modes

Normalized Distance: $X = k_0 x$

$$\frac{U_{i-1} - 2U_i + U_{i+1}}{(\Delta X)^2} + n_i^2 U_i = N^2 U_i, \qquad i = 0, 1, \cdots, M+1.$$

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FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES

TE Modes

$$\begin{bmatrix} a_1 & b & 0 & \dots & 0 & 0 \\ b & a_2 & b & 0 & \dots & 0 \\ 0 & b & a_3 & b & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & b & a_{M-1} & b \\ 0 & \dots & \dots & 0 & b & a_M \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ \vdots \\ U_{M-1} \\ U_M \end{bmatrix} = N^2 \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ \vdots \\ U_{M-1} \\ U_M \end{bmatrix}$$

$$a_i = n_i^2 - [2/(\Delta X)^2] \text{ for } i = 1, 2, \cdots, M \text{ and } b = 1/(\Delta X)^2$$

L. A. Coldren et al., "Diode Lasers & Photonic Integrated Circuits," J. Wiley & Sons (2012 -2nd Ed.)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES

TM Modes

$$\frac{1}{(\Delta X)^2}\frac{2n_i^2}{n_i^2+n_{i-1}^2}U_{i-1} + \left[n_i^2 - \frac{1}{(\Delta X)^2}\left(\frac{2n_i^2}{n_{i+1}^2+n_i^2}+\frac{2n_i^2}{n_i^2+n_{i-1}^2}\right)\right]U_i + \frac{1}{(\Delta X)^2}\frac{2n_i^2}{n_{i+1}^2+n_i^2}U_{i+1} = N^2U_i, \qquad i=1,2,\cdots,M.$$

$$\begin{bmatrix} a_1 & c_1 & 0 & \dots & 0 & 0 \\ b_2 & a_2 & c_2 & 0 & \dots & 0 \\ 0 & b_3 & a_3 & c_3 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & b_{M-1} & a_{M-1} & c_{M-1} \\ 0 & \dots & \dots & 0 & b_M & a_M \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ \vdots \\ U_{M-1} \\ U_M \end{bmatrix} = N^2 \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ \vdots \\ U_{M-1} \\ U_M \end{bmatrix}$$

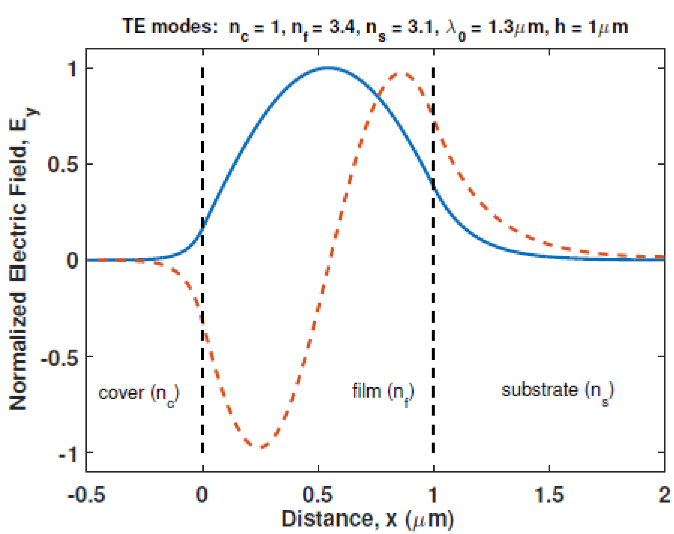
$$\begin{aligned} a_i &= n_i^2 - \frac{1}{(\Delta X)^2}\left(\frac{2n_i^2}{n_{i+1}^2+n_i^2}+\frac{2n_i^2}{n_i^2+n_{i-1}^2}\right), \quad (i=1,2,\cdots,M), \\ b_i &= \frac{1}{(\Delta X)^2}\frac{2n_i^2}{n_i^2+n_{i-1}^2}, \quad (i=1,2,\cdots,M) \\ c_i &= \frac{1}{(\Delta X)^2}\frac{2n_i^2}{n_{i+1}^2+n_i^2}, \quad (i=1,2,\cdots,M). \end{aligned}$$

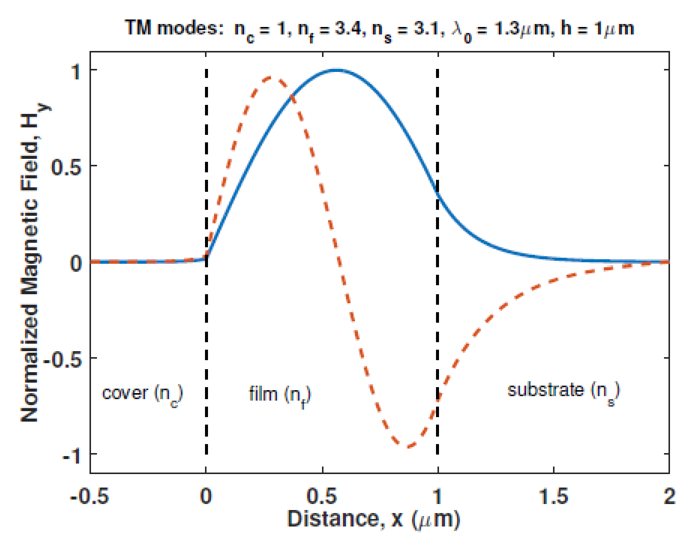
K. Kawano and T. Kitoh, "Introduction to Optical Waveguide Analysis," J. Wiley & Sons (2001)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES

Example: n_c= 1.0, n_f =3.4, n_s=3.1, h =1.0 μm, λ_0=1.3μm

	TE Modes		TM Modes	
Δx	N_0^{TE}	N_1^{TE}	N_0^{TM}	N_1^{TM}
$\lambda_0/20$	3.3585913	3.2362825	3.3518412	3.2129633
$\lambda_0/40$	3.3579455	3.2333453	3.3515178	3.2109972
$\lambda_0/60$	3.3578155	3.2327610	3.3514548	3.2106177
$\lambda_0/80$	3.3577736	3.2325729	3.3514346	3.2104963
$\lambda_0/100$	3.3577538	3.2324843	3.3514251	3.2104392
$\lambda_0/120$	3.3577424	3.2324333	3.3514196	3.2104064
$\lambda_0/150$	3.3577336	3.2323938	3.3514154	3.2103809
$\lambda_0/200$	3.3577268	3.2323632	3.3514215	3.2103612
Exact	**3.3577180**	**3.2323308**	**3.3514080**	**3.2103532**





L. A. Coldren et al., "Diode Lasers & Photonic Integrated Circuits," J. Wiley & Sons (2012 -2nd Ed.)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL (Solving Maxwell's Equations)

TE Modes

$$-\frac{\partial E_y}{\partial z'} = H'_x,$$
$$+\frac{\partial E_y}{\partial x'} = H'_z,$$
$$-\frac{\partial H'_z}{\partial x'} + \frac{\partial H'_x}{\partial z'} = \epsilon_r(x')E_y,$$

Normalized Magnetic Field

$$H'_w = -jZ_0 H_w$$

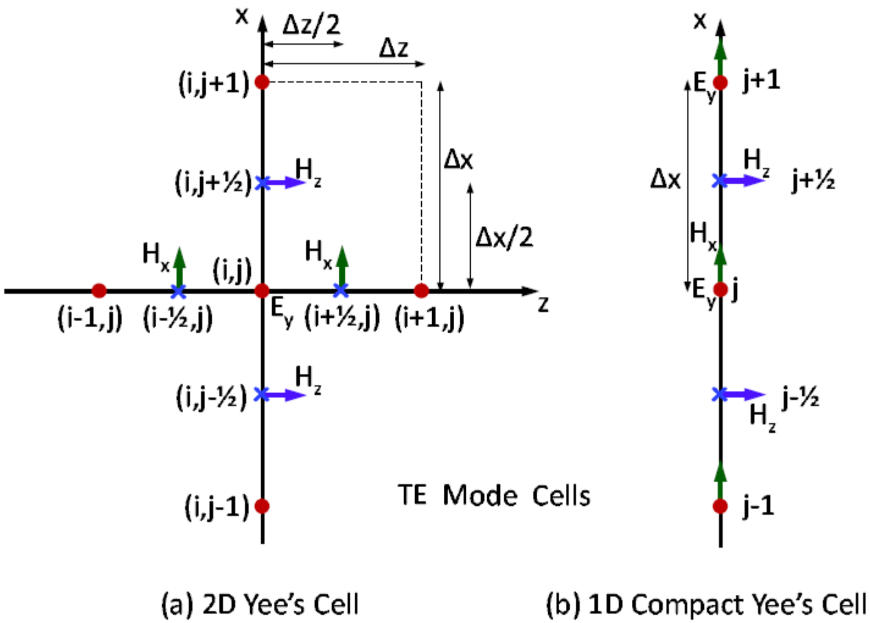
$x' = k_0 x$ and $z' = k_0 z$ are normalized coordinates.

R. C. Rumpf, PIERS B, vol. 36, 221-248 (2012)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL

(Solving Maxwell's Equations)

TE Modes



$$
\begin{aligned}
jN\tilde{E}_y &= \tilde{H}'_x, \\
\frac{1}{\Delta x'}\tilde{D}^E_{x'}\tilde{E}_y &= \tilde{H}'_z, \\
-\frac{1}{\Delta x'}\tilde{D}^H_{x'}\tilde{H}'_z - jN\tilde{H}'_x &= \tilde{\epsilon}_r\tilde{E}_y.
\end{aligned}
$$

$$
\left\{\tilde{\epsilon}_r + \frac{1}{(\Delta x')^2}\tilde{D}^H_{x'}\tilde{D}^E_{x'}\right\}\tilde{E}_y = N^2\tilde{E}_y
$$

$$
\tilde{E}_y = [E_y^1, E_y^2, \cdots, E_y^M]^T, \ \tilde{H}'_x = [H'^1_x, H'^2_x, \cdots, H'^M_x]^T,
$$

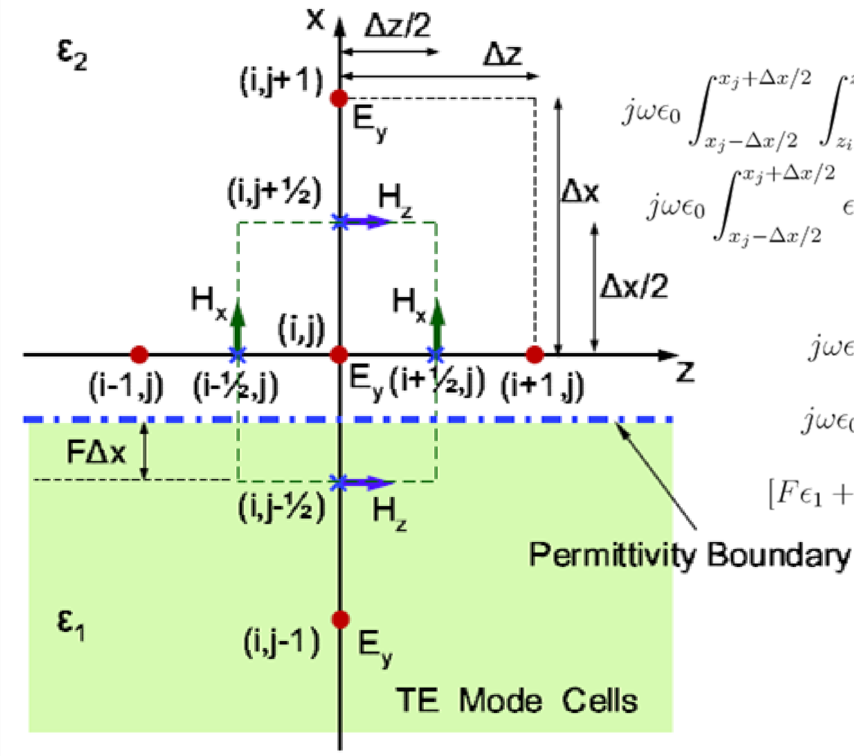
$$
\tilde{H}'_z = [H'^{1+1/2}_z, H'^{2+1/2}_z, \cdots, H'^{M+1/2}_z]^T
$$

$$
\tilde{\epsilon}^j_r = \epsilon_r(x_j)
$$

$$
\tilde{D}^E_{x'} = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & -1 & 1 \\ 0 & \dots & \dots & 0 & 0 & -1 \end{bmatrix}, \quad
\tilde{D}^H_{x'} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & -1 & 1 & 0 \\ 0 & \dots & \dots & 0 & -1 & 1 \end{bmatrix}
$$

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL
(Solving Maxwell's Equations)

TE Modes – Permittivity Averaging



$$j\omega \iint_S \vec{D}\cdot d\vec{S} = \oint_C \vec{H}\cdot d\vec{\ell} \implies$$

$$j\omega\epsilon_0 \int_{x_j-\Delta x/2}^{x_j+\Delta x/2}\int_{z_i-\Delta z/2}^{z_i+\Delta z/2} \epsilon_r(x)E_y(z,x)dxdz = \oint_C [H_x dx + H_z dz] \implies$$

$$j\omega\epsilon_0 \int_{x_j-\Delta x/2}^{x_j+\Delta x/2} \epsilon_r(x)E_y^j(x)dx[e^{-jk_0Nz_i}\Delta z] = -\left(H_z^{j+1/2} - H_z^{j-1/2}\right)[e^{-jk_0Nz_i}\Delta z] - H_x^j[-jk_0N\Delta z e^{-jk_0Nz_i}]\Delta x \implies$$

$$j\omega\epsilon_0 \int_{x_j-\Delta x/2}^{x_j+\Delta x/2} \epsilon_r(x)E_y^j(x)dx = -\left(H_z^{j+1/2} - H_z^{j-1/2}\right) - H_x^j[-jk_0N]\Delta x \implies$$

$$j\omega\epsilon_0 [F\epsilon_1 + (1-F)\epsilon_2]E_y^j\Delta x = -\left(H_z^{j+1/2} - H_z^{j-1/2}\right) - H_x^j[-jk_0N]\Delta x \implies$$

$$[F\epsilon_1 + (1-F)\epsilon_2]E_y^j = \epsilon_{r,eff}^j E_y^j = -\frac{H_z'^{j+1/2} - H_z'^{j-1/2}}{\Delta x'} - jNH_x'^j,$$

$$\epsilon_{r,eff}^j = [F\epsilon_1 + (1-F)\epsilon_2] = \epsilon_r^j = \left[F\epsilon_r^{j-1} + (1-F)\epsilon_r^{j+1}\right]$$

