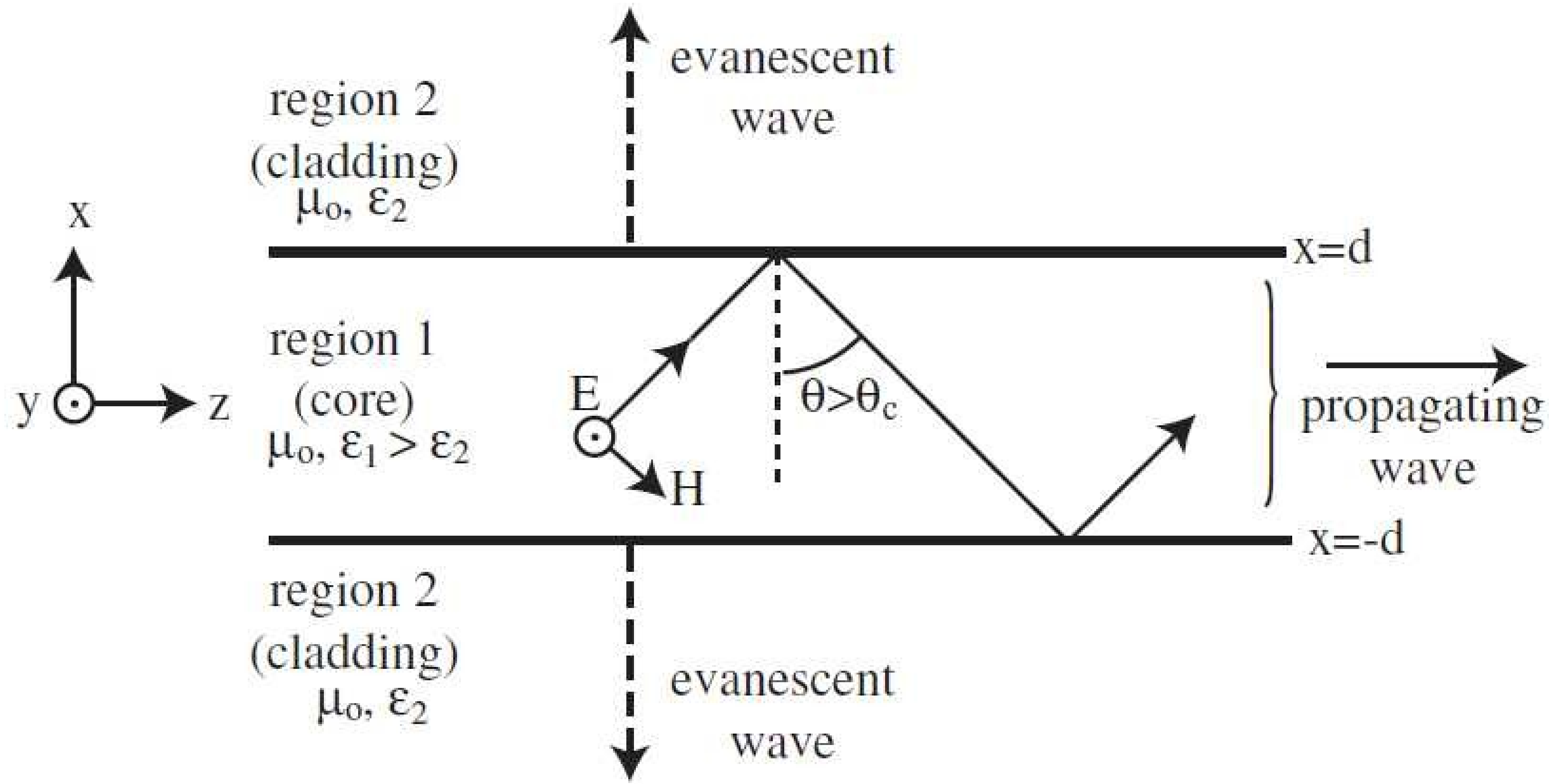
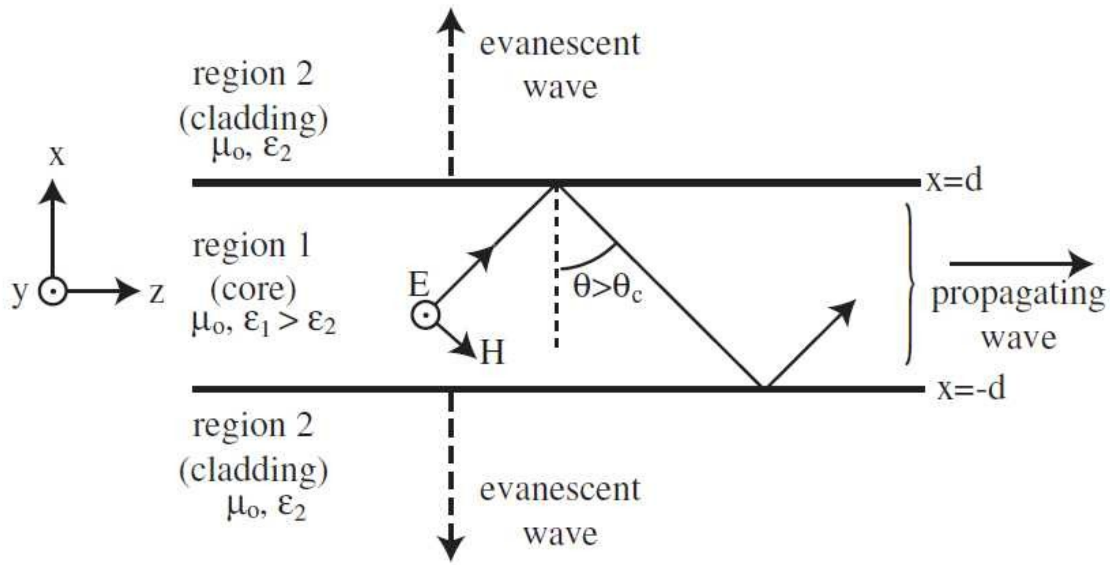
Dielectric Slab Waveguide



Dielectric Slab Waveguide



TE Modes

$$\nabla_t^{\,2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

$$\frac{\partial^2}{\partial x^2} H_z(x,y) + \frac{\partial^2}{\partial y^2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

= 0

$$k_c = \sqrt{k^2 - \beta_z^2}$$

= 0

$$E_x = \frac{-j}{(k_c^{\,2})}\left[\omega\mu \frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{j}{(k_c^{\,2})}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c^{\,2})}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

= 0

$$H_y = \frac{-j}{(k_c^{\,2})}\left[\beta_z \frac{\partial H_z}{\partial y}\right]$$

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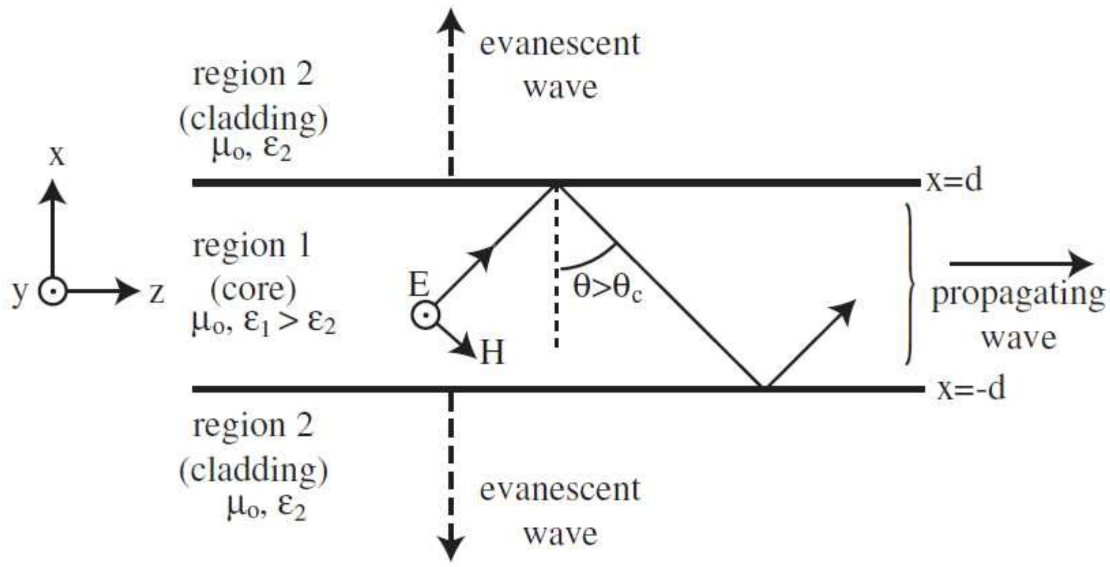

TE Modes

$$\nabla_t^{\,2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

$$\frac{\partial^2}{\partial x^2} H_z(x,y) + \frac{\partial^2}{\partial y^2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

= 0

$$\frac{\partial^2}{\partial x^2} H_z(x) + k_c^{\,2} H_z(x) = 0$$

$$E_y = \frac{j}{(k_c^{\,2})}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c^{\,2})}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$E_x = H_y = 0$$

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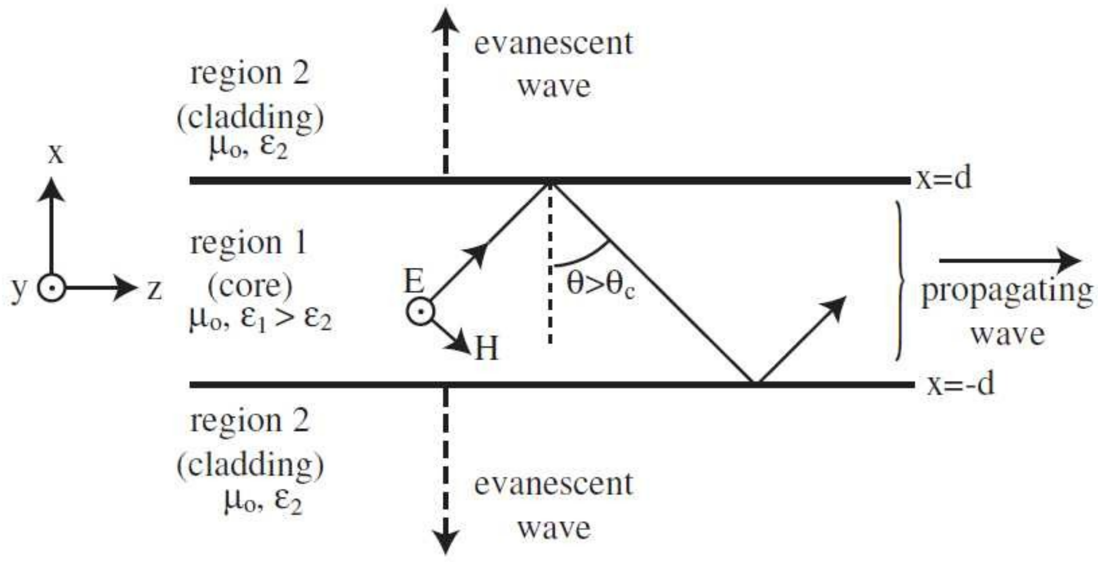