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING YEE's CELL

(Solving Maxwell's Equations)

Example: n_c= 1.0, n_f =3.4, n_s=3.1, h =1.0 μm, λ_0=1.3μm

TE Modes						
	Helmholtz		Yee's Cell No Averaging		Yee's Cell Averaging	
Δx	N_0^{TE}	N_1^{TE}	N_0^{TE}	N_1^{TE}	N_0^{TE}	N_1^{TE}
$\lambda_0/20$	3.3585913	3.2362825	3.3585914	3.2362911	3.3570617	3.2310872
$\lambda_0/40$	3.3579455	3.2333453	3.3579314	3.2332928	3.3575479	3.2319896
$\lambda_0/60$	3.3578155	3.2327610	3.3578157	3.2327737	3.3576393	3.2321748
$\boldsymbol{\lambda_0/80}$	**3.3577736**	**3.2325729**	**3.3577720**	**3.2325782**	**3.3576744**	**3.2322471**
$\lambda_0/100$	3.3577538	3.2324843	3.3577531	3.2324938	3.3576897	3.2322788
$\lambda_0/120$	3.3577424	3.2324333	3.3577426	3.2324471	3.3576981	3.2322964
$\lambda_0/150$	3.3577336	3.2323938	3.3577338	3.2324078	3.3577053	3.2323113
$\lambda_0/200$	3.3577268	3.2323632	3.3577268	3.2323769	3.3577109	3.2323231
$\lambda_0/500$	3.3577193	3.2323297	3.3577195	3.2323443	3.3577169	3.2323357
$\lambda_0/750$	3.3577185	3.2323262	3.3577187	3.2323409	3.3577176	3.2323371
$\lambda_0/1000$	3.3577183	3.2323250	3.3577184	3.2323397	3.3577178	3.2323376
Exact	**3.3577180**	**3.2323308**	**3.3577180**	**3.2323308**	**3.3577180**	**3.2323308**

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL

(Solving Maxwell's Equations)

Example: n_c= 1.0, n_f =3.4, n_s=3.1, h =1.0 μm, λ_0=1.3μm

TE Modes

	Helmholtz		Yee's Cell No Averaging		Yee's Cell Averaging	
Δx	N_0^{TE} Error (%)	N_1^{TE} Error (%)	N_0^{TE} Error (%)	N_1^{TE} Error (%)	N_0^{TE} Error (%)	N_1^{TE} Error (%)
$\lambda_0/20$	-0.0260	-0.1223	-0.0260	-0.1225	0.0195	0.0385
$\lambda_0/40$	-0.0068	-0.0314	-0.0064	-0.0298	0.0051	0.0106
$\lambda_0/60$	-0.0029	-0.0133	-0.0029	-0.0137	0.0023	0.0048
$\lambda_0/80$	**-0.0017**	**-0.0075**	**-0.0016**	**-0.0077**	**0.0013**	**0.0026**
$\lambda_0/100$	-0.0011	-0.0047	-0.0010	-0.0050	0.0008	0.0016
$\lambda_0/120$	-0.0007	-0.0032	-0.0007	-0.0036	0.0006	0.0011
$\lambda_0/150$	-0.0005	-0.0019	-0.0005	-0.0024	0.0004	0.0006
$\lambda_0/200$	-0.0003	-0.0010	-0.0003	-0.0014	0.0002	0.0002
$\lambda_0/500$	0.0000	0.0000	0.0000	-0.0004	0.0000	-0.0002
$\lambda_0/750$	0.0000	0.0001	0.0000	-0.0003	0.0000	-0.0002
$\lambda_0/1000$	0.0000	0.0002	0.0000	-0.0003	0.0000	-0.0002

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL
(Solving Maxwell's Equations)

TM Modes

$$
-\frac{\partial H'_y}{\partial z'} = \epsilon_r E_x,
$$

$$
\frac{\partial H'_y}{\partial x'} = \epsilon_r E_z,
$$

$$
\frac{\partial E_x}{\partial z'} - \frac{\partial E_z}{\partial x'} = H'_y,
$$

$$
H'_w = -j Z_0 H_w
$$

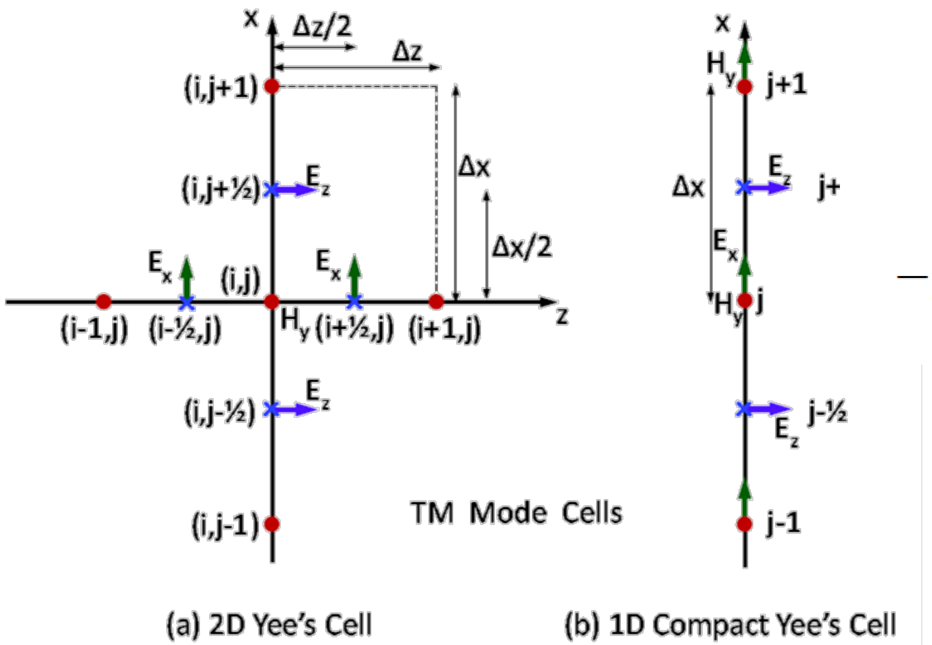
$x' = k_0 x$ and $z' = k_0 z$ are normalized coordinates.

R. C. Rumpf, PIERS B, vol. 36, 221-248 (2012)

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL

(Solving Maxwell's Equations)

TM Modes



$$
\begin{aligned}
jN\tilde{H}_y' &= \tilde{\epsilon}_{rxx}\tilde{E}_x, \\
\frac{1}{\Delta x'}\tilde{D}_{x'}^H \tilde{H}_y' &= \tilde{\epsilon}_{rzz}\tilde{E}_z, \\
-jN\tilde{E}_x - \frac{1}{\Delta x'}\tilde{D}_{x'}^E \tilde{E}_z &= \tilde{H}_y'.
\end{aligned}
$$

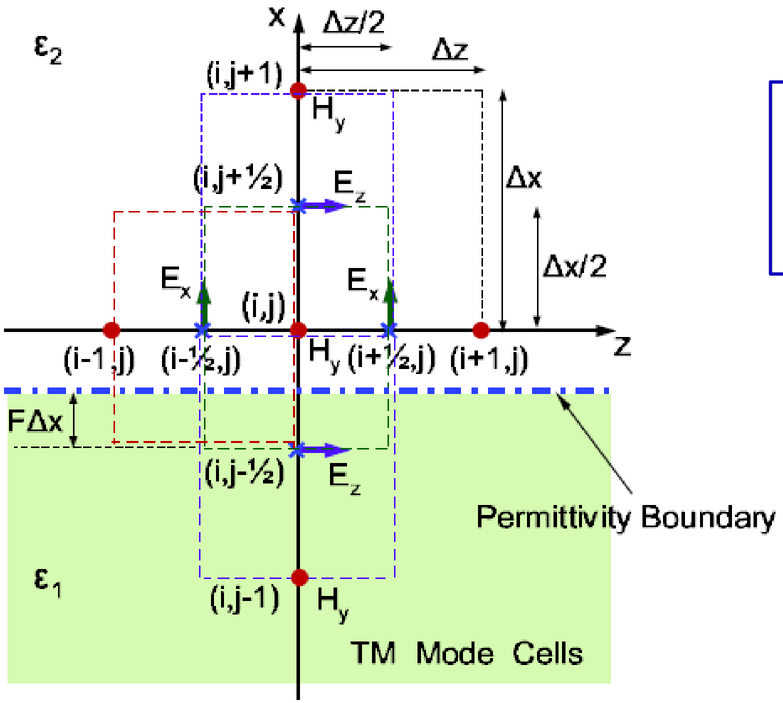
$$
\tilde{\epsilon}_{rxx}^j = \epsilon_r(x_j) \quad \tilde{\epsilon}_{rzz}^j = \epsilon_r(x_{j+1/2})
$$

$$
\left\{\tilde{\epsilon}_{rxx} + \frac{1}{(\Delta x')^2}\,\tilde{\epsilon}_{rxx}\tilde{D}_{x'}^E\ \tilde{\epsilon}_{rzz}^{-1}\ \tilde{D}_{x'}^H\right\}\tilde{H}_y = N^2\tilde{H}_y
$$

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL

(Solving Maxwell's Equations)

TM Modes



$$\epsilon_{r,eff} = \left[\frac{F}{\epsilon_{r1}} + \frac{1-F}{\epsilon_{r2}}\right]^{-1} = \epsilon_{rxx}^{j}$$

$$\epsilon_{r,eff} = \begin{cases} F_1\epsilon_{r1} + (1-F_1)\epsilon_{r2} = \epsilon_{rzz}^{j-1/2}, & F_1 = F + 1/2, \quad \text{for} \quad F \leq 1/2, \\ F_2\epsilon_{r1} + (1-F_2)\epsilon_{r2} = \epsilon_{rzz}^{j+1/2}, & F_2 = F - 1/2, \quad \text{for} \quad F > 1/2. \end{cases}$$

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD) ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL

(Solving Maxwell's Equations)

Example: n_c= 1.0, n_f =3.4, n_s=3.1, h =1.0 μm, λ_0=1.3μm

	TM Modes					
	Helmholtz Kawano [3]		Yee's Cell No Averaging		Yee's Cell Averaging	
Δx	N_0^{TM}	N_1^{TM}	N_0^{TM}	N_1^{TM}	N_0^{TM}	N_1^{TM}
$\lambda_0/20$	3.3518412	3.2129633	3.3496125	3.2041094	3.3512255	3.2110965
$\lambda_0/40$	3.3515178	3.2109972	3.3504428	3.2067131	3.3513622	3.2105083
$\lambda_0/60$	3.3514548	3.2106177	3.3507380	3.2077721	3.3513869	3.2104143
$\boldsymbol{\lambda_0/80}$	**3.3514346**	**3.2104963**	**3.3509033**	**3.2083858**	**3.3513963**	**3.2103793**
$\lambda_0/100$	3.3514251	3.2104392	3.3509982	3.2087442	3.3514004	3.2103641
$\lambda_0/120$	3.3514196	3.2104064	3.3510631	3.2089916	3.3514026	3.2103558
$\lambda_0/150$	3.3514154	3.2103809	3.3511305	3.2092505	3.3514045	3.2103487
$\lambda_0/200$	3.3514215	3.2103612	3.3511995	3.2095179	3.3514060	3.2103430
$\lambda_0/500$	3.3514086	3.2103396	3.3513237	3.2100031	3.3514076	3.2103368
$\lambda_0/750$	3.3514082	3.2103373	3.3513516	3.2101129	3.3514078	3.2103361
$\lambda_0/1000$	3.3514081	3.2103364	3.3513656	3.2101682	3.3514078	3.2103358
Exact	**3.3514080**	**3.2103532**	**3.3514080**	**3.2103532**	**3.3514080**	**3.2103532**

FINITE-DIFFERENCE FREQUENCY-DOMAIN (FDFD)
ANALYSIS OF SLAB WAVEGUIDES USING **YEE's** CELL
(Solving Maxwell's Equations)

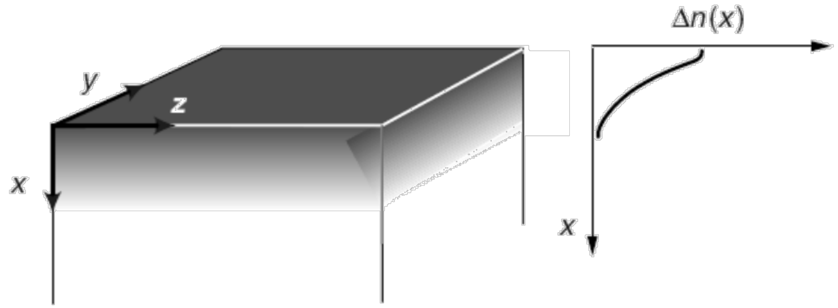
Example: n_c= 1.0, n_f =3.4, n_s=3.1, h =1.0 μm, λ_0=1.3μm

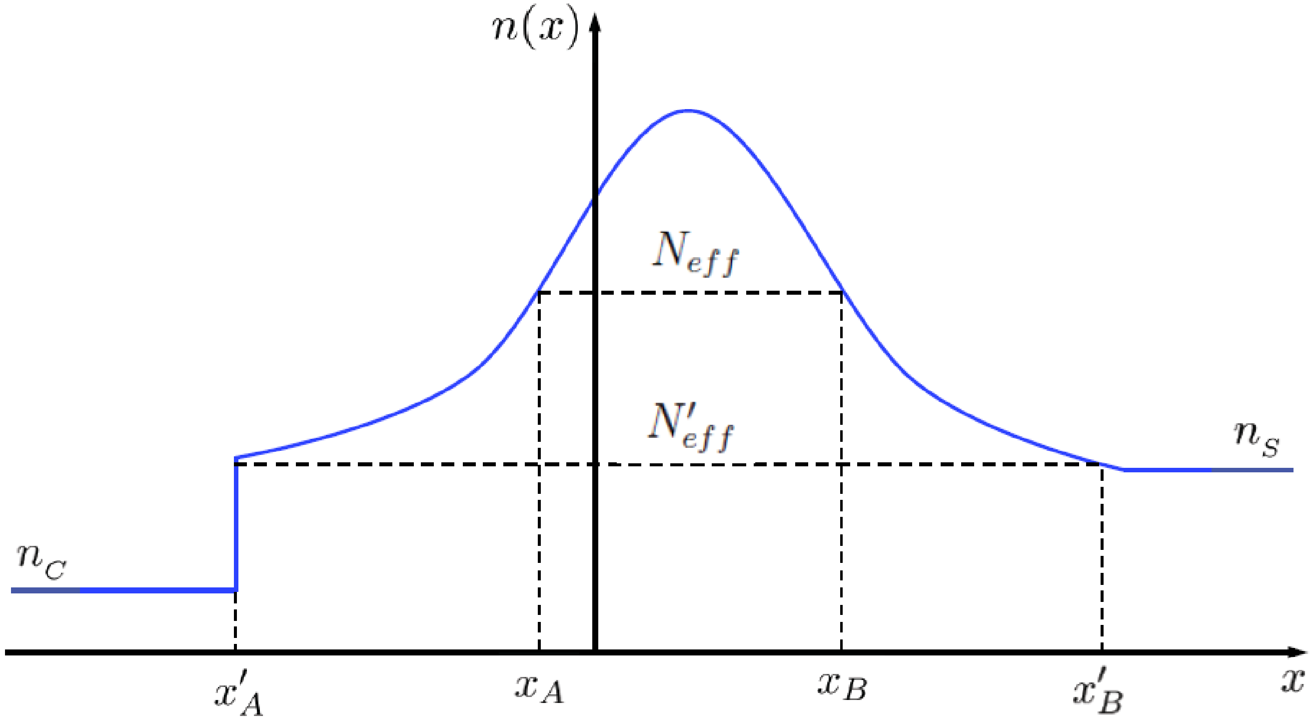
TM Modes

Δx	Helmholtz Kawano [3] N_0^{TM} Error (%)	N_1^{TM} Error (%)	Yee's Cell No Averaging N_0^{TM} Error (%)	N_1^{TM} Error (%)	Yee's Cell Averaging N_0^{TM} Error (%)	N_1^{TM} Error (%)
$\lambda_0/20$	-0.0129	-0.0813	0.0536	0.1945	0.0054	-0.0232
$\lambda_0/40$	-0.0033	-0.0201	0.0297	0.1134	0.0014	-0.0048
$\lambda_0/60$	-0.0014	-0.0082	0.0200	0.0804	0.0006	-0.0019
$\lambda_0/80$	**-0.0008**	**-0.0045**	**0.0153**	**0.0613**	**0.0003**	**-0.0008**
$\lambda_0/100$	-0.0005	-0.0027	0.0124	0.0501	0.0002	-0.0003
$\lambda_0/120$	-0.0003	-0.0017	0.0103	0.0424	0.0002	-0.0001
$\lambda_0/150$	-0.0002	-0.0009	0.0083	0.0343	0.0001	0.0001
$\lambda_0/200$	-0.0004	-0.0002	0.0063	0.0260	0.0001	0.0003
$\lambda_0/500$	0.0000	0.0004	0.0025	0.0109	0.0000	0.0005
$\lambda_0/750$	0.0000	0.0005	0.0017	0.0075	0.0000	0.0005
$\lambda_0/1000$	0.0000	0.0005	0.0013	0.0058	0.0000	0.0005

GRADED-INDEX SLAB WAVEGUIDES

Example of
Graded-index
Slab Waveguide





For TE Modes:

$$\frac{d^2E_y}{dx^2} + (k_0^2 n^2(x) - \beta^2)E_y(x) = 0.$$

WKB Method (Wentzel-Kramers-Brillouin) $E_y = \psi(x)$

$$\psi = \psi_0 \exp[jk_0 S(x)]$$

$$S(x) = S_0(x) + \frac{1}{k_0}S_1(x) + \frac{1}{k_0^2}S_2(x) + \cdots,$$

$$\psi(x) = \psi_0 \exp[jk_0 S_0(x) + jS_1(x) + j\frac{1}{k_0}S_2(x) + \cdots],$$

$$S_0(x) = \frac{1}{k_0}\int [k_0^2 n^2(x) - \beta^2]^{1/2} dx,$$

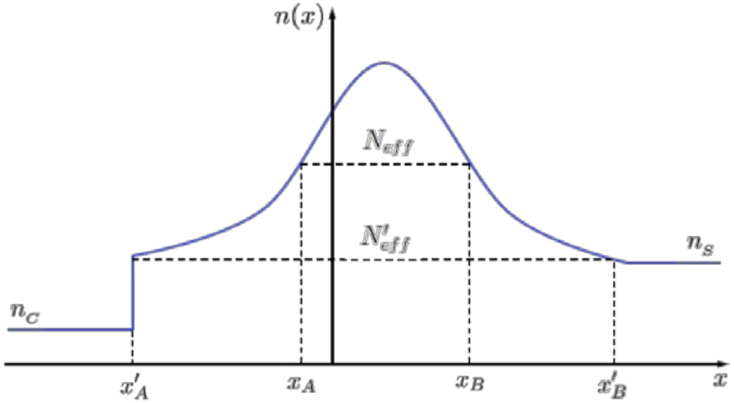
$$S_1(x) = \frac{j}{2}\ln\left|\frac{dS_0}{dx}\right|,$$

GRADED-INDEX SLAB WAVEGUIDES

WKB solution in general

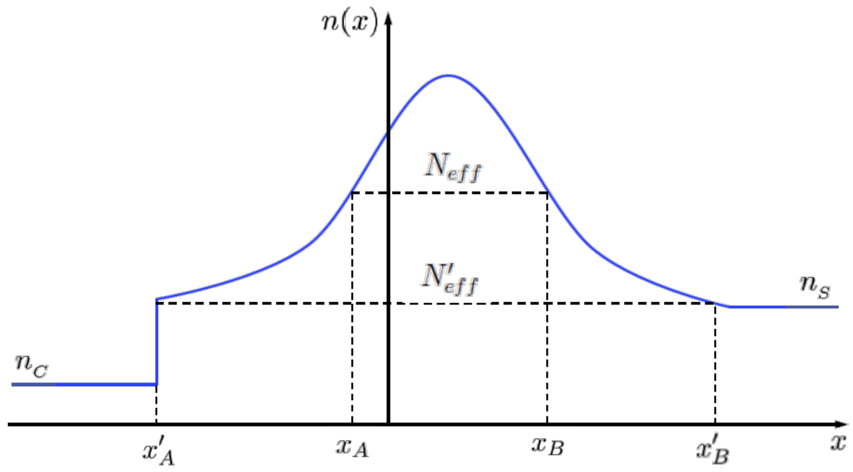
$$E_y(x) = \psi(x) = \begin{cases} \dfrac{\psi_0}{\sqrt{Q}} \exp[\pm j \int Q dx], & \text{if } [k_0^2 n^2(x) - \beta^2] = Q^2 > 0, \\ \dfrac{\psi_0}{\sqrt{P}} \exp[\pm \int P dx], & \text{if } [k_0^2 n^2(x) - \beta^2] = -P^2 < 0. \end{cases}$$

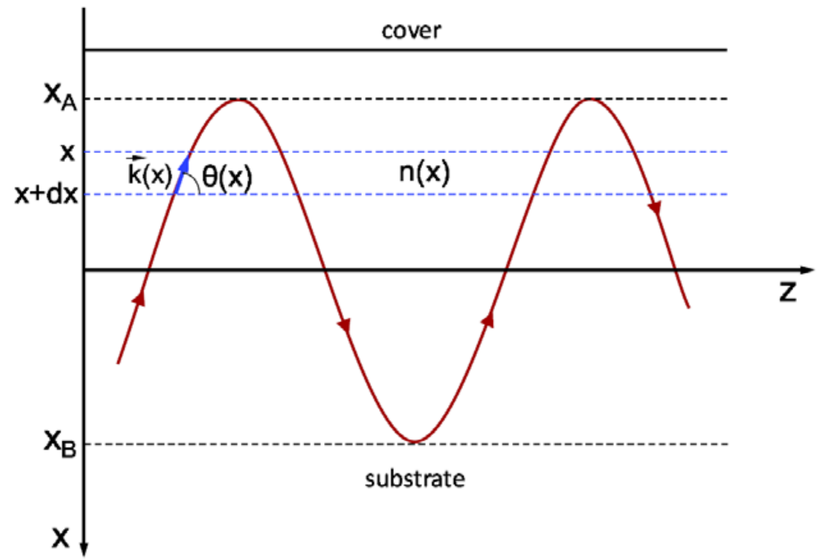
WKB for the profile shown



$$E_y(x) = \psi(x) = \begin{cases} \dfrac{\psi_0}{2[\beta^2 - k_0^2 n^2(x)]^{1/4}} \exp\left[-\int_x^{x_A} [\beta^2 - k_0^2 n^2(x')]^{1/2} dx'\right], & x < x_A, \\ \dfrac{\psi_0}{[k_0^2 n^2(x) - \beta^2]^{1/4}} \cos\left[\int_{x_A}^{x} [k_0^2 n^2(x') - \beta^2]^{1/2} dx' - \dfrac{\pi}{4}\right], & x_A < x < x_B, \\ \dfrac{\psi_0}{2[\beta^2 - k_0^2 n^2(x)]^{1/4}} \exp\left[-\int_{x_B}^{x} [\beta^2 - k_0^2 n^2(x')]^{1/2} dx'\right], & x > x_B, \end{cases}$$

GRADED-INDEX SLAB WAVEGUIDES





$$k_x^2(x) + \beta^2 = k_0^2 n^2(x) = |\vec{k}(x)|^2.$$

$$\boxed{-2\int_{x_A}^{x_B} k_x(x)dx + \phi_A + \phi_B = 2\nu\pi, \qquad \nu = 0, \pm 1, \pm 2, \cdots,}$$

GRADED-INDEX SLAB WAVEGUIDES

$$x_A - \Delta x < x < x_A + \Delta x$$

$$r = \frac{k_x(x_A + \Delta x) - k_x(x_A - \Delta x)}{k_x(x_A + \Delta x) + k_x(x_A - \Delta x)} = \frac{1 - \dfrac{k_x(x_A - \Delta x)}{k_x(x_A + \Delta x)}}{1 + \dfrac{k_x(x_A - \Delta x)}{k_x(x_A + \Delta x)}},$$

$$r = \frac{1 - \dfrac{[n^2(x_A - \Delta x) - N^2]^{1/2}}{[n^2(x_A + \Delta x) - N^2]^{1/2}}}{1 + \dfrac{[n^2(x_A - \Delta x) - N^2]^{1/2}}{[n^2(x_A + \Delta x) - N^2]^{1/2}}}.$$

$$\begin{aligned} n^2(x_A - \Delta x) &\simeq n^2(x_A) - \frac{dn}{dx}\Delta x = N^2 - \frac{dn}{dx}\Delta x, \\ n^2(x_A + \Delta x) &\simeq n^2(x_A) + \frac{dn}{dx}\Delta x = N^2 + \frac{dn}{dx}\Delta x, \end{aligned}$$

$$\begin{aligned} r &= \frac{1 - \left\{\dfrac{[n^2(x_A - \Delta x) - N^2]}{[n^2(x_A + \Delta x) - N^2]}\right\}^{1/2}}{1 + \left\{\dfrac{[n^2(x_A - \Delta x) - N^2]}{[n^2(x_A + \Delta x) - N^2]}\right\}^{1/2}} \simeq \frac{1 - \left\{\dfrac{-(dn/dx)\Delta x}{+(dn/dx)\Delta x}\right\}^{1/2}}{1 + \left\{\dfrac{-(dn/dx)\Delta x}{+(dn/dx)\Delta x}\right\}^{1/2}} = \\ &= \frac{1 - j}{1 + j} = 1 \exp\left(-j\frac{\pi}{2}\right) = 1 \exp(j\phi_A). \end{aligned}$$

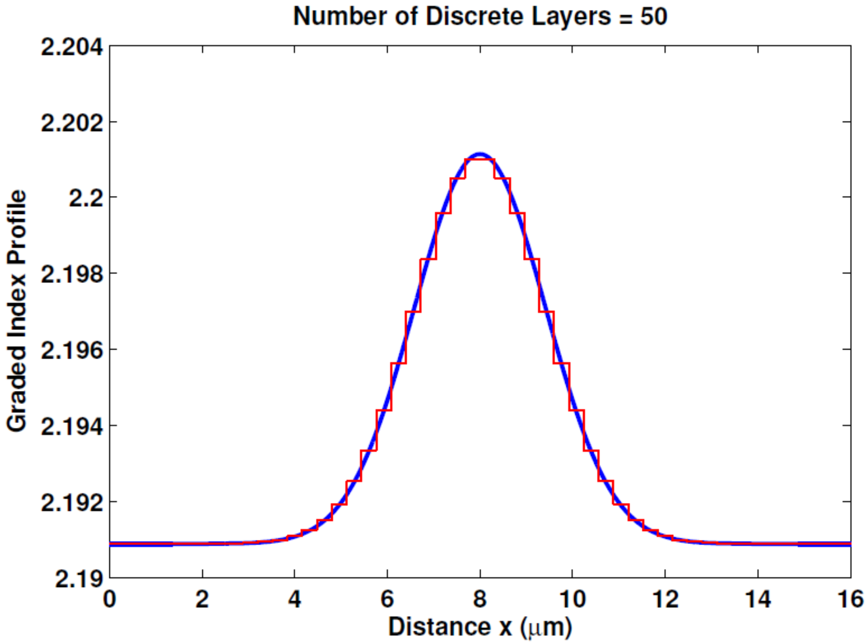
WKB - Dispersion Equation for Graded Index Slab Waveguide

$$k_0 \int_{x_A = n^{-1}(N)}^{x_B = n^{-1}(N)} \sqrt{(n^2(x') - N^2)}\, dx' = \left(\nu + \frac{1}{2}\right)\pi, \qquad \nu = 0, 1, 2, \cdots.$$

GRADED-INDEX SLAB WAVEGUIDES

Example Case 1: ε_s=4.80, $\Delta\varepsilon$=0.045, x_0 = 8μm, w_0 = 2μm, λ_0 = 0.6328μm