TE Modes

$$\frac{\partial^2}{\partial x^2} H_z(x) + k_c{}^2 H_z(x) = 0$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$E_x = H_y = 0$$

Region II: x>d and x<-d

$$H_z(x,z) = \begin{cases} H_o\, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o\, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$H_z(x,z) = H_o\left(A_1 \cos(\beta_x x) + A_2 \sin(\beta_x x)\right) e^{-j\beta_z z}$$

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TE Modes

Region II: x>d and x<-d

$$H_z(x,z) = \begin{cases} H_o\, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o\, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

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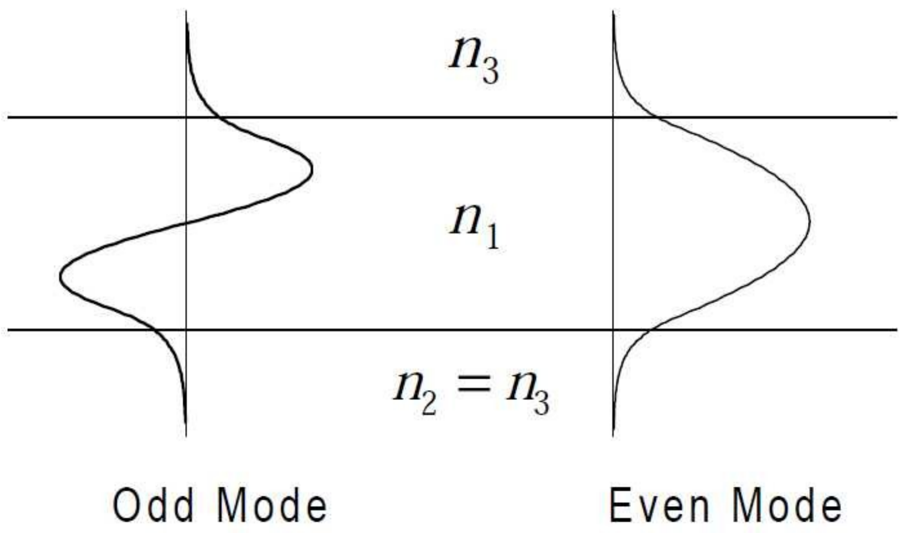

Lets look just first at the Even TE Modes

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$E_x = H_y = 0$$

Dielectric Slab Waveguide

TE Even Modes

Region II: x>d and x<-d

$$E_x(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$E_z(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$H_x(x,z) = \begin{cases} H_o \dfrac{j\alpha\beta_z}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{-j\alpha\beta_z}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$H_y(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$H_z(x,z) = \begin{cases} H_o\, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o\, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$E_x(x,z) = 0$$

$$E_y(x,z) = \frac{-j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

$$E_z(x,z) = 0$$

$$H_x(x,z) = \frac{j\beta_x\beta_z H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

$$H_y(x,z) = 0$$

$$H_z(x,z) = H_o \cos(\beta_x x) e^{-j\beta_z z}$$

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TE Even Modes

Region II: x>d and x<-d

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$H_z(x,z) = \begin{cases} H_o \, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$E_y(x,z) = \frac{-j\omega\mu\beta_x H_o}{\beta_z^{\;2} - \omega^2\mu_o\varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

$$H_z(x,z) = H_o \cos(\beta_x x) e^{-j\beta_z z}$$

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

Dispersion Relationships:

$$\beta_x^{\;2} + \beta_z^{\;2} = \omega^2\mu_o\varepsilon_1$$

$$-\alpha^2 + \beta_z^{\;2} = \omega^2\mu_o\varepsilon_2$$

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TE Even Modes

Region II: x>d and x<-d

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2 \mu_o \varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2 \mu_o \varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

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Region I: -d < x < d

$$E_y(x,z) = \frac{-j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2 \mu_o \varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

$$H_z(x,z) = H_o \cos(\beta_x x) e^{-j\beta_z z}$$

<u>Boundary Conditions:</u>

$$E_y{}^{(1)}(-d,z) = E_y{}^{(2)}(-d,z)$$

$$E_y{}^{(1)}(d,z) = E_y{}^{(2)}(d,z)$$

$$H_z{}^{(1)}(-d,z) = H_z{}^{(2)}(-d,z)$$

$$H_z{}^{(1)}(d,z) = H_z{}^{(2)}(d,z)$$

$$E_y{}^{(1)}(d,z) = E_y{}^{(2)}(d,z)$$

$$\frac{-j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2 \mu_o \varepsilon_1} \sin(\beta_x d) e^{-j\beta_z z} = \frac{-j\alpha\omega\mu H_o}{\beta_z{}^2 - \omega^2 \mu_o \varepsilon_1} e^{-\alpha d} e^{-j\beta_z z}$$

$$H_z{}^{(1)}(d,z) = H_z{}^{(2)}(d,z)$$

$$H_o \cos(\beta_x d) e^{-j\beta_z z} = H_o e^{-\alpha d} e^{-j\beta_z z}$$

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TE Even Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

(1) $\beta_x \sin(\beta_x d) = \alpha e^{-\alpha d}$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

(2) $\cos(\beta_x d) = e^{-\alpha d}$

(1)/(2): $\dfrac{\beta_x \sin(\beta_x d)}{\cos(\beta_x d)} = \dfrac{\alpha e^{-\alpha d}}{e^{-\alpha d}}$

⇨ $\tan(\beta_x d) = \dfrac{\alpha}{\beta_x}$

Dielectric Slab Waveguide

TE Even Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$
$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$
$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$
$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$\Rightarrow \quad \tan(\beta_x d) = \frac{\alpha}{\beta_x}$$

Dispersion Relationships:

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$
$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

$$\Rightarrow \quad \beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$
$$\alpha = \sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}$$

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TE Even Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$
$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$
$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$
$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$\Rightarrow \quad \tan(\beta_x d) = \frac{\alpha}{\beta_x}$$

Dispersion Relationships:

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$
$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

$$\Rightarrow \quad \beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$
$$\alpha = \sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}$$

$$\Rightarrow \quad \tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}}{\beta_x}$$

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TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\,2}}}{\beta_x}$$

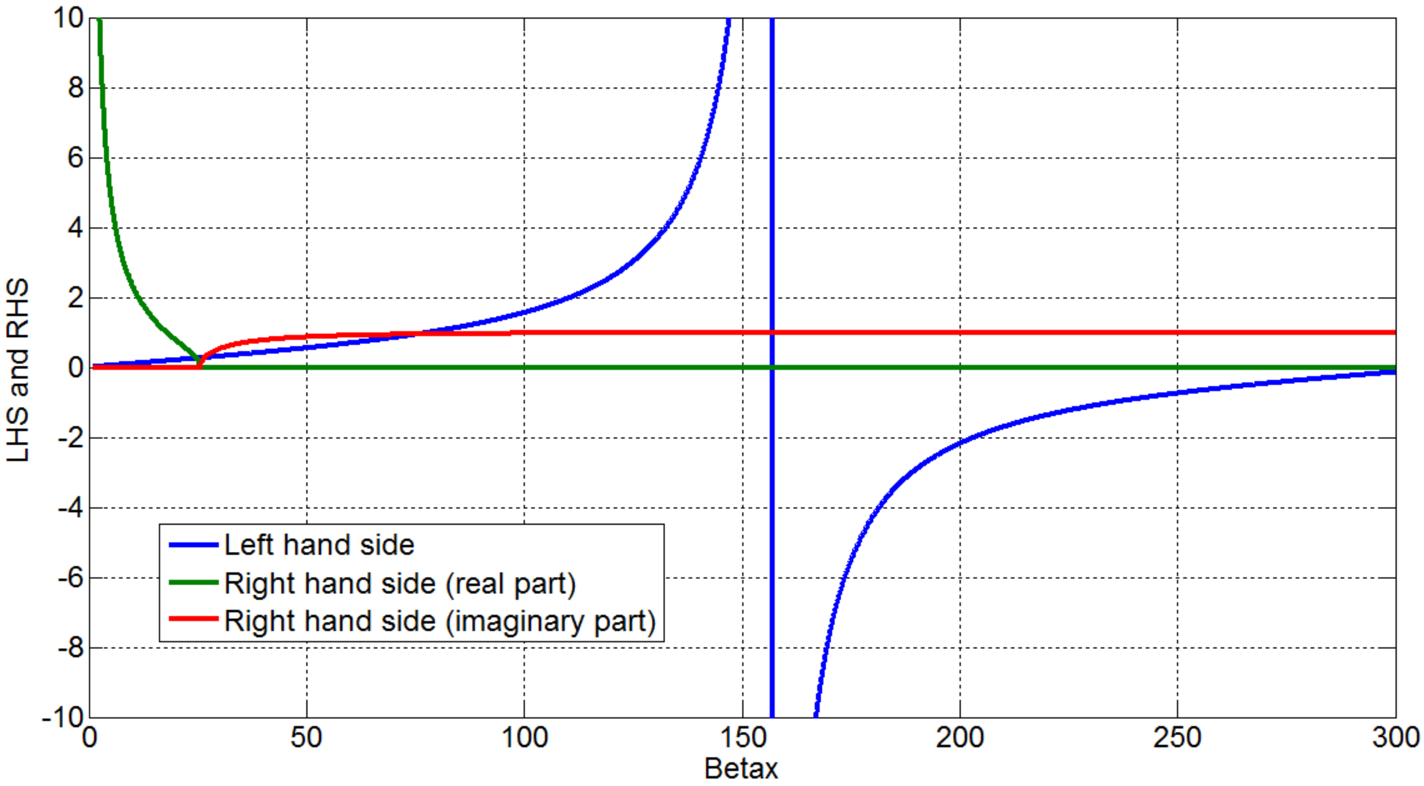
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 1.0 \quad GHz$

$d = 1 \quad cm$



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TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