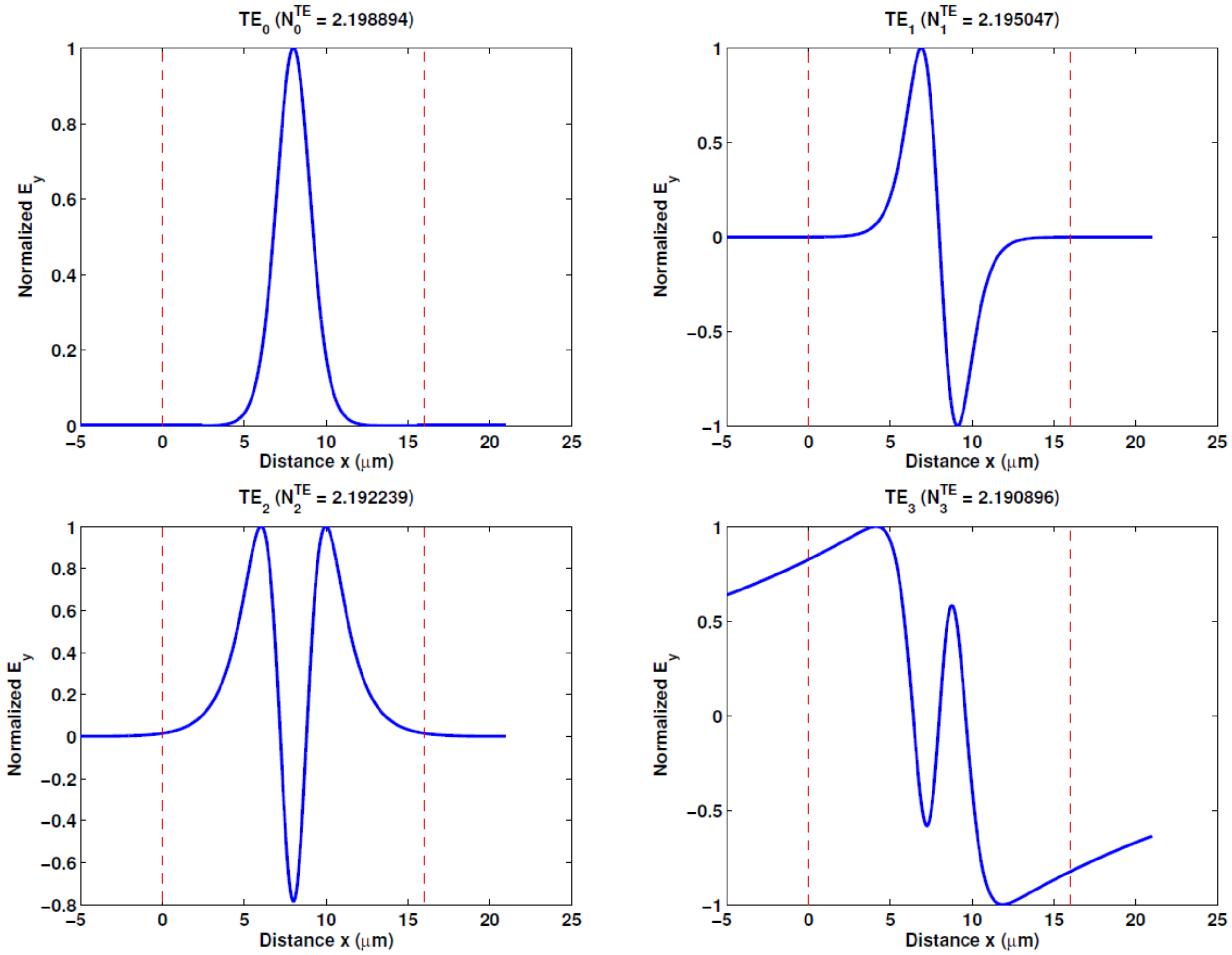
$$n^2(x) = \varepsilon_s + \Delta\varepsilon \exp\left[-\frac{(x-x_0)^2}{w_0^2}\right],$$



TE_ν	Multilayered Waveguide Approximation *Number of Layers* = 50 N_ν	Finite Difference $\Delta x = 0.05\mu$m N_ν	Finite Difference $\Delta x = 0.025\mu$m N_ν	WKB Approximation N_ν
TE_0	2.198894532	2.198926188	2.198925969	2.198818251
TE_1	2.195047282	2.194992344	2.194991579	2.194884141
TE_2	2.192239107	2.192152744	2.192151661	2.192049954
TE_3	2.190896246	Not Found	Not Found	Not Found

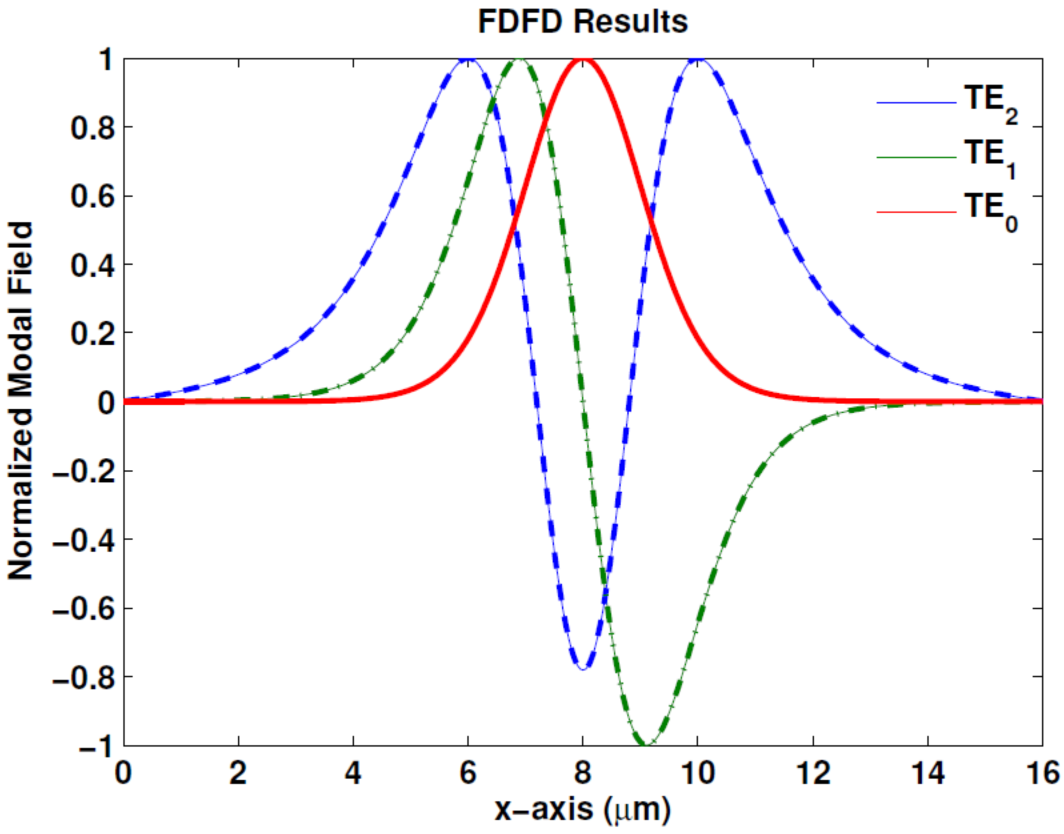
GRADED-INDEX SLAB WAVEGUIDES

Example Case 1: ε_s=4.80, $\Delta\varepsilon$=0.045, x_0 = 8μm, w_0 = 2μm, λ_0 = 0.6328μm



GRADED-INDEX SLAB WAVEGUIDES

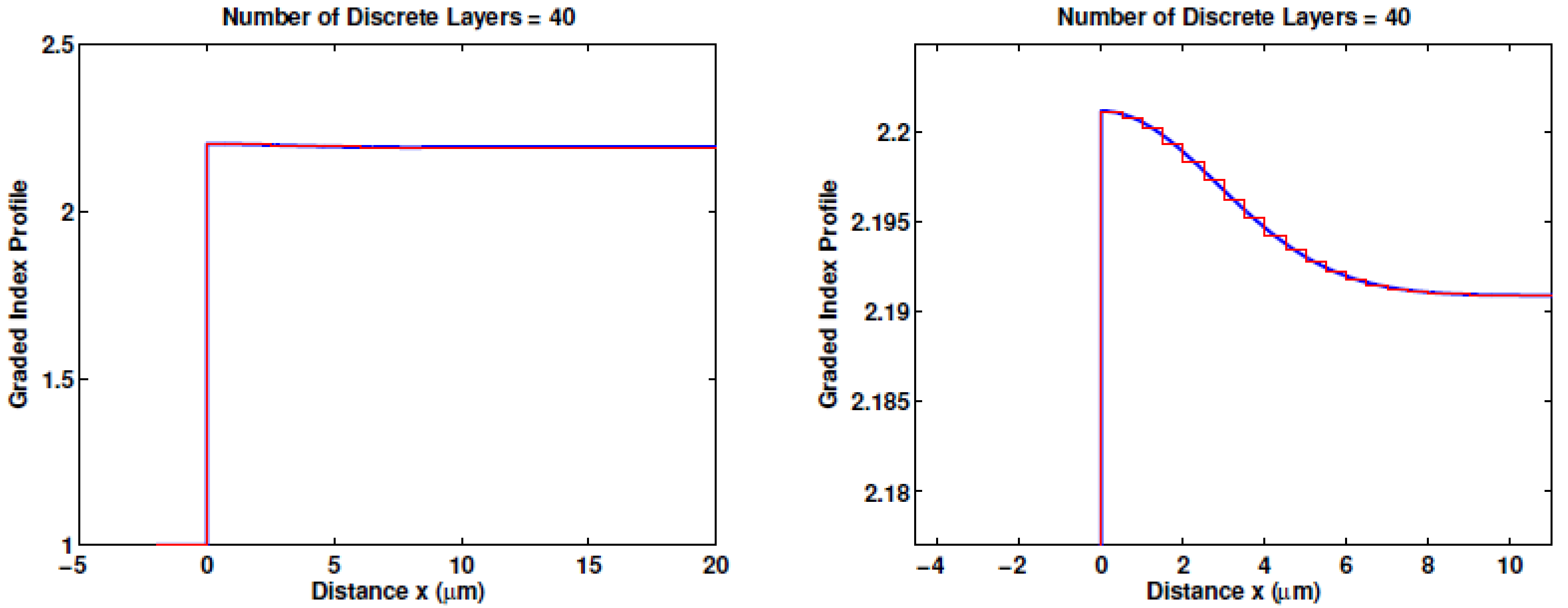
Example Case 1: ε_s=4.80, $\Delta\varepsilon$=0.045, x_0 = 8μm, w_0 = 2μm, λ_0 = 0.6328μm



GRADED-INDEX SLAB WAVEGUIDES

Example Case 2: ε_s=4.80, $\Delta\varepsilon$=0.045, w_0 = 4μm, λ_0 = 0.6328μm

$$n^2(x) = \begin{cases} \varepsilon_s + \Delta\varepsilon \exp\left[-\dfrac{x^2}{w_0^2}\right], & \text{if } x > 0, \\ n_c^2, & \text{if } x < 0. \end{cases}$$



Example Case 2: ε_s=4.80, $\Delta\varepsilon$=0.045, w_0 = 4μm, λ_0 = 0.6328μm

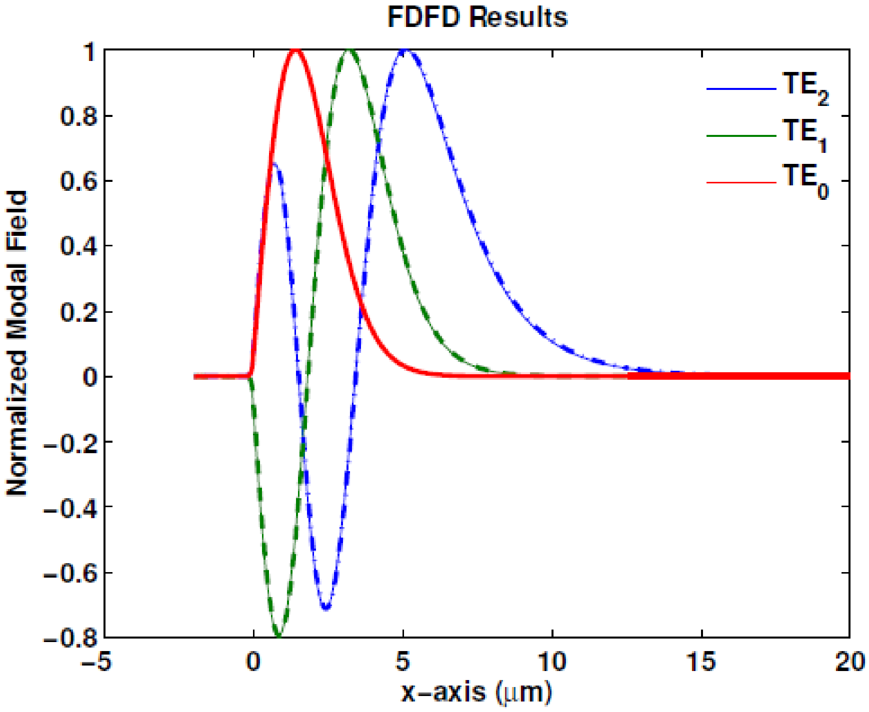
WKB Modified Dispersion Equation

$$k_0 \int_{x_A=0}^{x_B=n^{-1}(N)} \sqrt{(n^2(x') - N^2)}\, dx' - \tan^{-1}\left\{\left[\frac{N^2 - n_c^2}{\varepsilon_s + \Delta\varepsilon - N^2}\right]^{1/2}\right\} - \frac{\pi}{4} = \nu\pi, \qquad \nu = 0, 1, 2, \cdots .$$

TE_ν	Multilayered Waveguide Approximation *Number of Layers* = 40 N_ν	Finite Difference $\Delta x = 0.05\mu$m N_ν	Finite Difference $\Delta x = 0.025\mu$m N_ν	WKB Approximation N_ν
TE_0	2.197868744	2.197905044	2.197877837	2.197825988
TE_1	2.194253274	2.194235177	2.194204855	2.194152293
TE_2	2.191721704	2.191680005	2.191658955	2.191615558

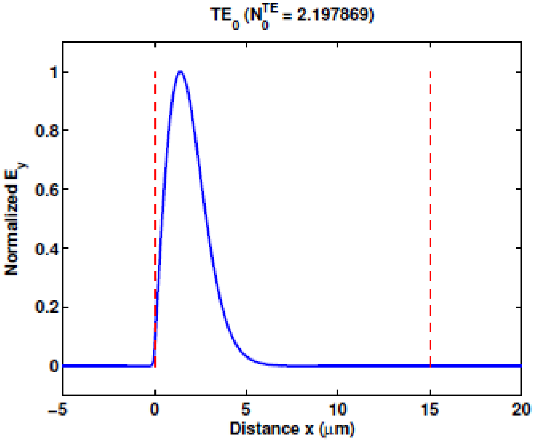
Example Case 2: ε_s=4.80, $\Delta\varepsilon$=0.045, w_0 = 4μm, λ_0 = 0.6328μm