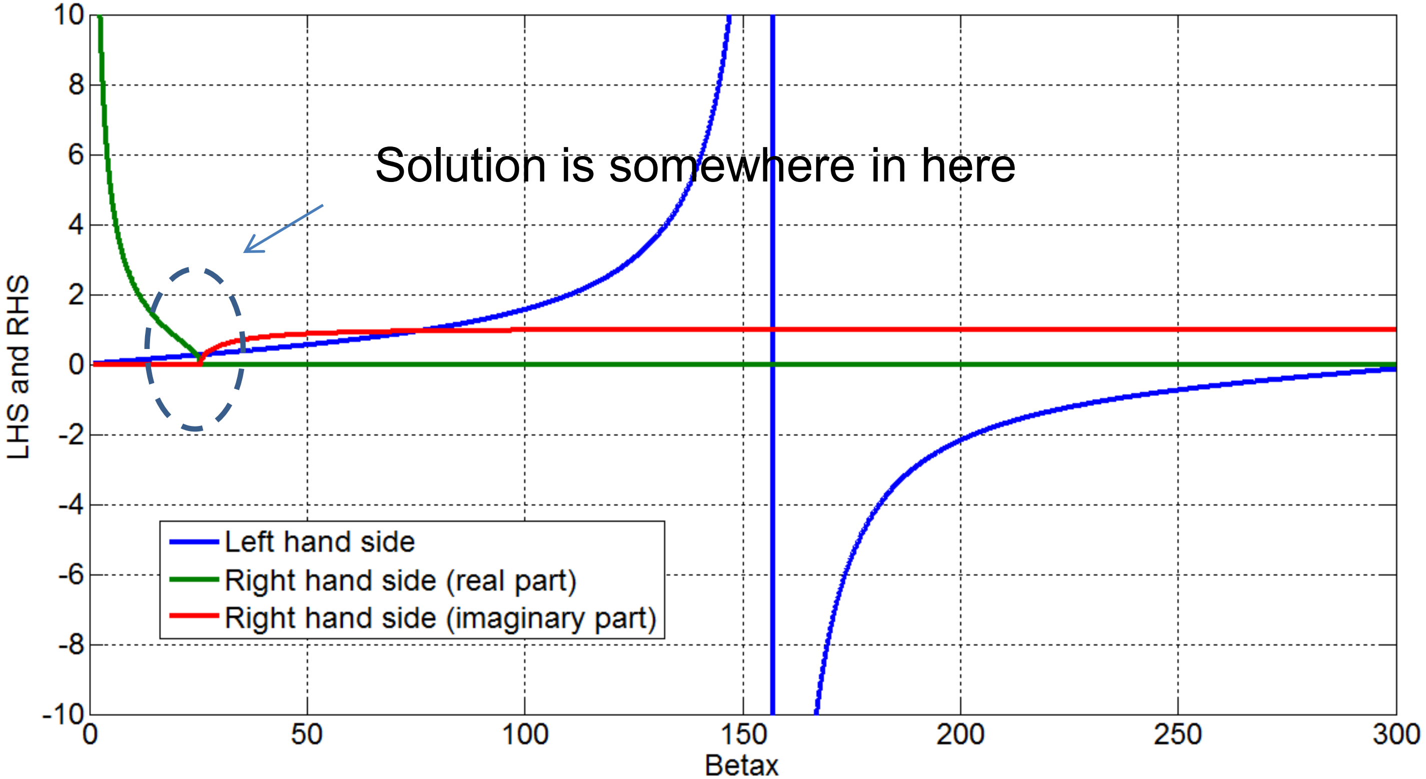
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 1.0 \quad GHz$

$d = 1 \quad cm$



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TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

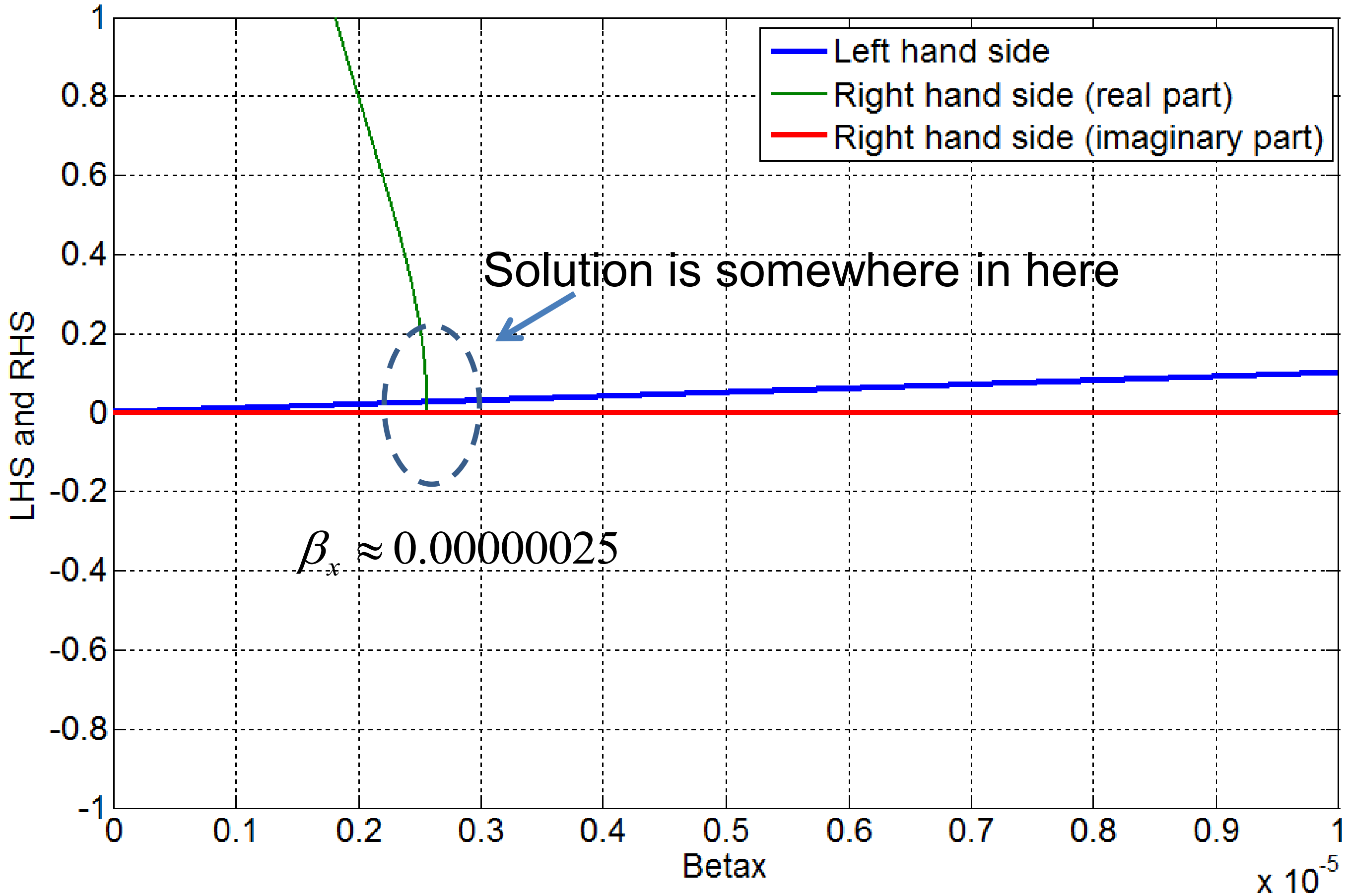
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 100 \quad Hz$

$d = 1 \quad cm$



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TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

Example

$\varepsilon_{r1} = 2.5$

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$f = 100 \quad Hz$

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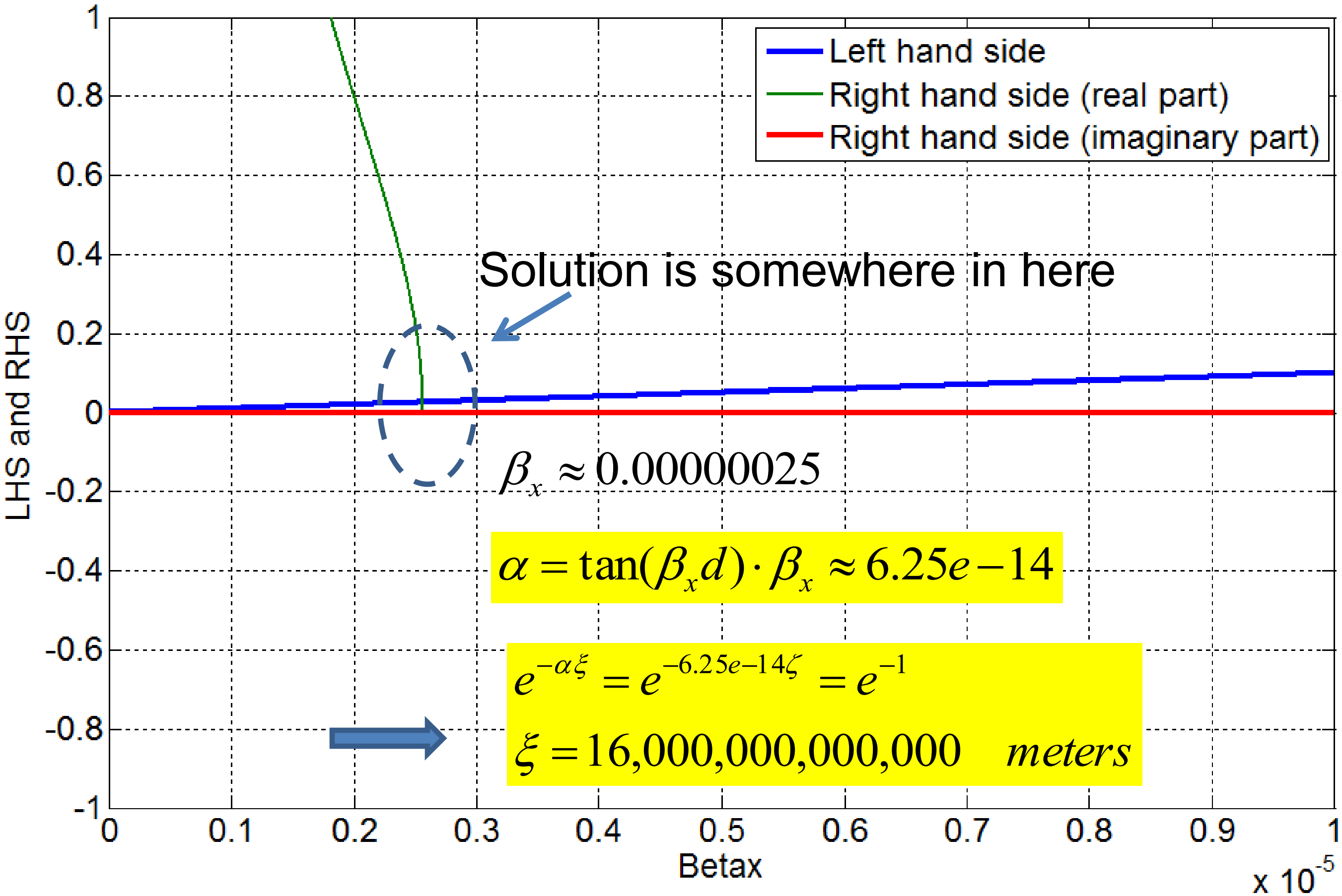


$\beta_x \approx 0.00000025$

$\alpha = \tan(\beta_x d) \cdot \beta_x \approx 6.25e-14$

$e^{-\alpha \xi} = e^{-6.25e-14 \zeta} = e^{-1}$

$\xi = 16{,}000{,}000{,}000{,}000 \quad meters$

Dielectric Slab Waveguide

TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

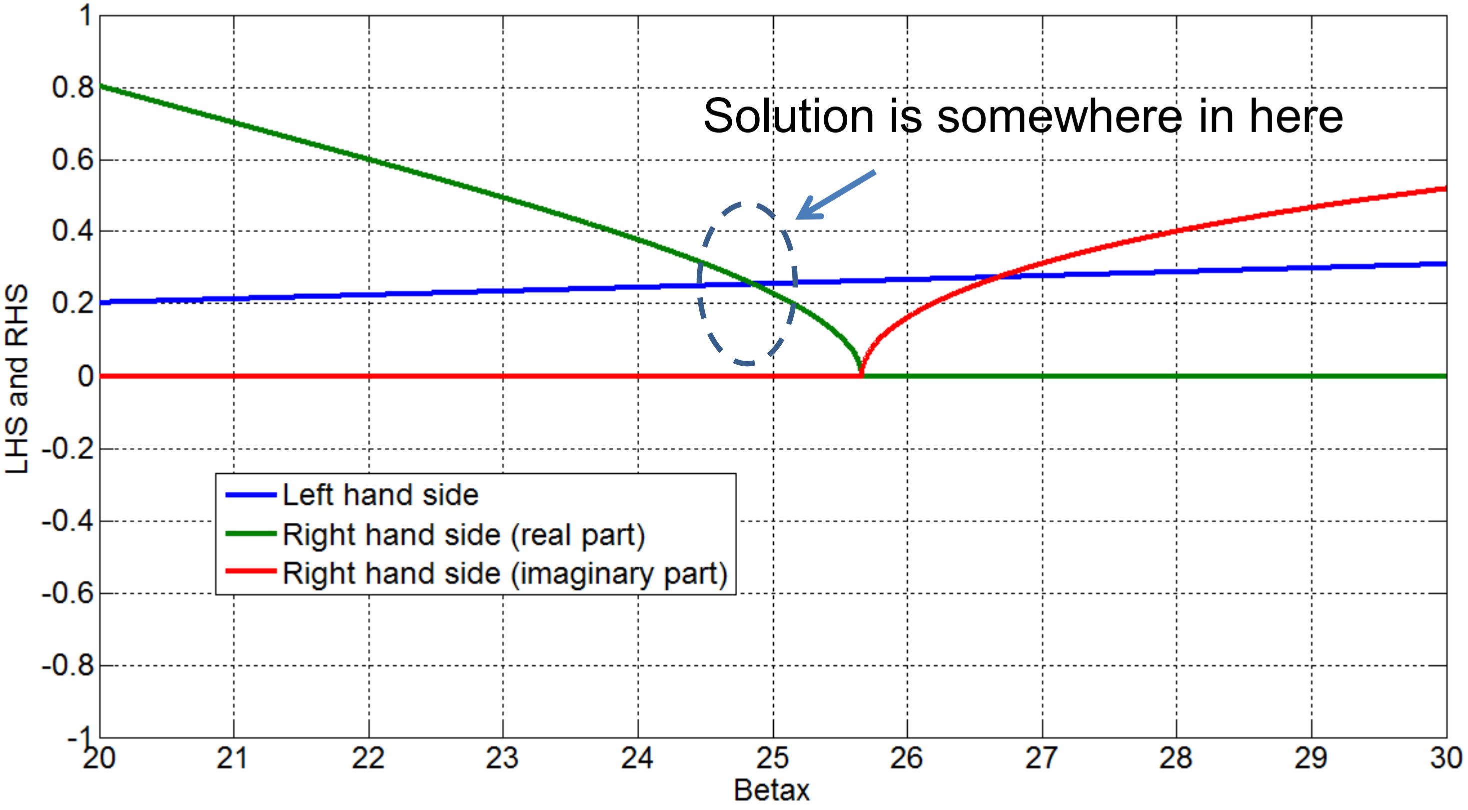
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 1.0 \quad GHz$

$d = 1 \quad cm$



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TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

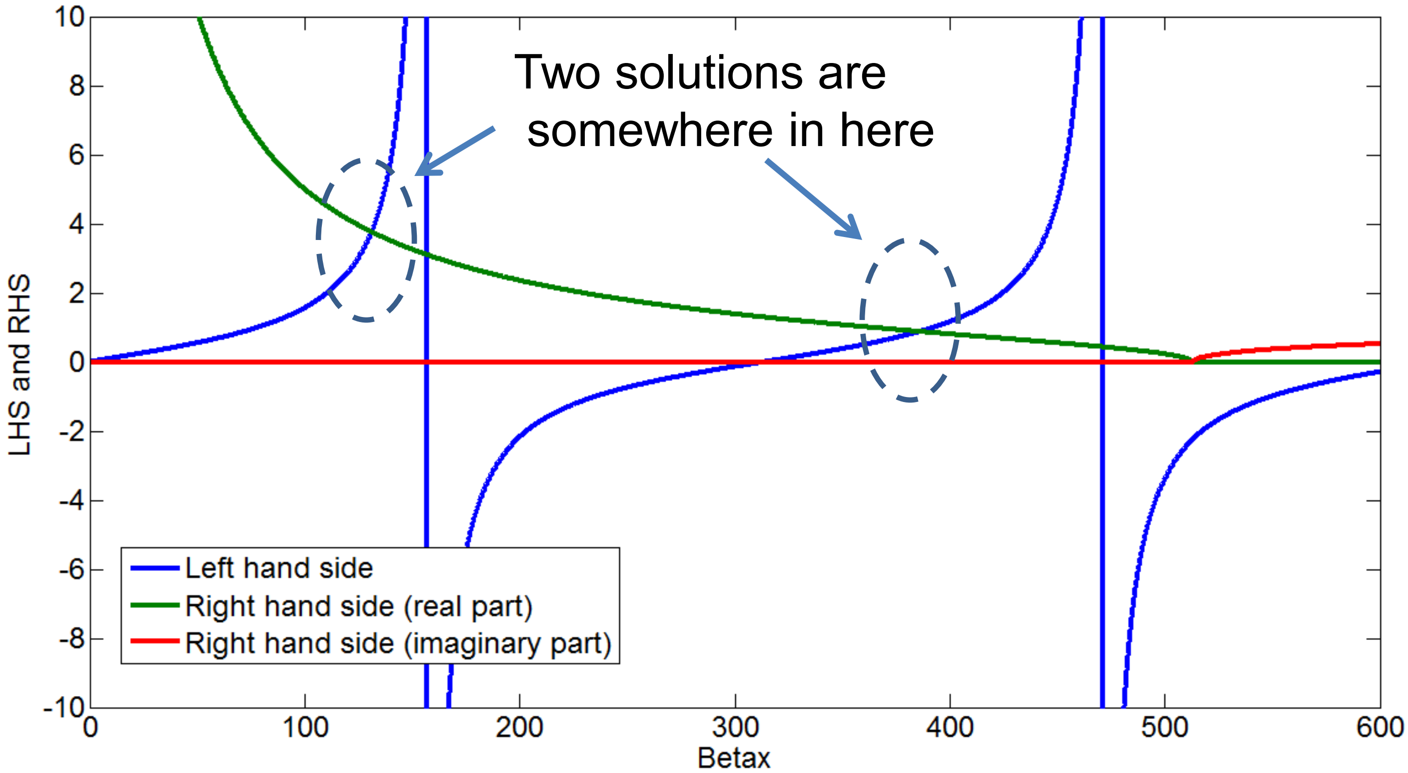
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 20 \quad GHz$

$d = 1 \quad cm$



Dielectric Slab Waveguide

$$\beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$
$$\alpha = \sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}$$

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$
$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

For a lossless waveguide mode to propagate we need:

(1) β_z needs to be real
(2) β_x needs to be real
(3) α needs to be real

Look at various cases:

(a) $\beta_z < \omega\sqrt{\mu_o \varepsilon_1} < \omega\sqrt{\mu_o \varepsilon_2}$

$$\beta_x = \pm\sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\,2}} \quad real$$
$$\alpha = \pm\sqrt{\beta_z^{\,2} - \omega^2 \mu_o \varepsilon_2} \quad imaginary$$

(b) $\beta_z > \omega\sqrt{\mu_o \varepsilon_1} > \omega\sqrt{\mu_o \varepsilon_2}$

$$\beta_x = \pm\sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\,2}} \quad imaginary$$
$$\alpha = \pm\sqrt{\beta_z^{\,2} - \omega^2 \mu_o \varepsilon_2} \quad real$$

(c) $\omega\sqrt{\mu_o \varepsilon_1} > \beta_z > \omega\sqrt{\mu_o \varepsilon_2}$

$$\beta_x = \pm\sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\,2}} \quad real$$
$$\alpha = \pm\sqrt{\beta_z^{\,2} - \omega^2 \mu_o \varepsilon_2} \quad real$$

Dielectric Slab Waveguide

$$\beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$

$$\alpha = \sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}$$

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$

$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

For a lossless waveguide mode to propagate we need:

(1) β_z needs to be real
(2) β_x needs to be real
(3) α needs to be real

(c) $\omega\sqrt{\mu_o \varepsilon_1} > \beta_z > \omega\sqrt{\mu_o \varepsilon_2}$

$$\beta_x = \pm\sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\,2}} \quad real$$

$$\alpha = \pm\sqrt{\beta_z^{\,2} - \omega^2 \mu_o \varepsilon_2} \quad real$$

Cutoff Frequency

$$\beta_z^{\,c} = \omega_c \sqrt{\mu_o \varepsilon_2} \quad \Rightarrow \quad \alpha = 0$$

$$\tan(\beta_x d) = \frac{\alpha}{\beta_x} \quad \Rightarrow \quad \tan(\beta_x d) = 0$$

$$\beta_x = \sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\,2}} \quad \Rightarrow \quad \beta_x^{\,c} = \sqrt{\omega_c^{\,2} \mu_o (\varepsilon_1 - \varepsilon_2)}$$