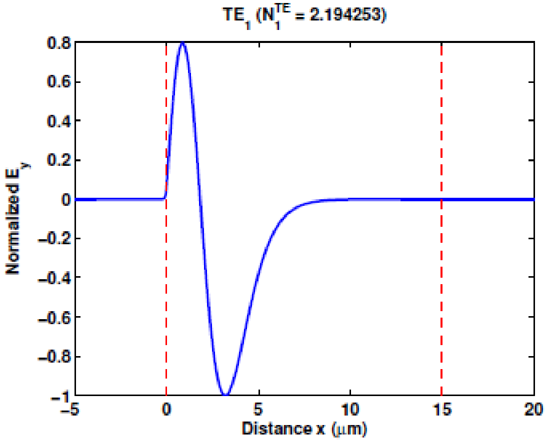
Finite-Difference Frequency-Domain Results

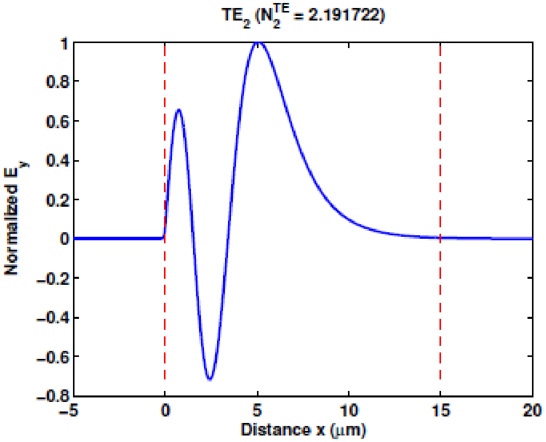


Example Case 2: ε_s=4.80, $\Delta\varepsilon$=0.045, w_0 = 4μm, λ_0 = 0.6328μm

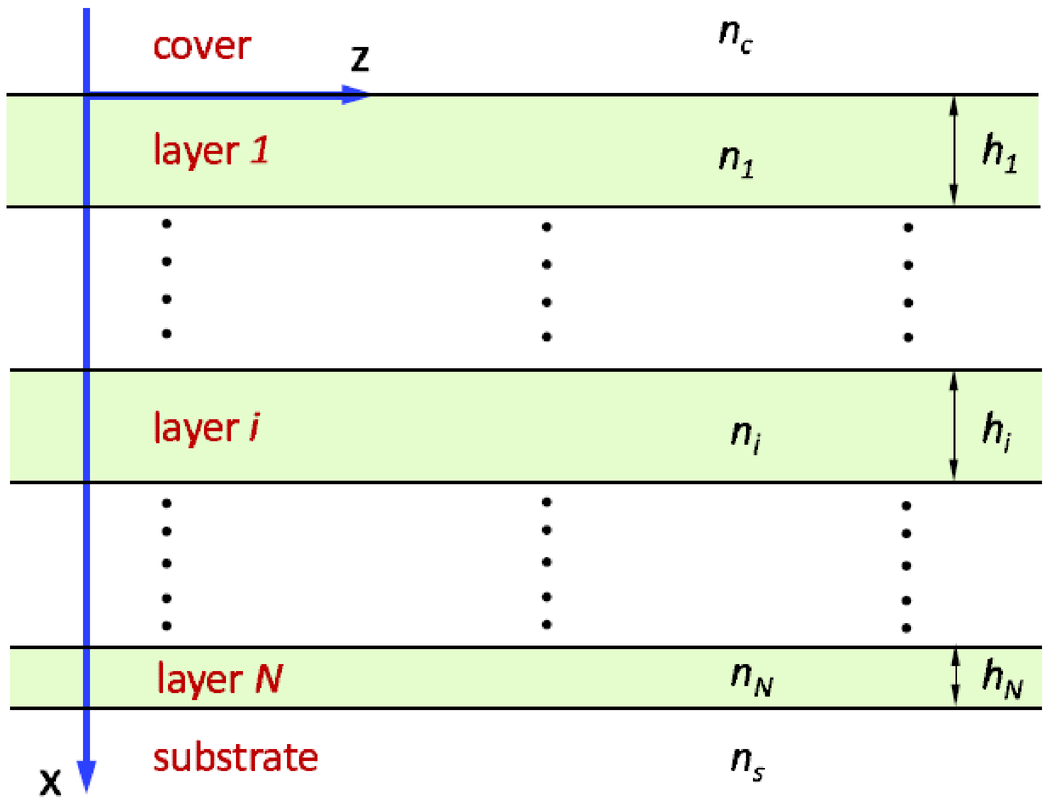
Multilayer Approximation Results (N=40)







LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

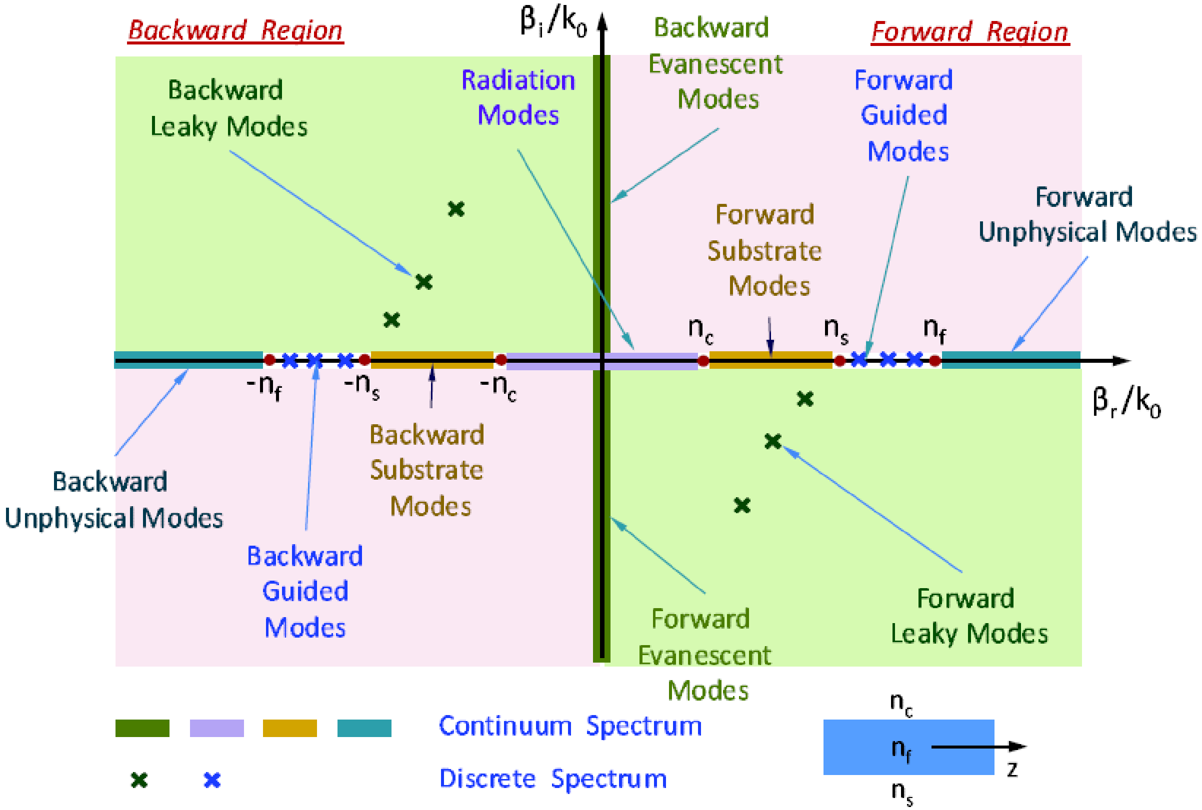


- Some materials could have loss or gain. Their corresponding refractive Indices will be in general complex.
- Then the propagation constant **β** will be complex for guided modes.
- Leaky modes is an approximation of radiation field and will have complex propagation constants.

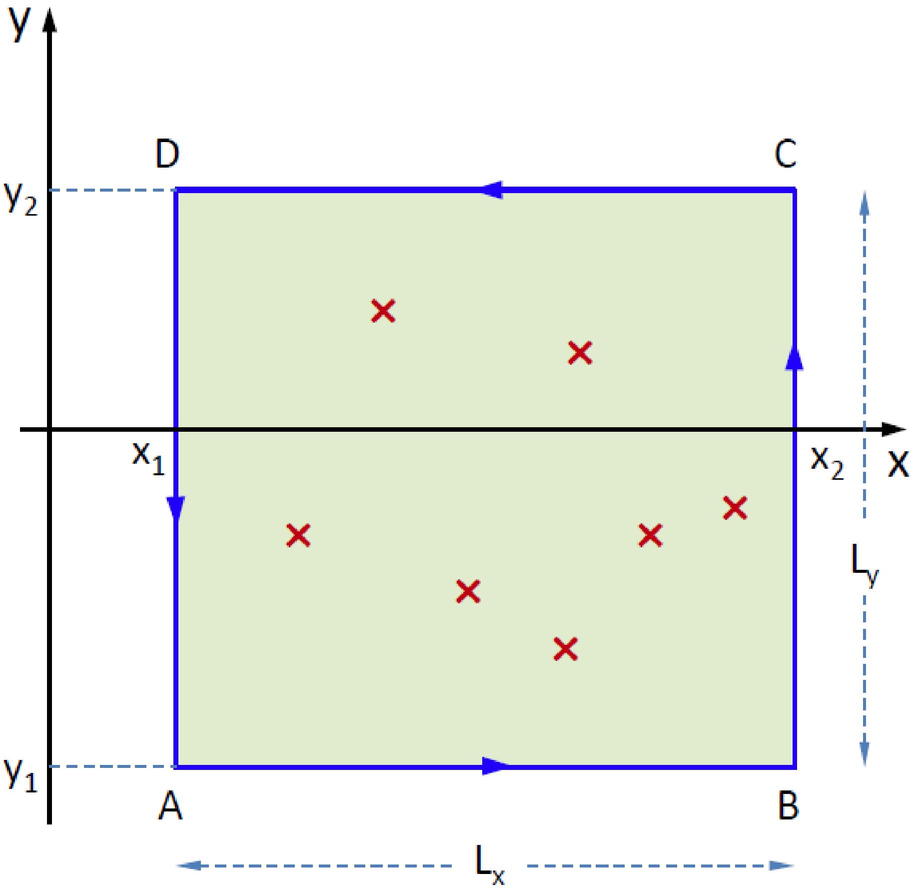
LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Analysis Methods Summarized

- Argument Principle Method (APM).
- Abd-ellal, Delves, Reid Method (ADR).
- Derivative-Free Zero-Extraction by Phase-based Enclosure Method (DFZEPE)



Argument Principle Method (APM)



$$\sigma_k = \frac{1}{j2\pi} \oint_C z^k \, \frac{f'(z)}{f(z)} \, dz = \sum_{i=1}^{N_z} \zeta_i^k$$

$$f'(z_0) = \frac{1}{j2\pi} \oint_D \frac{f(z)}{(z - z_0)^2} \, dz$$

$$S(z) = \sum_{\ell=0}^{N_z} c_\ell \, z^\ell$$

Müller’s Refinement Process
is used with guess the polynomial roots

$$(N_z - \ell) \, c_\ell + \sigma_1 c_{\ell+1} + \sigma_2 c_{\ell+2} + \cdots + \sigma_{N_z-\ell} c_{N_z} = 0 \quad \text{for} \quad \ell = N_z - 1, \cdots, 0 \, .$$

E. Anemogiannis and E. N. Glytsis, JLT vol. 10, 1344-1352 (1992)

Argument Principle Method (APM) - Example

$$f(z) = e^{3z} + 2z\cos z - 1$$

$$f'(z) = 3e^{3z} + 2\cos z - 2z\sin z$$

APM σ_k's of function $f(z) = e^{3z} + 2z\cos z - 1$

k	σ_k Coefficients
0	$6.0000000000000 - j0.000000000000000$
1	$2.0467702626400 - j0.000000000000000$
2	$-14.1574340854587 - j0.000000000000001$
3	$-84.7999601511681 - j0.000000000000007$
4	$-30.4653454987963 + j0.000000000000006$
5	$696.0895474735294 - j0.000000000000028$
6	$2527.5240087190520 + j0.000000000000123$

$\sigma_0 = N_z = 6$

APM zeros estimates and refined zeros

APM Zeros Estimates	Zeros after Müller's Refinement
$-1.844233953262218 - j0.000000000000002$	$-1.844233953262213 + j0.000000000000000$
$0.000000000000019 + j0.000000000000007$	$0.000000000000000 - j0.000000000000000$
$0.530894930292922 - j1.331791876751122$	$0.530894930292931 - j1.331791876751121$
$0.530894930292921 + j1.331791876751117$	$0.530894930292931 + j1.331791876751121$
$1.414607177658184 - j3.047722062627170$	$1.414607177658184 - j3.047722062627173$
$1.414607177658189 + j3.047722062627174$	$1.414607177658184 + j3.047722062627173$

C. J. Gillan et al.,,Comp. Phys. Commun., 175 (,304-313), 2006

Argument Principle Method (APM) - Waveguides

$$F(\beta) = \frac{\gamma_s}{a_s} m_{22} + \frac{\gamma_c}{a_c} m_{11} - \frac{\gamma_c}{a_c}\frac{\gamma_s}{a_s} m_{12} - m_{21} = 0$$

$a_c = a_s = 1$ for TE polarization, and $a_c = n_c^2$, $a_s = n_s^2$ for TM polarization

$$\beta = k_0 N_{eff} = k_0 (N_{eff,r} + j N_{eff,i})$$

$$\begin{aligned} \frac{dF}{dN_{eff}} = & \frac{1}{a_s}\frac{d\gamma_s}{dN_{eff}} m_{22} + \frac{\gamma_s}{a_s}\frac{dm_{22}}{dN_{eff}} + \frac{1}{a_c}\frac{d\gamma_c}{dN_{eff}} m_{11} + \frac{\gamma_c}{a_c}\frac{dm_{11}}{dN_{eff}} \\ & - \frac{\gamma_s}{a_s a_c}\frac{d\gamma_c}{dN_{eff}} m_{12} - \frac{\gamma_c}{a_s a_c}\frac{d\gamma_s}{dN_{eff}} m_{12} - \frac{\gamma_c \gamma_s}{a_c a_s}\frac{dm_{12}}{dN_{eff}} - \frac{dm_{21}}{dN_{eff}} \end{aligned}$$