Dielectric Slab Waveguide

Cutoff Frequency

$$\beta_z^{\ c} = \omega_c \sqrt{\mu_o \varepsilon_2} \quad \Rightarrow \quad \alpha^c = 0$$

$$\tan(\beta_x d) = \frac{\alpha}{\beta_x} \quad \Rightarrow \quad \tan(\beta_x^{\ c} d) = 0$$

$$\beta_x = \sqrt{\omega^2 \mu_o \varepsilon_1 - \beta_z^{\ 2}} \quad \Rightarrow \quad \beta_x^{\ c} = \sqrt{\omega_c^{\ 2} \mu_o (\varepsilon_1 - \varepsilon_2)}$$

$$\Rightarrow \quad \tan(\sqrt{\omega_c^{\ 2} \mu_o (\varepsilon_1 - \varepsilon_2)}\, d) = 0$$

$$\Rightarrow \quad (f_c)_m = \frac{m}{4d\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2)})} \qquad m = 0,2,4....$$

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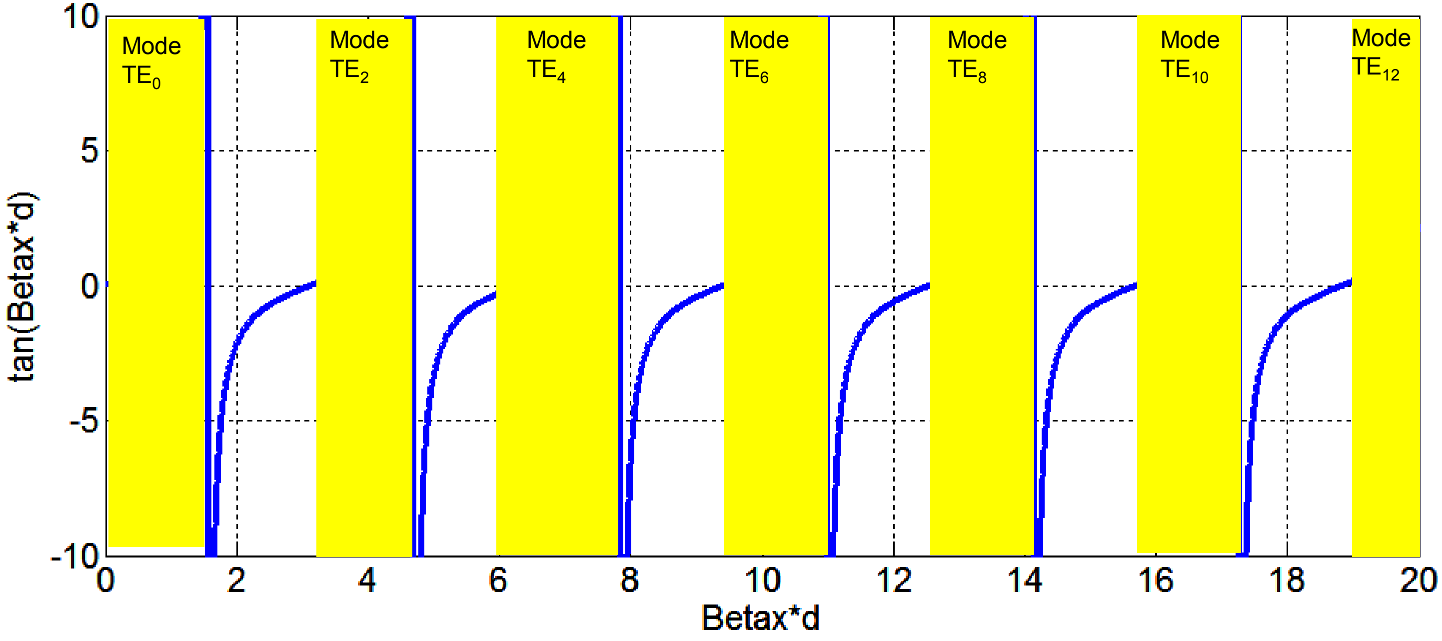
TE Odd Modes

$$\tan(\beta_x d) = \frac{\alpha}{\beta_x}$$



Dielectric Slab Waveguide

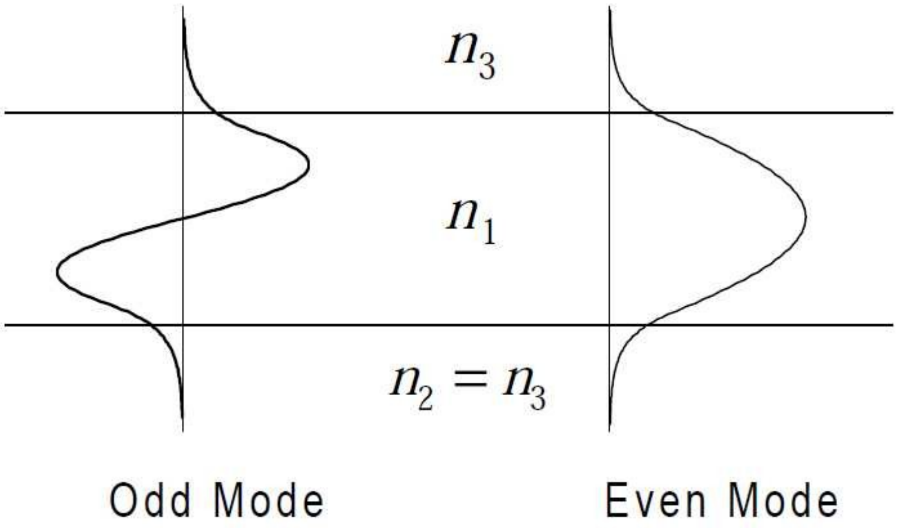
TE Modes Odd

Region II: x>d and x<-d

$$H_z(x,z) = \begin{cases} H_o\, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o\, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$H_z(x,z) = H_o\left(A_1 \cos(\beta_x x) + A_2 \sin(\beta_x x)\right) e^{-j\beta_z z}$$



Lets now look at the Odd TE Modes

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$E_x = H_y = 0$$

Dielectric Slab Waveguide

TE Odd Modes

Region II: x>d and x<-d

$$E_x(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$H_x(x,z) = \begin{cases} H_o \dfrac{-\alpha\beta_z}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{-\alpha\beta_z}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$H_y(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$E_z(x,z) = \begin{cases} 0 & x > d \\ 0 & x < -d \end{cases}$$

$$H_z(x,z) = \begin{cases} H_o e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$E_x(x,z) = 0$$

$$E_y(x,z) = \frac{j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1}\cos(\beta_x x) e^{-j\beta_z z}$$

$$E_z(x,z) = 0$$

$$H_x(x,z) = \frac{-j\beta_x\beta_z H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1}\cos(\beta_x x) e^{-j\beta_z z}$$

$$H_y(x,z) = 0$$

$$H_z(x,z) = H_o \sin(\beta_x x) e^{-j\beta_z z}$$

Dielectric Slab Waveguide

TE Odd Modes

Region II: x>d and x<-d

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2 \mu_o \varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2 \mu_o \varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$H_z(x,z) = \begin{cases} H_o\, e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o\, e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$E_y(x,z) = \frac{j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2 \mu_o \varepsilon_1} \cos(\beta_x x) e^{-j\beta_z z}$$

$$H_z(x,z) = H_o \sin(\beta_x x) e^{-j\beta_z z}$$

Boundary Conditions:

$$E_y{}^{(1)}(-d,z) = E_y{}^{(2)}(-d,z)$$

$$E_y{}^{(1)}(d,z) = E_y{}^{(2)}(d,z)$$

$$H_z{}^{(1)}(-d,z) = H_z{}^{(2)}(-d,z)$$

$$H_z{}^{(1)}(d,z) = H_z{}^{(2)}(d,z)$$

Dispersion Relationships:

$$\beta_x{}^2 + \beta_z{}^2 = \omega^2 \mu_o \varepsilon_1$$

$$-\alpha^2 + \beta_z{}^2 = \omega^2 \mu_o \varepsilon_2$$

Dielectric Slab Waveguide

TE Odd Modes

Region II: x>d and x<-d

$$E_y(x,z) = \begin{cases} H_o \dfrac{-j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o \dfrac{j\alpha\omega\mu}{\beta_z^2 - \omega^2\mu_o\varepsilon_2} e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

$$H_z(x,z) = \begin{cases} H_o e^{-\alpha x} e^{-j\beta_z z} & x > d \\ H_o e^{\alpha x} e^{-j\beta_z z} & x < -d \end{cases}$$

Region I: -d < x < d

$$E_y(x,z) = \frac{j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1} \cos(\beta_x x) e^{-j\beta_z z}$$

$$H_z(x,z) = H_o \sin(\beta_x x) e^{-j\beta_z z}$$

Boundary Conditions:

$$E_y{}^{(1)}(-d,z) = E_y{}^{(2)}(-d,z)$$

$$E_y{}^{(1)}(d,z) = E_y{}^{(2)}(d,z)$$

$$H_z{}^{(1)}(-d,z) = H_z{}^{(2)}(-d,z)$$

$$H_z{}^{(1)}(d,z) = H_z{}^{(2)}(d,z)$$

$$E_y{}^{(1)}(d,z) = E_y{}^{(2)}(d,z)$$

$$\frac{j\omega\mu\beta_x H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1} \cos(\beta_x d) e^{-j\beta_z z} = \frac{j\alpha\omega\mu H_o}{\beta_z{}^2 - \omega^2\mu_o\varepsilon_1} e^{-\alpha d} e^{-j\beta_z z}$$

$$H_z{}^{(1)}(d,z) = H_z{}^{(2)}(d,z)$$

$$H_o \sin(\beta_x d) e^{-j\beta_z z} = H_o e^{-\alpha d} e^{-j\beta_z z}$$

Dielectric Slab Waveguide

TE Odd Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

(1) $\beta_x \cos(\beta_x d) = \alpha e^{-\alpha d}$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

(2) $\sin(\beta_x d) = e^{-\alpha d}$

(2)/(1): $$\frac{\sin(\beta_x d)}{\beta_x \cos(\beta_x d)} = \frac{e^{-\alpha d}}{\alpha e^{-\alpha d}}$$

$$\Rightarrow \quad \cot(\beta_x d) = \frac{\alpha}{\beta_x}$$

Dielectric Slab Waveguide

TE Odd Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$\Rightarrow \quad \cot(\beta_x d) = \frac{\alpha}{\beta_x}$$

Dispersion Relationships:

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$

$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

$$\Rightarrow \quad \beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o \left(\varepsilon_1 - \varepsilon_2\right)$$

$$\alpha = \sqrt{\omega^2 \mu_o \left(\varepsilon_1 - \varepsilon_2\right) - \beta_x^{\,2}}$$

Dielectric Slab Waveguide

TE Odd Modes

Boundary Conditions:

$$E_y^{(1)}(-d,z) = E_y^{(2)}(-d,z)$$

$$E_y^{(1)}(d,z) = E_y^{(2)}(d,z)$$

$$H_z^{(1)}(-d,z) = H_z^{(2)}(-d,z)$$

$$H_z^{(1)}(d,z) = H_z^{(2)}(d,z)$$

$$\Rightarrow \quad \cot(\beta_x d) = \frac{\alpha}{\beta_x}$$

Dispersion Relationships:

$$\beta_x^{\,2} + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_1$$

$$-\alpha^2 + \beta_z^{\,2} = \omega^2 \mu_o \varepsilon_2$$

$$\Rightarrow \quad \beta_x^{\,2} + \alpha^2 = \omega^2 \mu_o (\varepsilon_1 - \varepsilon_2)$$

$$\alpha = \sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}$$

$$\Rightarrow \quad \cot(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o (\varepsilon_1 - \varepsilon_2) - \beta_x^{\,2}}}{\beta_x}$$

Dielectric Slab Waveguide

TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

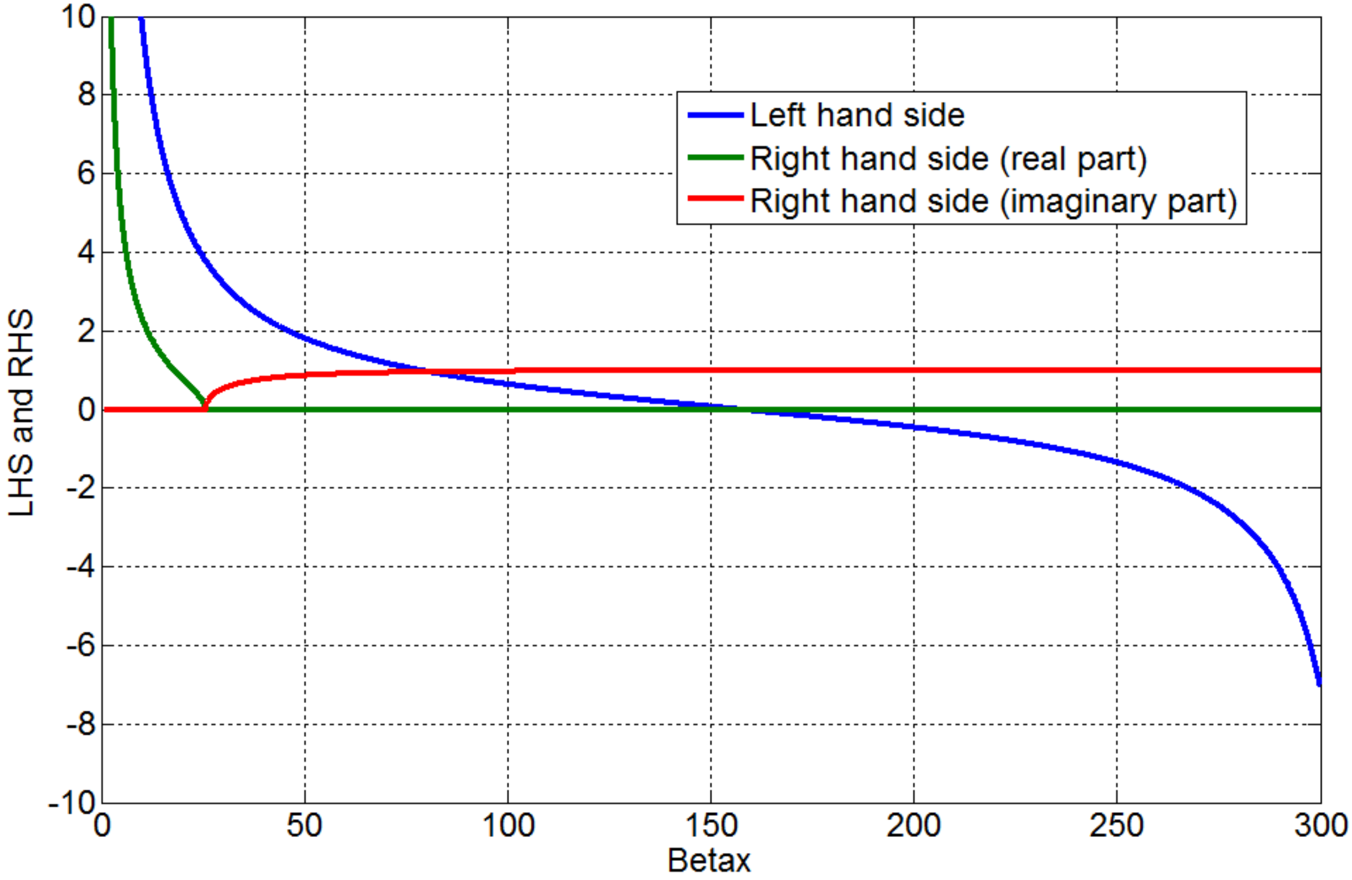
Example

$\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 1.0$

$f = 1.0 \quad GHz$

$d = 1 \quad cm$



Dielectric Slab Waveguide

TE Even Modes

$$\tan(\beta_x d) = \frac{\sqrt{\omega^2 \mu_o \varepsilon_o (\varepsilon_{r1} - \varepsilon_{r2}) - \beta_x^{\ 2}}}{\beta_x}$$

Example

$\varepsilon_{r1} = 2.5$

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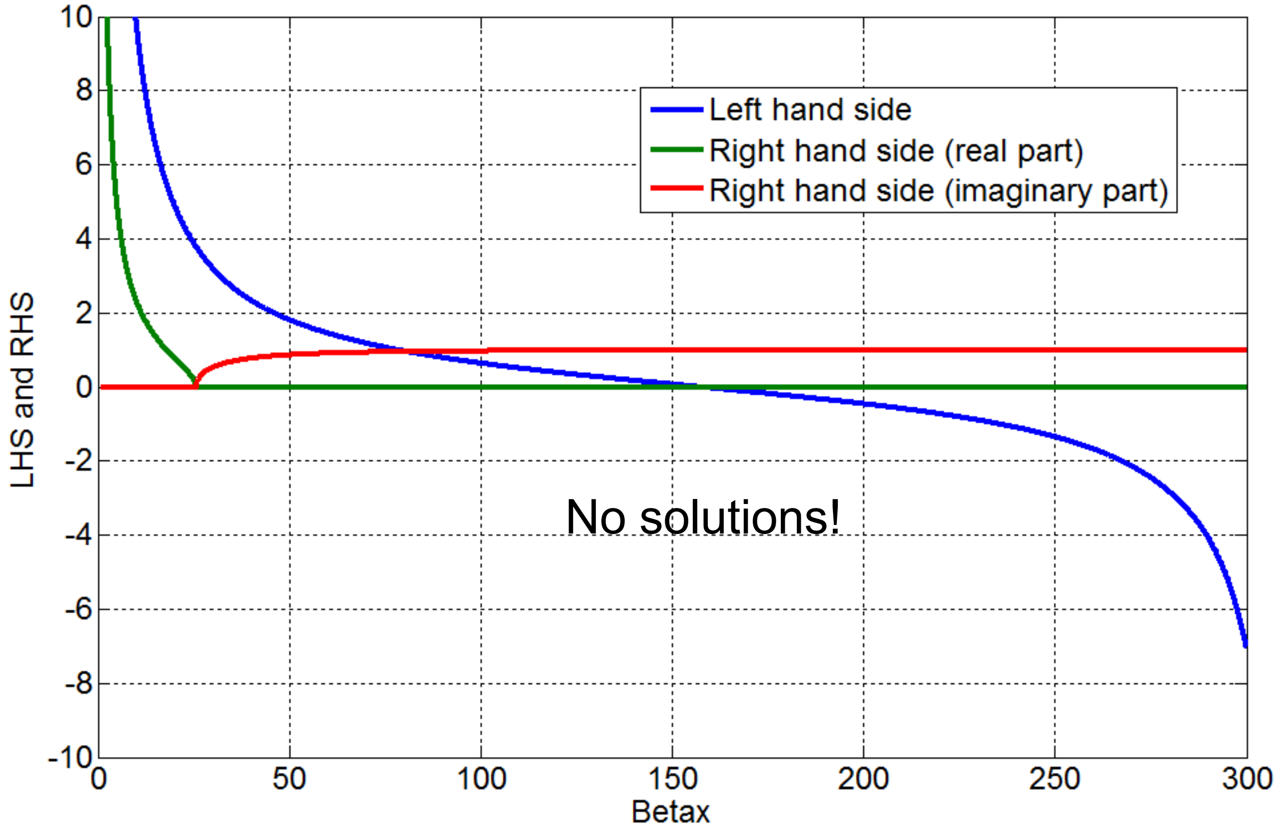

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Cutoff Frequency

$$\beta_z{}^c = \omega_c\sqrt{\mu_o\varepsilon_2} \quad \Rightarrow \quad \alpha^c = 0$$

$$\cot(\beta_x d) = \frac{\alpha}{\beta_x} \quad \Rightarrow \quad \cot(\beta_x{}^c d) = 0$$

$$\beta_x = \sqrt{\omega^2\mu_o\varepsilon_1 - \beta_z{}^2} \quad \Rightarrow \quad \beta_x{}^c = \sqrt{\omega_c{}^2\mu_o(\varepsilon_1 - \varepsilon_2)}$$

$$\Rightarrow \quad \cot(\sqrt{\omega_c{}^2\mu_o(\varepsilon_1 - \varepsilon_2)}\, d) = 0$$

$$\Rightarrow \quad (f_c)_m = \frac{m}{4d\sqrt{\mu_o(\varepsilon_1 - \varepsilon_2))}} \qquad m = 1,3,5\ldots.$$