Argument Principle Method (APM) - Waveguides

$$\tilde{M}_i = \begin{bmatrix} \cos(k_{xi}h_i) & -\dfrac{\sin(k_{xi}h_i)}{k_{xi}} \\ k_{xi}\sin(k_{xi}h_i) & \cos(k_{xi}h_i) \end{bmatrix} \quad \text{if} \quad k_{xi} \neq 0,$$

$$\tilde{M}_i = \begin{bmatrix} 1 & -h_i \\ 0 & 1 \end{bmatrix}, \quad \text{if} \quad k_{xi} = 0,$$

$$\frac{d\gamma_c}{dN_{eff}} = k_0^2 \frac{N_{eff}}{\gamma_c},$$

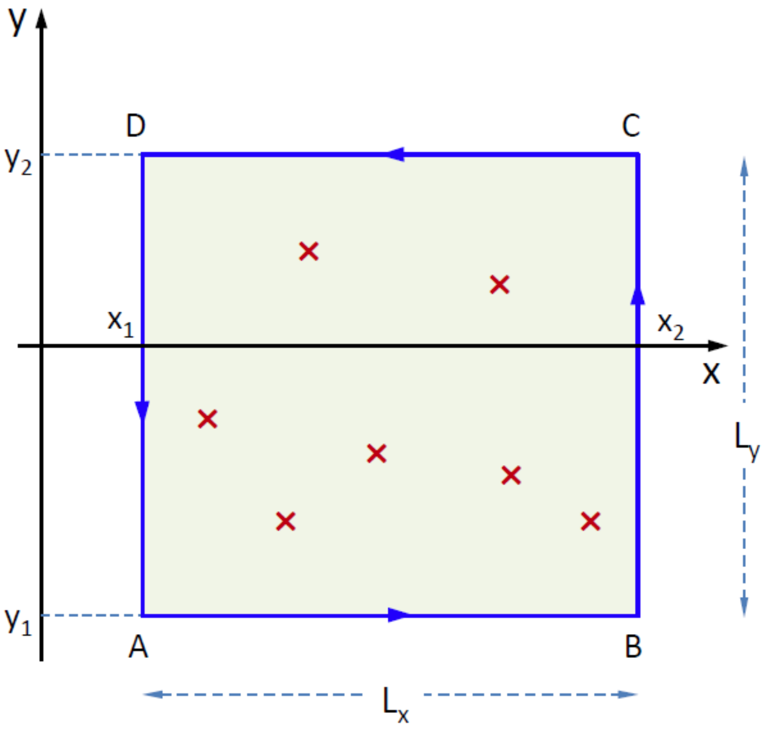
$$\frac{d\gamma_s}{dN_{eff}} = k_0^2 \frac{N_{eff}}{\gamma_s},$$

$$\frac{d\tilde{M}}{dN_{eff}} = \sum_{i=1}^{N} \left(\frac{d\tilde{M}_i}{dN_{eff}} \prod_{\substack{j=1 \\ j\neq i}}^{N} \tilde{M}_j \right)$$

$$\frac{d\tilde{M}_i}{dN_{eff}} = k_0^2 N_{eff} \begin{bmatrix} \dfrac{h_i \sin(k_{xi}h_i)}{k_{xi}} & \dfrac{a_i h_i \cos(k_{xi}h_i)}{k_{xi}^2} - \dfrac{a_i \sin(k_{xi}h_i)}{k_{xi}^3} \\ -\dfrac{\sin(k_{xi}h_i)}{a_i k_{xi}} - \dfrac{h_i}{a_i}\cos(k_{xi}h_i) & \dfrac{h_i \sin(k_{xi}h_i)}{k_{xi}} \end{bmatrix}$$

LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Abd-ellal, Delves, Reid Method (ADR)



$$P_{N_z}(z) = c_0 + c_1 z + c_2 z^2 + \cdots + c_{N_z-1} z^{N_z-1} + z^{N_z}$$

$$\sum_{j=0}^{N_z-1} c_j \, G_{r+j} + G_{r+N_z} = 0, \qquad \text{for} \;\; r = 0, 1, 2, \cdots, (N_z - 1)$$

$$G_k = \frac{1}{j2\pi} \oint_C \frac{z^k}{f(z)} \, dz \quad \text{for} \;\; k = 0, 1, 2, \cdots, (2N_z - 1)$$

L. F. Abd-ellal, L. M. Delves, and J. K. Reid, Numerical Methods for Nonlinear Algebraic Equations, ch. 3, (47-59), Gordon & Breach Ltd. (1970)

LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Abd-ellal, Delves, Reid Method (ADR)

$$N_z = \frac{1}{j2\pi}\oint \frac{f'(z)}{f(z)}dz = \frac{1}{j2\pi}\Delta_C\left\{\ln\left[f(z)\right]\right\} = \frac{1}{2\pi}\Delta_C\left\{\arg\left[f(z)\right]\right\}$$

$$\boldsymbol{H}^{<} = \begin{bmatrix} G_1 & G_2 & G_3 & \dots & G_{N_z} \\ G_2 & G_3 & G_4 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{N_z} & \dots & \dots & \dots & G_{2N_z} \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} G_0 & G_1 & G_2 & \dots & G_{N_z-1} \\ G_1 & G_2 & G_3 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{N_z-1} & \dots & \dots & \dots & G_{2N_z-2} \end{bmatrix}$$

Generalized Eigenvalue Problem

$$\left(\boldsymbol{H}^{<} - \zeta'\boldsymbol{H}\right)\boldsymbol{V} = 0$$

$\zeta'_1, \cdots, \zeta'_{N_z}$ are the estimates of the zeros of the function

Müller's Refinement Process
is used with guess the polynomial roots

C. J. Gillan et al.,,Comp. Phys. Commun., 175 (,304-313), 2006

Abd-ellal, Delves, Reid Method (ADR) - Example

$$f(z) = e^{3z} + 2z\cos z - 1$$

$N_z = 6$

ADR G_k's of function $f(z) = e^{3z} + 2z\cos z - 1$

k	G_k Coefficients
0	$-0.07519739 - j0.00000000$
1	$0.27503184 + j0.00000000$
2	$-0.90593139 - j0.00000000$
3	$2.07560724 + j0.00000000$
4	$-2.01447904 + j0.00000000$
5	$2.44932797 + j0.00000000$
6	$-22.63060397 + j0.00000000$
7	$14.84101715 + j0.00000000$
8	$99.50440329 + j0.00000000$
9	$456.78080464 - j1.832 \times 10^{-13}$
10	$-483.81132715 - j9.047 \times 10^{-14}$
11	$-5297.84498932 - j2.099 \times 10^{-12}$

Abd-ellal, Delves, Reid Method (ADR) - Example

ADR zeros estimates and refined zeros of function $f(z) = e^{3z} + 2z\cos z - 1$

ADR Zeros Estimates	Zeros after Müller's Refinement
$-1.844233953262213 - j0.000000000000000$	$-1.844233953262213 + j0.000000000000000$
$-0.000000000000017 - j0.000000000000032$	$0.000000000000000 - j0.000000000000000$
$0.530894930292934 - j1.331791876751112$	$0.530894930292930 - j1.331791876751121$
$0.530894930292934 + j1.331791876751058$	$0.530894930292931 + j1.331791876751121$
$1.414607177658188 - j3.047722062627168$	$1.414607177658184 - j3.047722062627173$
$1.414607177658189 + j3.047722062627171$	$1.414607177658184 + j3.047722062627173$

LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Abd-ellal, Delves, Reid Method (ADR) - Waveguides

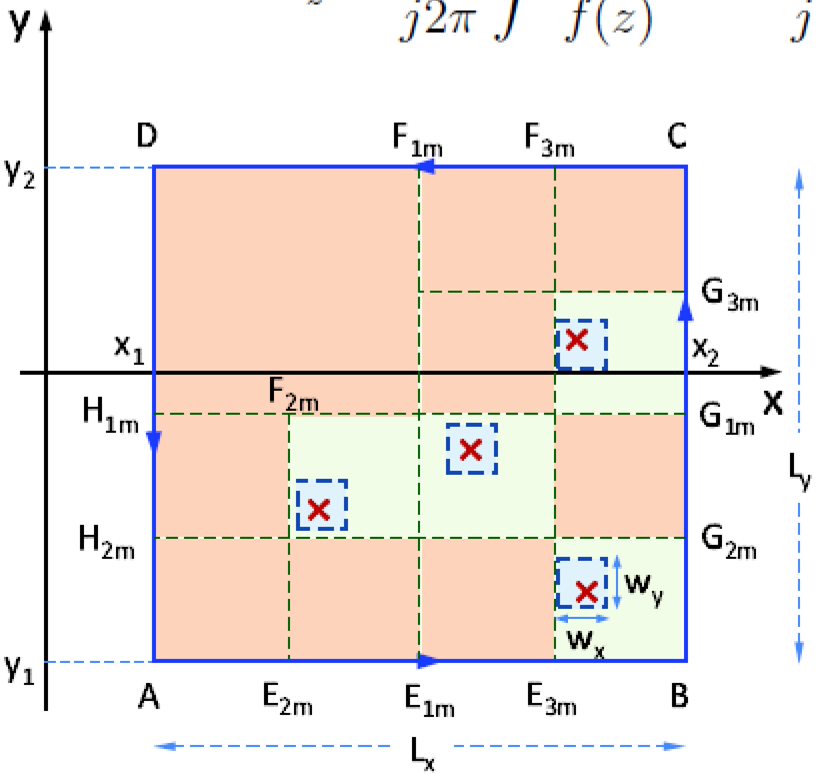
$$F(\beta) = \frac{\gamma_s}{a_s} m_{22} + \frac{\gamma_c}{a_c} m_{11} - \frac{\gamma_c}{a_c}\frac{\gamma_s}{a_s} m_{12} - m_{21} = 0$$

$a_c = a_s = 1$ for TE polarization, and $a_c = n_c^2$, $a_s = n_s^2$ for TM polarization

$$\beta = k_0 N_{eff} = k_0 (N_{eff,r} + jN_{eff,i})$$

Derivative-Free Zero-Extraction by Phase-based Enclosure Method (DFZEPE)

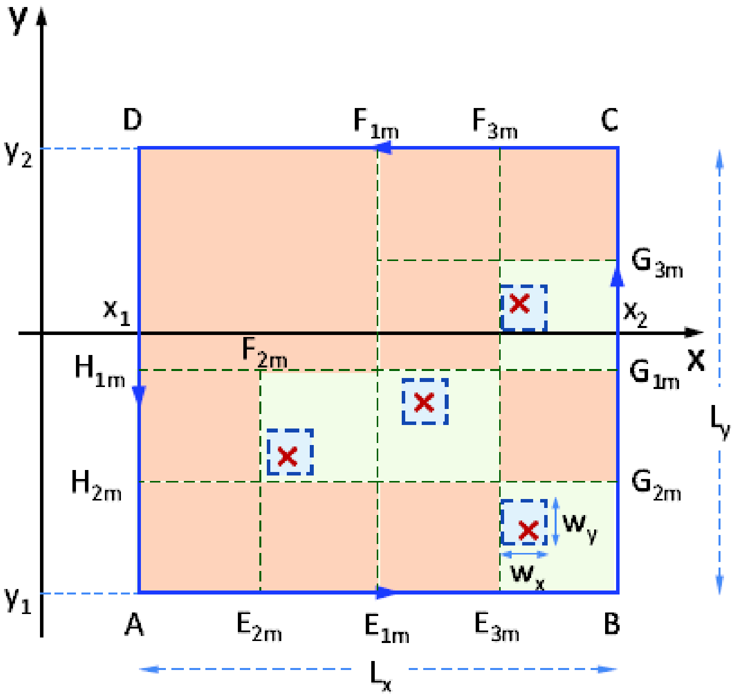
$$N_z = \frac{1}{j2\pi}\oint \frac{f'(z)}{f(z)}dz = \frac{1}{j2\pi}\Delta_C\{\ln[f(z)]\} = \frac{1}{2\pi}\Delta_C\{\arg[f(z)]\}$$



$$N_z = \frac{1}{2\pi}\sum_{i=1}^{4M}\arg\left[\frac{f(z_{i+1})}{f(z_i)}\right] = \frac{1}{2\pi}\Phi_t$$

E. N. Glytsis and E. Anemogiannis, Appl. Opt. 2018 (submitted)

Derivative-Free Zero-Extraction by Phase-based Enclosure Method (DFZEPE)