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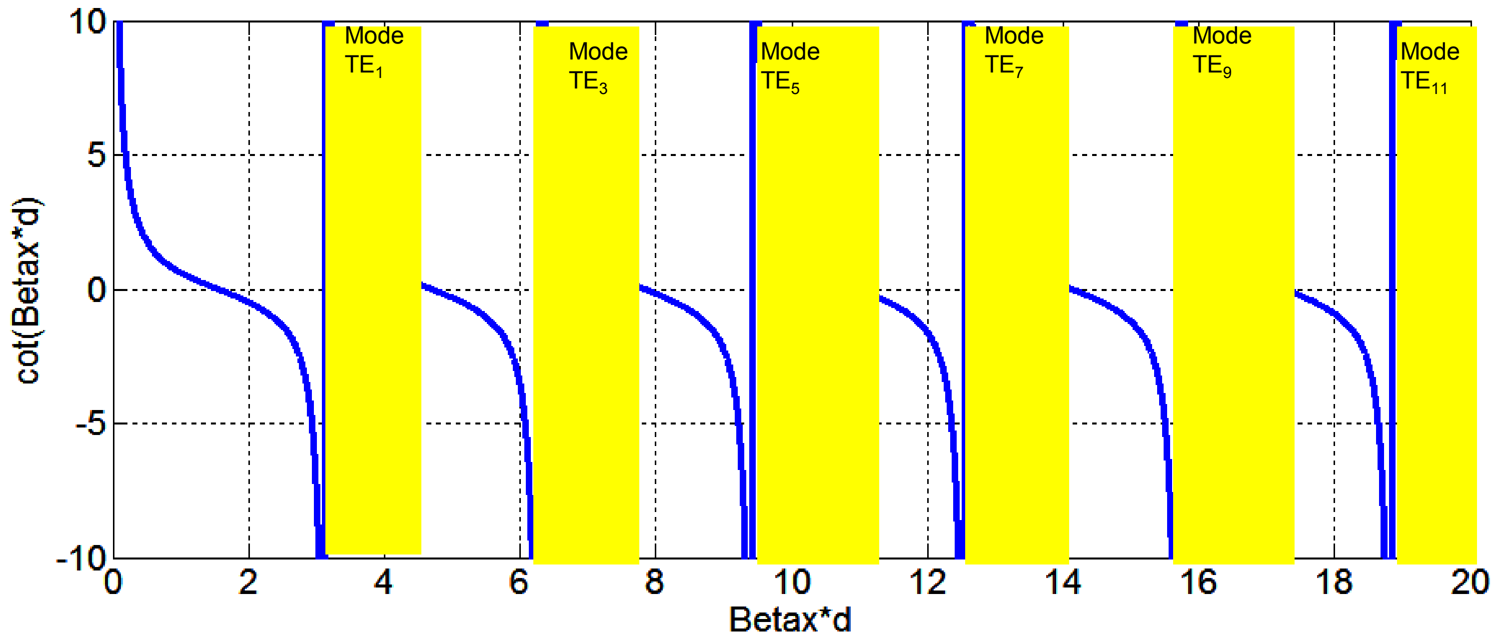
TE Even Modes

$$\cot(\beta_x d) = \frac{\alpha}{\beta_x}$$

α>0 mode propagates
α<0 mode does not propagate



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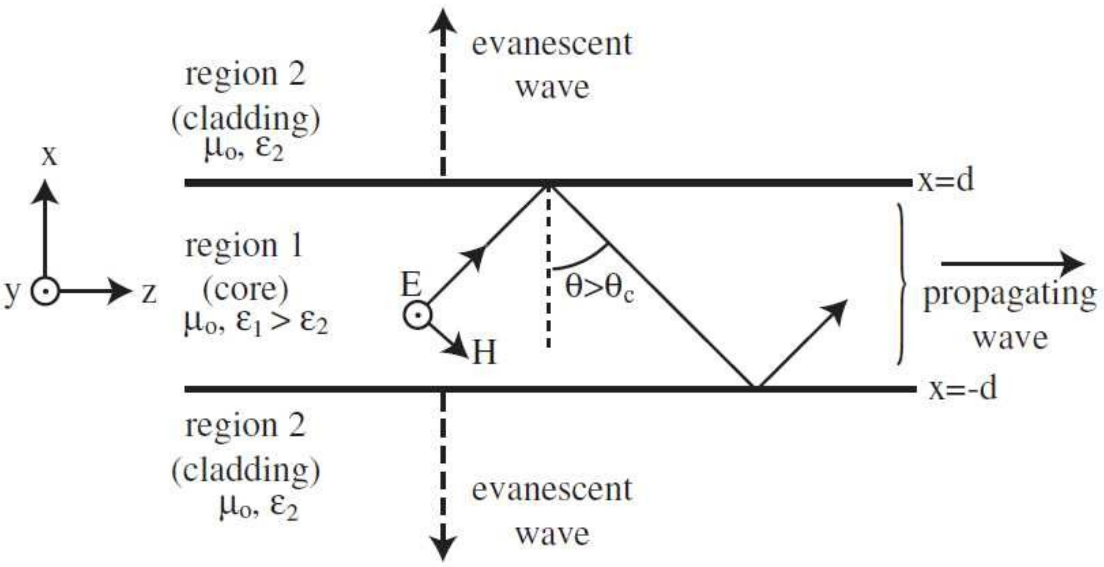

$$\theta > \theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$k_I \sin(\theta_c) < \beta_z < k_I \sin(90^o)$$

$$k_I = \omega\sqrt{\mu_o \varepsilon_1}$$

Ray optics interpretation!

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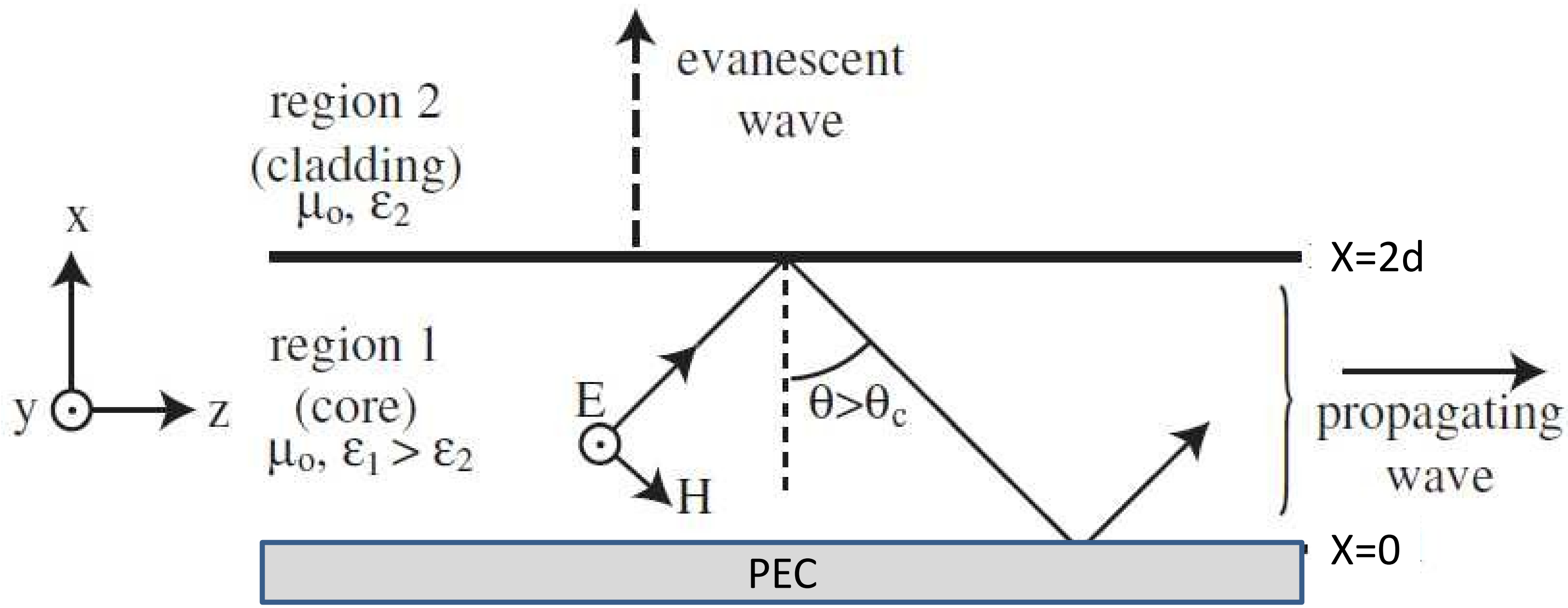

How would you solve this problem?

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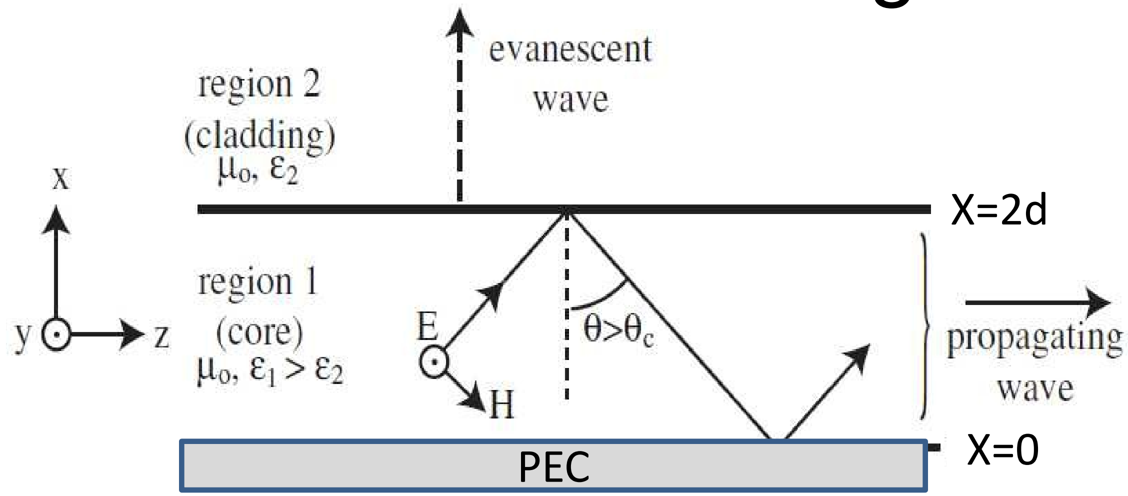