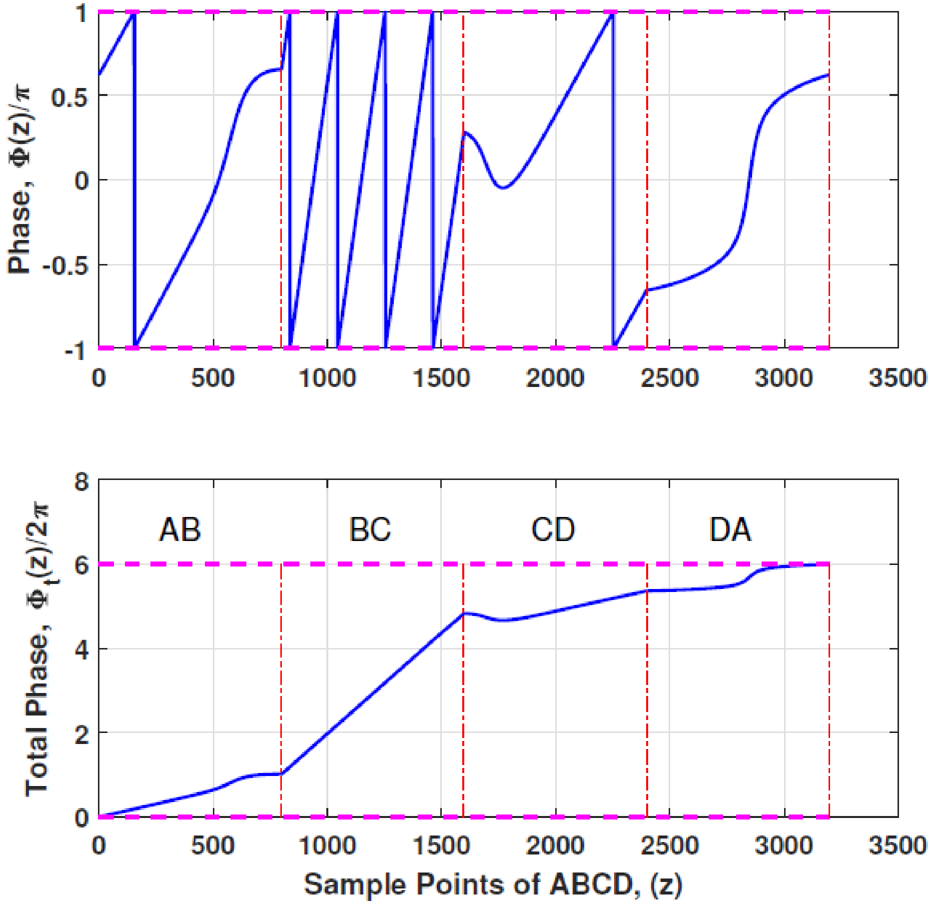
DFZEPE Algorithm

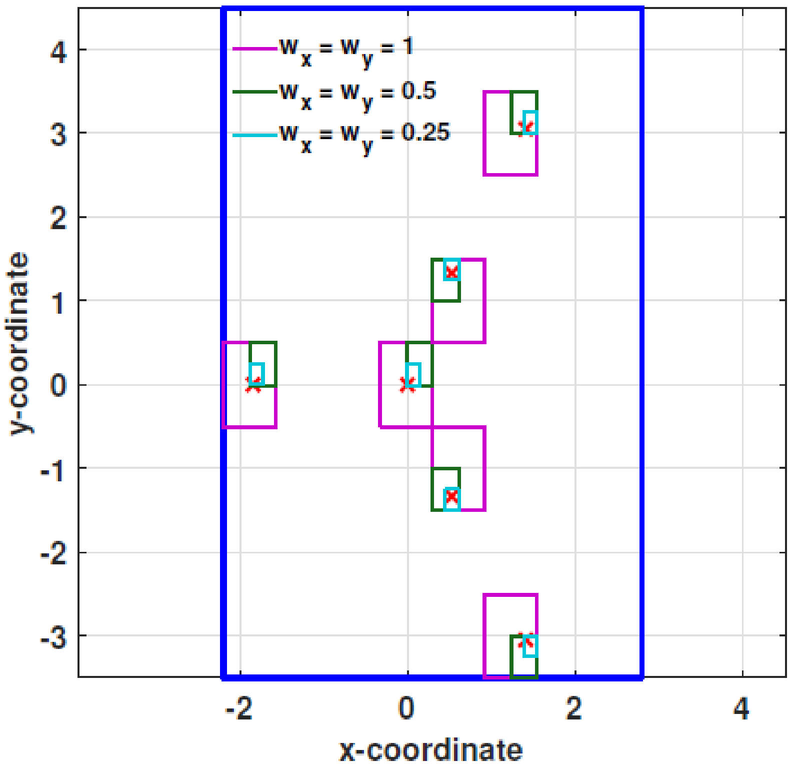
- Define M
- Subdivide ABCD into 4 equal sub-rectangles
- Check N_z in each sub-rectangle (adjust dividers)
- If $N_z = 0$ Disregard sub-rectangle
- If $N_z > 0$ Continue subdivision process while "*width*" > w_x and "*height*" > w_y
- Use converged sub-rectangles centers as estimates in the Müller's (with deflation) refinement

E. N. Glytsis and E. Anemogiannis, Appl. Opt. 2018 (submitted)

DFZEPE - Example

$$f(z) = e^{3z} + 2z\cos z - 1$$





E. N. Glytsis and E. Anemogiannis, Appl. Opt. 2018 (submitted)

DFZEPE - Example

$$f(z) = e^{3z} + 2z\cos z - 1$$

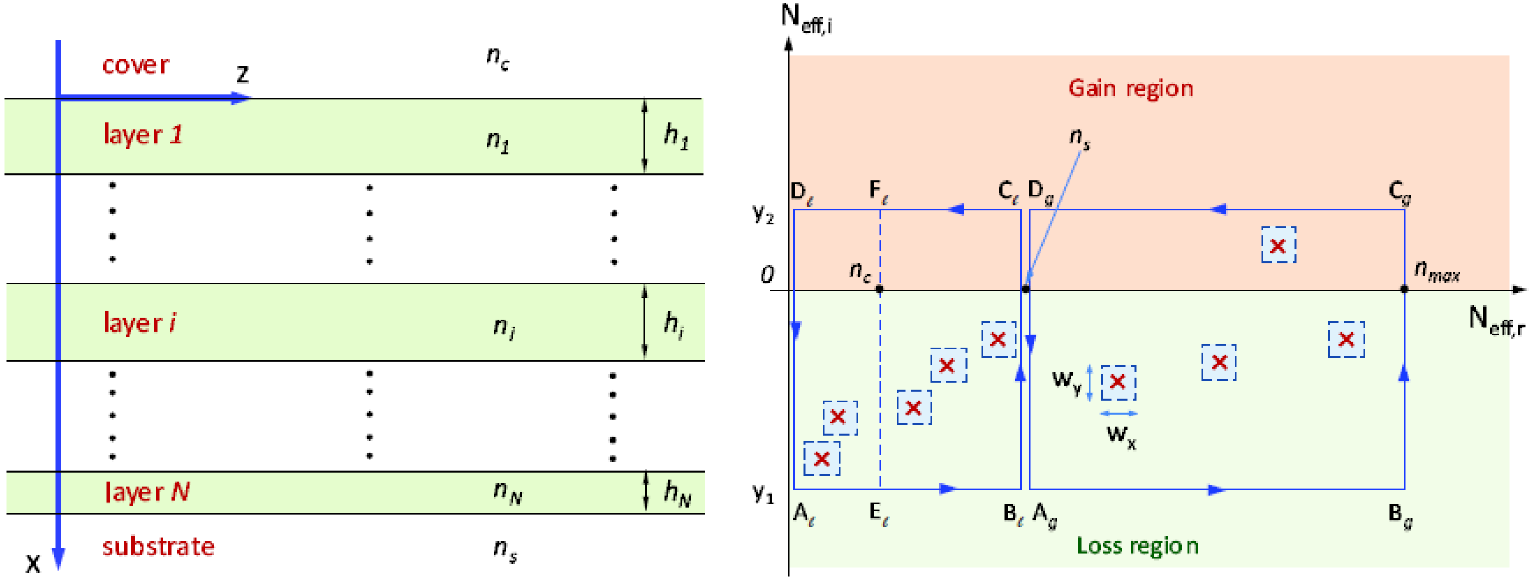
DFZEPE zeros estimates and refined zeros of function $f(z) = e^{3z} + 2z\cos z - 1$

DFZEPE Zeros Estimates	Zeros after Müller's Refinement
$-1.844165039062500 + j0.000976562500000$	$-1.844233953262213 + j0.000000000000000$
$0.000317382812500 + j0.000260416666667$	$0.000000000000000 - j0.000000000000000$
$0.531323242187500 - j1.331054687500000$	$0.530894930292931 - j1.331791876751121$
$0.531323242187500 + j1.331054687500000$	$0.530894930292931 + j1.331791876751121$
$1.415112304687500 - j3.047851562500000$	$1.414607177658184 - j3.047722062627173$
$1.415112304687500 + j3.047851562500000$	$1.414607177658184 + j3.047722062627173$

$$F(\beta) = \frac{\gamma_s}{a_s} m_{22} + \frac{\gamma_c}{a_c} m_{11} - \frac{\gamma_c}{a_c}\frac{\gamma_s}{a_s} m_{12} - m_{21} = 0$$

$a_c = a_s = 1$ for TE polarization, and $a_c = n_c^2$, $a_s = n_s^2$ for TM polarization

$$\beta = k_0 N_{eff} = k_0 (N_{eff,r} + j N_{eff,i})$$



Guided Modes: $Re\{\gamma_c\} > 0$ and $Re\{\gamma_s\} > 0$

Leaky Modes: $Re\{\gamma_c\} > 0$ and $Im\{\gamma_s\} > 0$ (substrate radiation/cover confinement)

$Im\{\gamma_c\} > 0$ and $Im\{\gamma_s\} > 0$ (substrate and cover radiation)

E. N. Glytsis and E. Anemogiannis, Appl. Opt. 2018 (submitted)

Lossless Waveguide

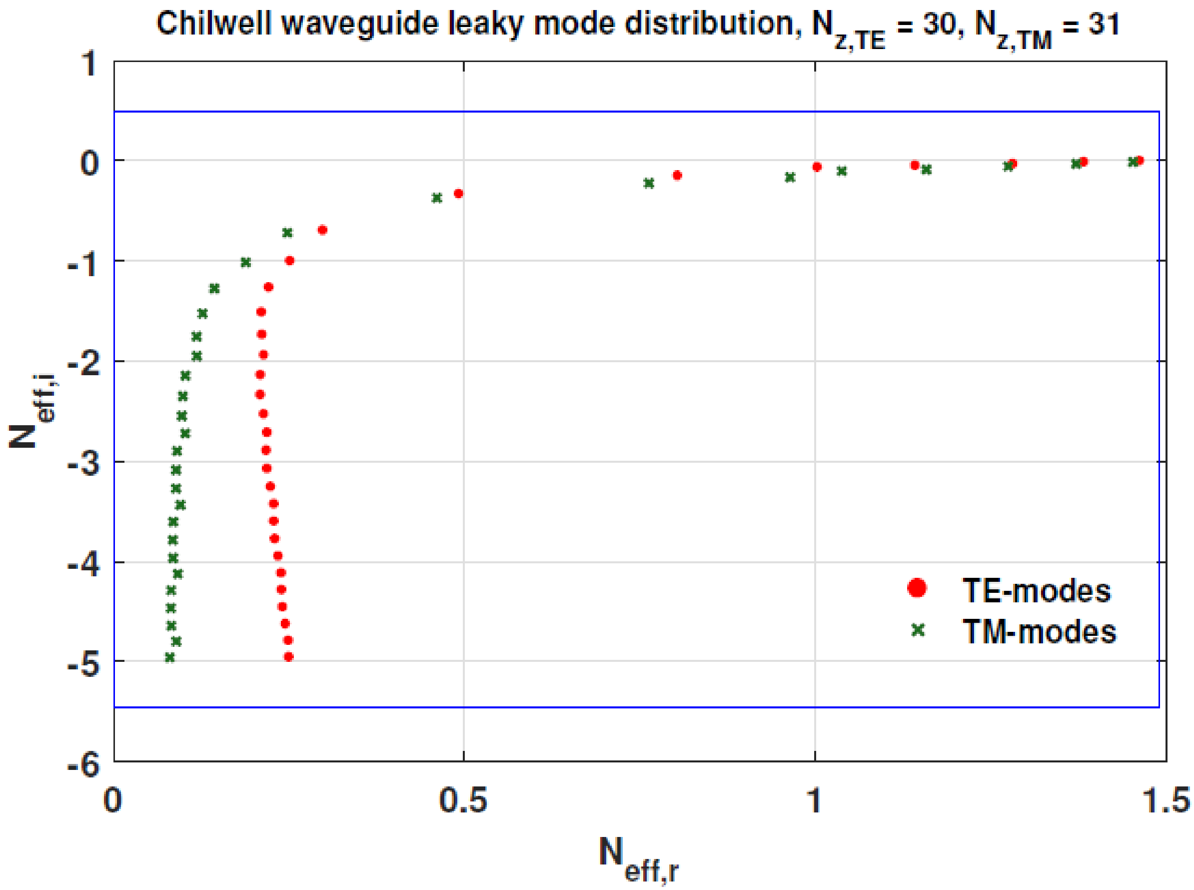
Effective indices of guided and leaky modes of a multilayer lossless waveguide from **Chilwell** λ_0 = 0.6328 μm, n_c = 1.0, n_s = 1.45, n_1 = 1.66, n_2 = 1.53, n_3 = 1.60, n_4 = 1.66, $h_1 = h_2 = h_3 = h_4$ = 0.5 μm.

Mode	$N_{eff} = N_{eff,r} + jN_{eff,i}$	Mode	$N_{eff} = N_{eff,r} + jN_{eff,i}$
Guided Modes			
TE_0	$1.62272868 + j0$	TM_0	$1.62003132 + j0$
TE_1	$1.60527569 + j0$	TM_1	$1.59478848 + j0$
TE_2	$1.55713615 + j0$	TM_2	$1.55498069 + j0$
TE_3	$1.50358711 + j0$	TM_3	$1.50181780 + j0$
Leaky Modes - Substrate Radiating			
TE_4	$1.46185664 - j0.00715587$	TM_4	$1.45153498 - j0.01192359$
TE_5	$1.38248922 - j0.01816588$	TM_5	$1.37066437 - j0.03014206$
TE_6	$1.28136443 - j0.03587739$	TM_6	$1.27373706 - j0.05679177$
TE_7	$1.14231446 - j0.05287607$	TM_7	$1.15731285 - j0.08757849$
TE_8	$1.00303702 - j0.07077094$	TM_8	$1.03695026 - j0.10307808$
First Leaky Mode - Substrate/Cover Radiating			
TE_9	$0.80402477 - j0.15549191$	TM_9	$0.96341519 - j0.16525032$

J. Chilwell and I. Hodhkinson. JOSA A, 1 (742-753), 1984

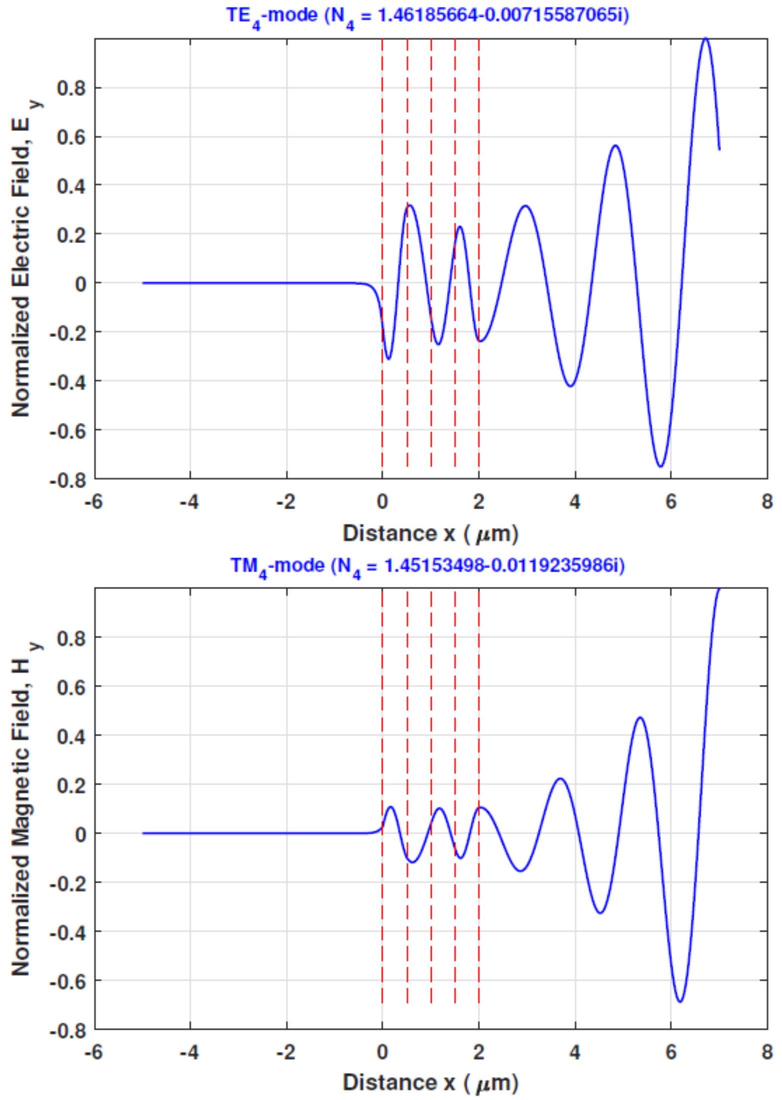
Lossless Waveguide

Distribution of effective indices of leaky modes of a multilayer lossless waveguide from **Chilwell** λ_0 = 0.6328 μm, n_c = 1.0, n_s = 1.45, n_1 = 1.66, n_2 = 1.53, n_3 = 1.60, n_4 = 1.66, $h_1 = h_2 = h_3 = h_4$ = 0.5 μm.



LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

First TE and first TM leaky modes profiles of a multilayer lossless waveguide from **Chilwell** $\lambda_0 = 0.6328$ μm, $n_c = 1.0$, $n_s = 1.45$, $n_1 = 1.66$, $n_2 = 1.53$, $n_3 = 1.60$, $n_4 = 1.66$, $h_1 = h_2 = h_3 = h_4 = 0.5$ μm.



LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Lossy Waveguide

Effective indices of guided and leaky modes of a multilayer lossy waveguide from **Chilwell**
λ_0 = 0.6328 μm, n_c = 1.0, n_s = 1.45, $n_1 = 1.66 - j1.66 \times 10^{-4}$, $n_2 = 1.53 - j1.53 \times 10^{-4}$, n_3 = 1.60, n_4 = 1.66, $h_1 = h_2 = h_3 = h_4$ = 0.5 μm.

Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$	Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$
Guided Modes			
TE_0	$1.62272868 - j0.00673727$	TM_0	$1.62003131 - j0.00892759$
TE_1	$1.60527569 - j1.66244285$	TM_1	$1.59478847 - j1.65565266$
TE_2	$1.55713612 - j0.20880097$	TM_2	$1.55498066 - j0.23704828$
TE_3	$1.50358696 - j0.55032495$	TM_3	$1.50181764 - j0.42530043$
Leaky Modes - Substrate Radiating			
TE_4	$1.46185448 - j0.00726710$	TM_4	$1.45153751 - j0.01202887$
TE_5	$1.38249997 - j0.01827662$	TM_5	$1.37068384 - j0.03024261$
TE_6	$1.28137151 - j0.03596266$	TM_6	$1.27375077 - j0.05687731$
TE_7	$1.14233026 - j0.05299360$	TM_7	$1.15732794 - j0.08766890$
TE_8	$1.00303470 - j0.07087449$	TM_8	$1.03694118 - j0.10316486$

J. Chilwell and I. Hodhkinson. JOSA A, 1 (742-753), 1984

LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Active Semiconductor Waveguide

Effective indices of guided modes and first leaky mode of a multilayer active waveguide from Visser λ_0 = 1.30 μm, n_c = 1.0, n_s = 3.16, n_1 = 0.18 − j10.2, n_2 = 3.16 − j0.0001, n_3 = 3.6 + j0.002, n_4 = 3.16 − j0.0001, h_1 = 0.04 μm, h_2 = 1.0 μm, h_3 = 0.15 μm, h_4 = 3 μm.

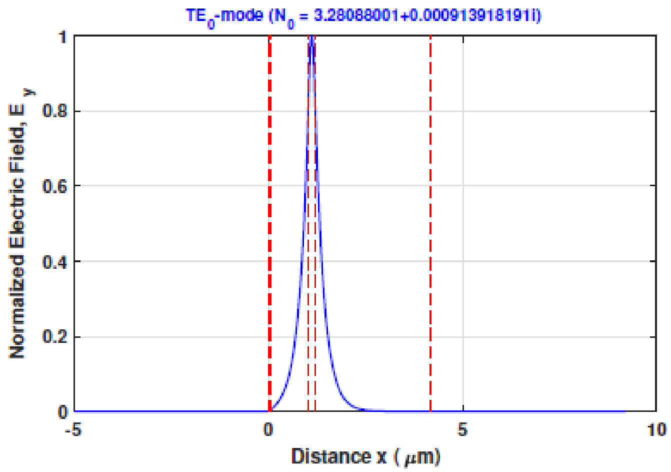
Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$	Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$
Guided Modes			
TE_0	$3.28088001 + j\ 9.13918191$	TM_0	$3.33449848 - j\ 75.18872326$
		TM_1	$3.24809848 + j\ 5.46307013$
First Leaky Mode			
TE_1	$3.13650356 - j376.20259075$	TM_2	$3.13622674 - j377.75840427$

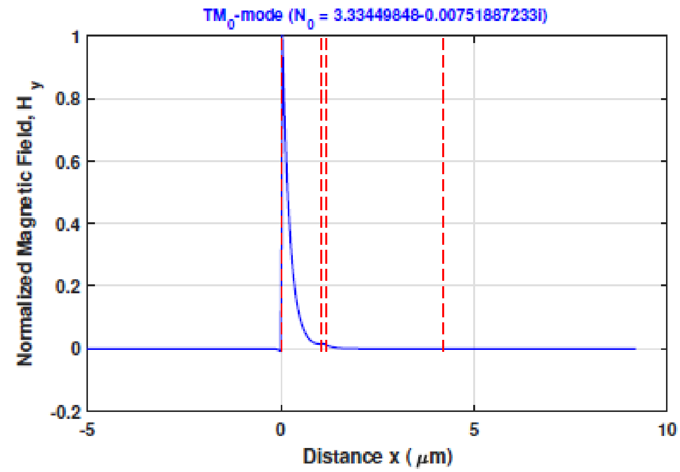
T. D. Visser et al., IEEE J. Quantum Eletron., 31 (1803-1810), 1995

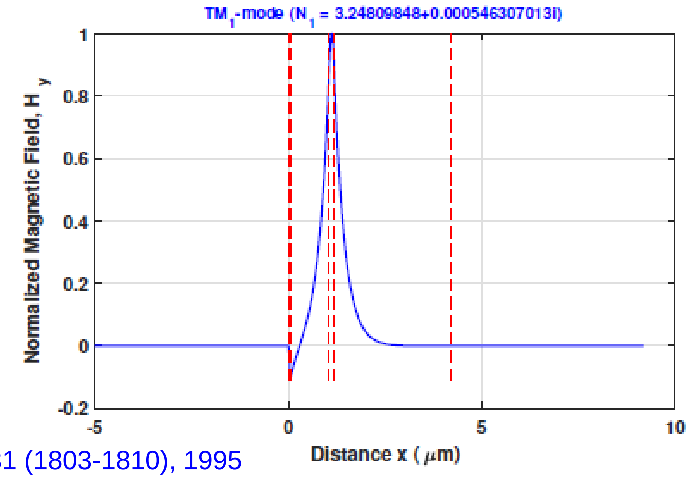
LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Active Semiconductor Waveguide

Effective indices of guided modes and first leaky mode of a multilayer active waveguide from Visser $\lambda_0 = 1.30$ μm, $n_c = 1.0$, $n_s = 3.16$, $n_1 = 0.18 - j10.2$, $n_2 = 3.16 - j0.0001$, $n_3 = 3.6 + j0.002$, $n_4 = 3.16 - j0.0001$, $h_1 = 0.04$ μm, $h_2 = 1.0$ μm, $h_3 = 0.15$ μm, $h_4 = 3$ μm.







T. D. Visser et al., IEEE J. Quantum Eletron., 31 (1803-1810), 1995

Antiresonant Reflecting Optical Waveguide (ARROW)

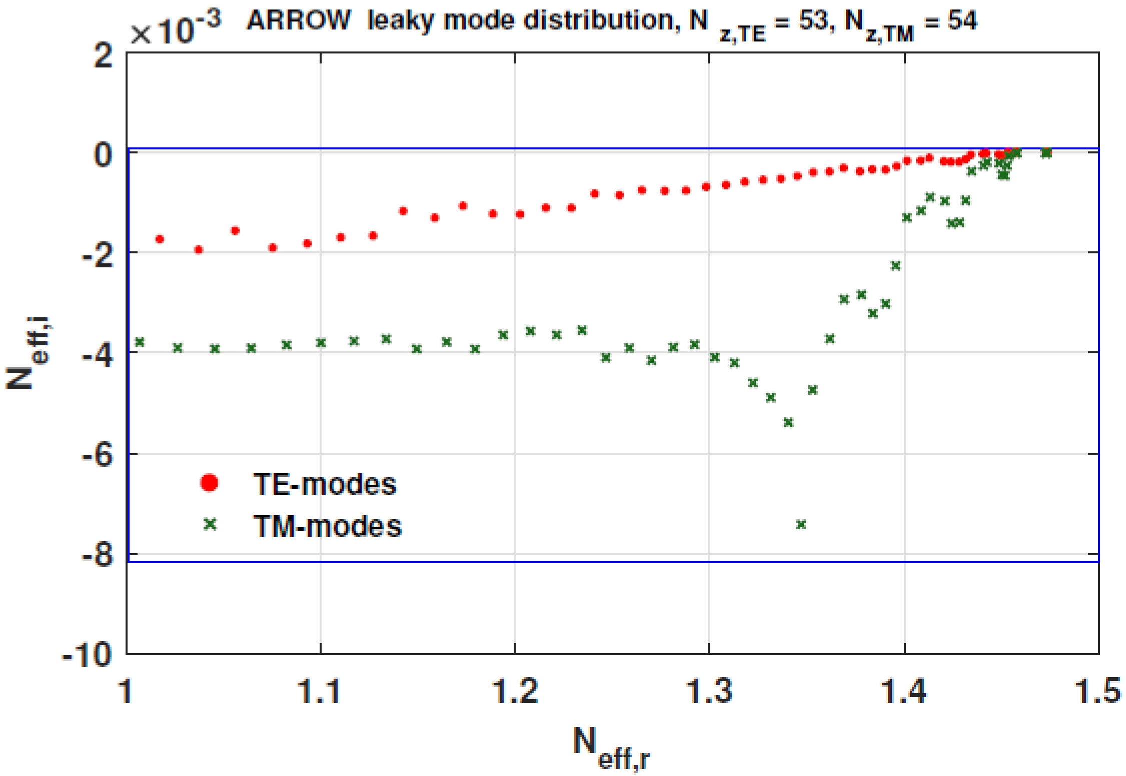
Effective indices of leaky modes of a multilayer ARROW waveguide from Semwal λ_0 = 0.6328 μm, n_c = 1.0, n_s = 3.50, n_1 = 1.46, n_2 = 1.50, n_3 = 1.46, n_4 = 1.50, n_5 = 1.46, n_6 = 1.50, n_7 = 1.46, n_8 = 1.50, n_9 = 1.46, h_1 = 2.00 μm, h_2 = 0.448 μm, h_3 = 4.00 μm, h_4 = 0.448 μm, h_5 = 2.00 μm, h_6 = 0.448 μm, h_7 = 4.00 μm, h_8 = 0.448 μm h_9 = 2.00 μm.

Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$	Mode	$N_{eff} = N_{eff,r} + jN_{eff,i} \times 10^{+4}$
TE_0	$1.473925808 - j0.000000801$	TM_0	$1.473275805 - j0.000005809$
TE_1	$1.473697976 - j0.000017405$	TM_1	$1.473027205 - j0.032900856$
TE_2	$1.473696644 - j0.005452261$	TM_2	$1.473026854 - j0.000035036$
TE_3	$1.473459693 - j0.000001142$	TM_3	$1.472767027 - j0.000008508$
TE_4	$1.457920191 - j0.007106241$	TM_4	$1.457925423 - j0.045880488$
TE_5	$1.457791244 - j0.009053396$	TM_5	$1.457782773 - j0.057163274$
TE_6	$1.453780369 - j0.114698816$	TM_6	$1.453795448 - j0.645756672$
TE_7	$1.453045406 - j0.420121480$	TM_7	$1.452928429 - j2.555862981$
TE_8	$1.451864807 - j0.693651857$	TM_8	$1.451781628 - j4.567101184$
TE_9	$1.450269491 - j0.732515868$	TM_9	$1.450247659 - j4.357488809$

G. Semwal and V. Rastogi , Opt. & Quantum Eletron., 46 (423-438), 2014

Antiresonant Reflecting Optical Waveguide (ARROW)

Distribution of effective indices of leaky modes of a multilayer ARROW waveguide from Semwal λ_0 = 0.6328 μm, n_c = 1.0, n_s = 3.50, n_1 = 1.46, n_2 = 1.50, n_3 = 1.46, n_4 = 1.50, n_5 = 1.46, n_6 = 1.50, n_7 = 1.46, n_8 = 1.50, n_9 = 1.46, h_1 = 2.00 μm, h_2 = 0.448 μm, h_3 = 4.00 μm, h_4 = 0.448 μm, h_5 = 2.00 μm, h_6 = 0.448 μm, h_7 = 4.00 μm, h_8 = 0.448 μm h_9 = 2.00 μm.



LOSSY/ACTIVE and LEAKY SLAB WAVEGUIDES

Antiresonant Reflecting Optical Waveguide (ARROW)

Modal profiles of leaky modes of a multilayer ARROW waveguide from Semwal λ_0 = 0.6328 μm, n_c = 1.0, n_s = 3.50, n_1 = 1.46, n_2 = 1.50, n_3 = 1.46, n_4 = 1.50, n_5 = 1.46, n_6 = 1.50, n_7 = 1.46, n_8 = 1.50, n_9 = 1.46, h_1 = 2.00 μm, h_2 = 0.448 μm, h_3 = 4.00 μm, h_4 = 0.448 μm, h_5 = 2.00 μm, h_6 = 0.448 μm, h_7 = 4.00 μm, h_8 = 0.448 μm h_9 = 2.00 μm.