TE Even

$$E_x(x,z) = 0$$

$$E_y(x,z) = \frac{-j\omega\mu\beta_x H_o}{\beta_z^{\,2} - \omega^2\mu_o\varepsilon_1}\sin(\beta_x x)e^{-j\beta_z z}$$

$$E_z(x,z) = 0$$

TE Odd

$$E_x(x,z) = 0$$

$$E_y(x,z) = \frac{j\omega\mu\beta_x H_o}{\beta_z^{\,2} - \omega^2\mu_o\varepsilon_1}\cos(\beta_x x)e^{-j\beta_z z}$$

$$E_z(x,z) = 0$$

TM Even

$$E_x(x,z) = \frac{j\beta_z\beta_x H_o}{\beta_z^{\,2} - \omega^2\mu_o\varepsilon_1}\sin(\beta_x x)e^{-j\beta_z z}$$

$$E_y(x,z) = 0$$

$$E_z(x,z) = E_o\cos(\beta_x x)e^{-j\beta_z z}$$

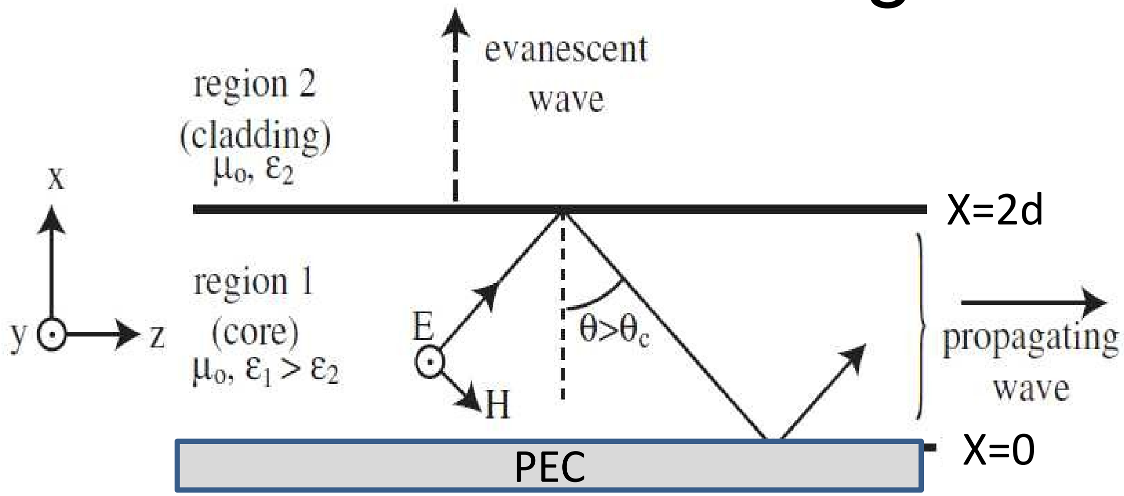
TM Odd

$$E_x(x,z) = \frac{j\omega\mu\beta_x H_o}{\beta_z^{\,2} - \omega^2\mu_o\varepsilon_1}\cos(\beta_x x)e^{-j\beta_z z}$$

$$E_y(x,z) = 0$$

$$E_z(x,z) = E_o\sin(\beta_x x)e^{-j\beta_z z}$$

Dielectric Slab Waveguide



TE Even

$$E_x(x,z) = 0$$

$$E_y(x,z) = \frac{-j\omega\mu\beta_x H_o}{\beta_z^{\ 2} - \omega^2 \mu_o \varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

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TM Even

$$E_x(x,z) = \frac{j\beta_z \beta_x H_o}{\beta_z^{\ 2} - \omega^2 \mu_o \varepsilon_1} \sin(\beta_x x) e^{-j\beta_z z}$$

$$E_y(x,z) = 0$$

$$E_z(x,z) = E_o \cos(\beta_x x) e^{-j\beta_z z}$$

TE Odd

$$E_x(x,z) = 0$$

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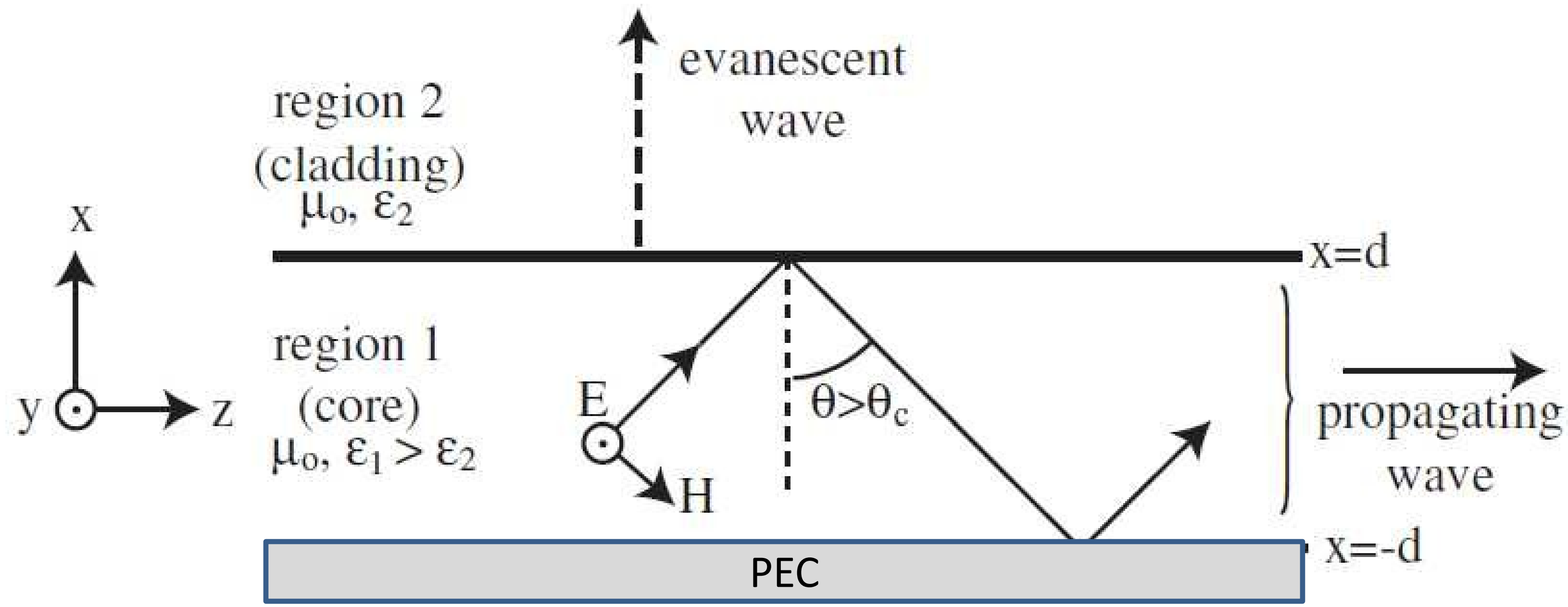
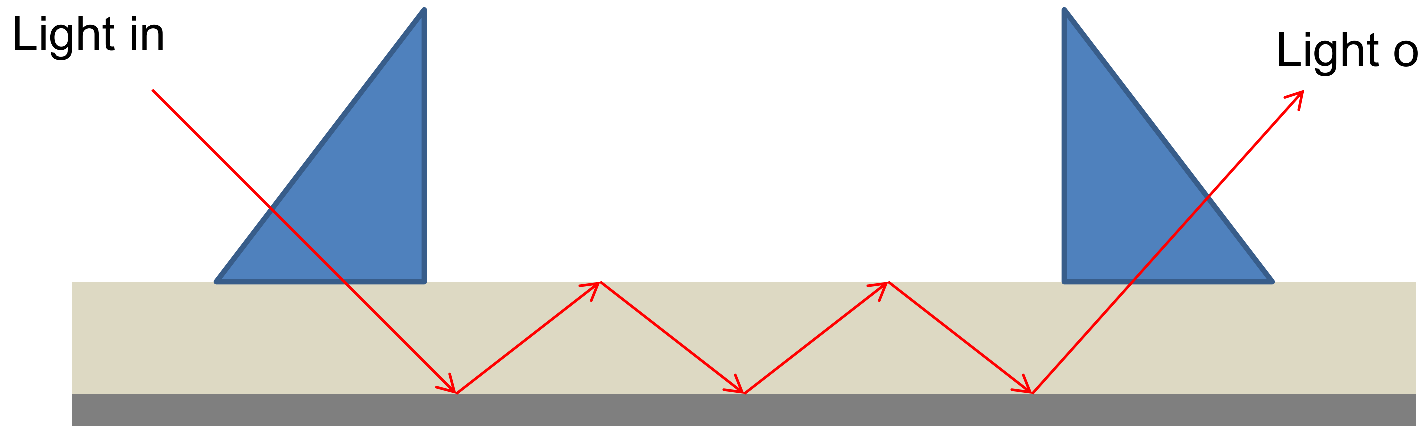
Dielectric Slab Waveguide



How can you get energy into a guided mode within the dielectric slab?

Dielectric Slab Waveguide



Prism Coupling

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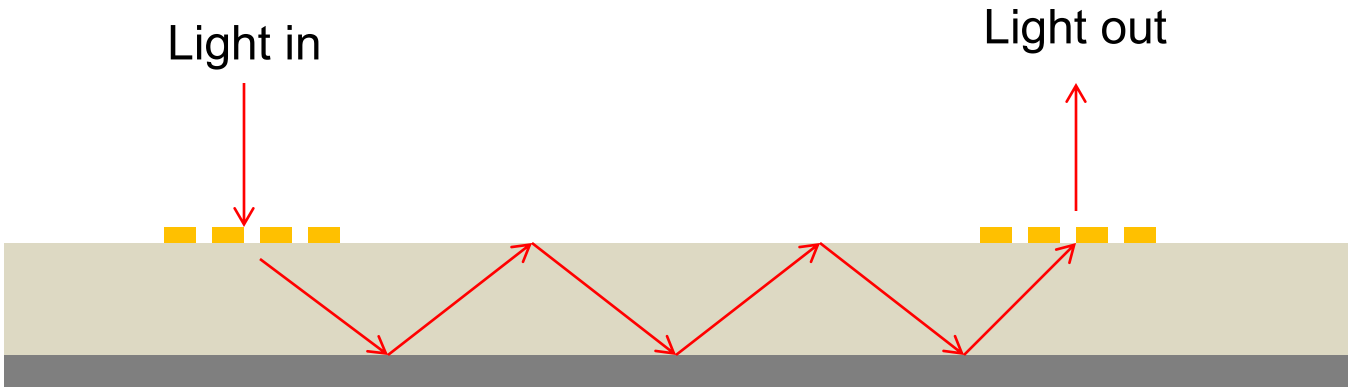

Grating Coupling